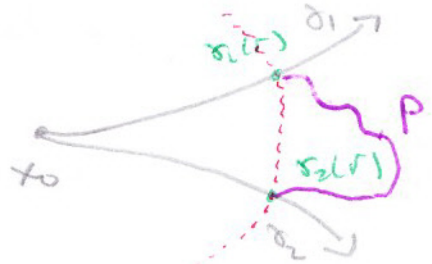
Divergence in Groups - Pallavi Dani

X = metric space, x_0 basept



rate of divergence?

measure paths btw them

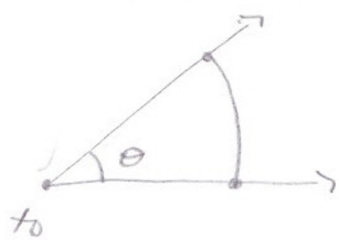
outside $B_r(x_0)$ - "r-advoidant"

def: divergence of γ_1, γ_2 is

$$\mathrm{Div}_{\gamma_1,\gamma_2}(r) = \inf_P \{ \ell(P) \}$$

egs

① $\mathbb{R}^2$



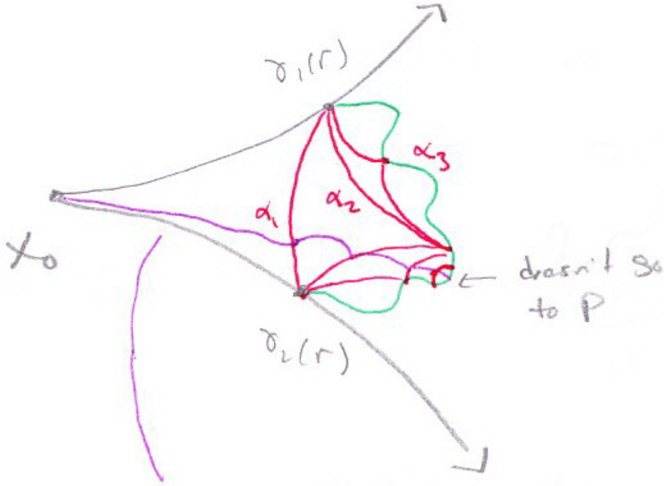
$\mathrm{Div}(r) = \theta r$

② δ-hyperbolic space (all Δ's δ-slim)

"negative curvature condition"

- hyperbolic space
- trees

WTS: $\mathrm{Div} \geq b e^{cr}$ f.s. $b, c > 0$



α_n = pieces of length

$1 \le$ length ≤ 2

α_n is $(r-1)$-avoidant

$\Rightarrow \ell(P) \ge 2^n$

$$\left.\begin{array}{l} \text{length} \le n\delta + A \\ \text{length} \ge r-1 \end{array}\right\} \quad \ell(P) \ge 2^{\frac{r-1-A}{\delta}}$$

So Euclidean $\to$ linear

hyperbolic $\to$ exponential

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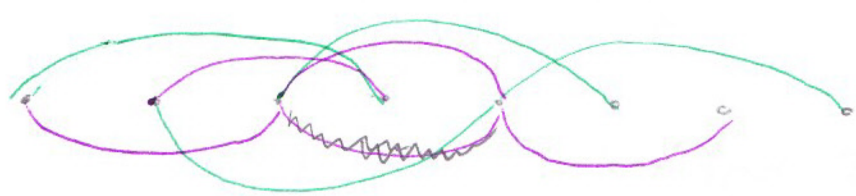
any f.g. gp G is a metric space via the word metric

$\mathbb{Z} = \langle 2, 3 \rangle$

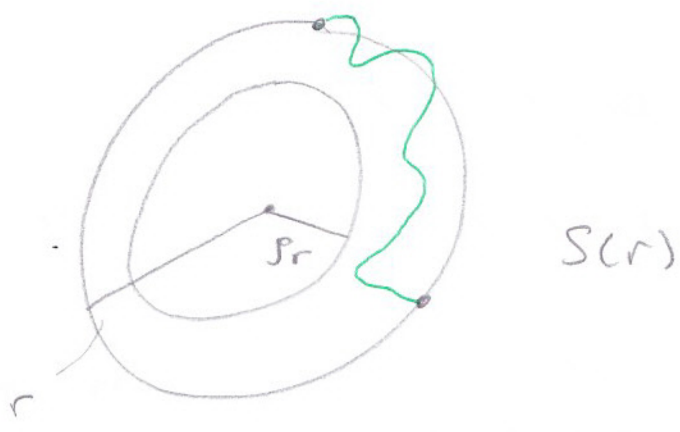


$C(G,S)$ QI to $C(G,S')$

<u>Gromov's Program</u>: classify gps up to QI

<u>def</u>: G a one-ended gp.

(complements of compact sets are connected)



$0 < \rho \leq 1$

$$\delta_\rho(r) = \sup_{x,y \in S(r)} \inf \{ \ell(P) \} \quad , \quad P = \rho r\text{-avoidant}$$

$\mathrm{Div}_G(r) = \{ \delta_\rho(r) \}$; this is a QI-invt

In suff. nice examples, $\delta_\rho(r)$ are

"coarse bi-Lipschitz equivalent"

- polys of same degree are $\simeq$
- " dif " " $\not\simeq$
- $e^r \simeq \lambda^r$

For the sequel $\rho = 1$, $\mathrm{Div}_G(r) \simeq \delta_1(r)$

egs.

① $G = \mathbb{Z}^n$, $\mathrm{Div}_G(r) \simeq r$

② $G =$ one ended hyperbolic gp, $\mathrm{Div}_G(r) \simeq e^r$

③ {Symmetric space of non compact type} ← G acts on

$\Rightarrow \mathrm{Div}_G(r) \simeq e^r$

$\cap$

④ CAT(0) space "=" Δ's no fatter than Euclidean

generalize: symmetric spaces, $\pi_1(M) = 0$ of sectional curvature ≤ 0, "Euclidean buildings", piecewise Euclidean complexes w/ some gluing condition

Thm (Gersten): $\exists$ a $G \in \mathrm{CAT}(0)$ w/ $\mathrm{Div}(r) \simeq r^2$

Thm (Kapovich - Leeb, Gersten):

$G =$ 3-mfold gp

$$\mathrm{Div}_G(r) \simeq \begin{cases} r^2, & G \text{ a graph mfd gp.} \\ r \text{ or } e^r & \end{cases}$$

Thm (Macura, Behrstock – Charney):

$$\forall\, d \quad \exists\, G \quad \text{s.t.} \quad \mathrm{Div}(G) \simeq r^d$$

Thm: Dani & others

G = right-angled Artin gp

$$\mathrm{Div}_G(r) \simeq \begin{cases} r & \iff \text{direct product} \\ r^2 & \text{otherwise} \end{cases}$$

Divergence in Right-Angled Coxeter gps

def: the RACG G_Γ based on graph Γ

$$V(\Gamma) = \{g_1, \dots, g_n\}$$

$$G_\Gamma = \langle g_1, \dots, g_n \mid \overbrace{[g_i, g_j] = 1 \iff (g_i, g_j) \in E(\Gamma)}^{\text{just this: RAAG}},\ \underline{g_i^2 = 1} \rangle$$

add this: Coxeter

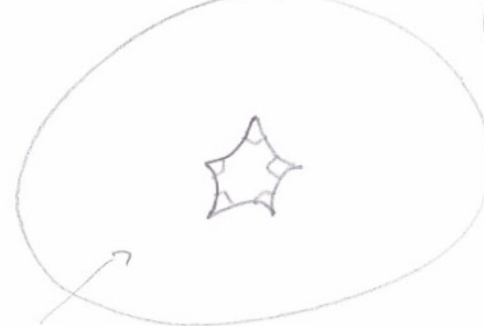
Egs.

Γ $\qquad$ G_Γ

• • $\qquad$ $\mathbb{Z}_2 * \mathbb{Z}_2 = D_\infty$

 $\qquad$ $D_\infty \times D_\infty$

$\mathbb{H}^2$



G_Γ = reflection gp

Thm (Dani - Thomas)

G_Γ = RACG

① $\mathrm{Div}_G(r) \simeq r \iff G_\Gamma$ a direct product

$\iff \Gamma$ "splits" as join of

2 subgraphs

② $\forall\ d \in \mathbb{Z}^+,\ \exists\ \Gamma$ s.t. $\mathrm{Div}_{G_\Gamma}(r) \simeq r^d$

③ $\exists\ \Gamma$ s.t. $\mathrm{Div}_{G_\Gamma}(r) \simeq e^r$

Q. Which Γ give $\mathrm{Div}_{G_\Gamma}(r) = r^2$?

X = union of 2-flats