

Σ Manifold Mutation and Heegaard Floor Hom

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Q: Does Heegaard Floor detect Σ manifold mutation?

Def: (Ozváth-Szabo) Take M^3 's closed, connected,

orient. $M^3 \rightarrow \hat{HF}(M) \rightarrow \text{rk } \hat{HF}(M)$

abelian group \mathbb{Z}
 \mathbb{N}

mutation \equiv cut along embedded surf. glue back together.

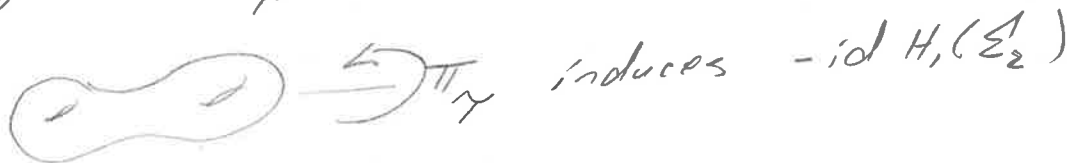
Four choices: manifold M , surf Σ , embedding $f: \Sigma \hookrightarrow M$,

gluing $\varphi: \Sigma^k$

take Σ closed, conn., orient., genus ≥ 2 and let

gluing maps preserve orient.

Def: genus 2 hyperelliptic involution



Thm: Given $\varphi: \Sigma^k$ s.t. $\varphi \neq id$ and $\varphi \neq \tau$

$\exists M^3$ and $f: \Sigma \hookrightarrow M^3$ s.t. $\text{rk } \hat{HF}(M) \neq \text{rk } \hat{HF}(M_f^\varphi)$.

Remark: Mutation by elem of Torelli group pres. $H_*(M)$.

Pf steps: ① reformulate as MCG question

② reduce to pseudo-Anosov elem of Torelli group.

③ show f elem can change rank.

MCB formulation

$$\text{Mod}(\Sigma) = \text{Diff}^+(\Sigma) / \text{isotop.}$$

Def: $N = \{ [\varphi] \in \text{Mod}(\Sigma) \mid \text{for all } M, A \}$
 $\text{rk } \widehat{HF}(M) = \text{rk } \widehat{HF}(M_{\varphi}^{\circ}) \}$

Goal: $N = \{ \text{id} \}$ or $\{ \text{id}, \gamma \}$.

Lemma: N is a normal subgroup.

Def: Torelli group $\mathcal{I}(\Sigma) \triangleleft \text{Mod}(\Sigma)$

maps that induce id $H_1(\Sigma)$

pseudo-Anosov elem. $[\varphi] \in \text{Mod}(\Sigma)$

st. \exists rep. $\varphi: \Sigma^{\circ} \rightarrow \Sigma^{\circ}$, pair of T -measured

foliations, const.

- ∞ order
- violent, understood

Facts: • $\mathcal{I}(\Sigma)$ is torsion free

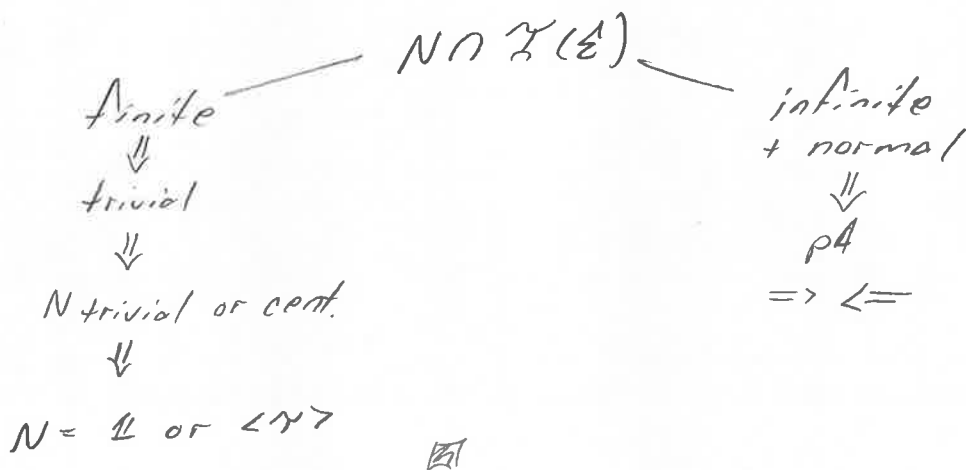
• ∞ normal subgroups of $\text{Mod}(\Sigma)$ contain pA elem.

Thm (Long) normal, non-trivial, non-central subgroups intersect non-trivially.

$$\mathcal{Z}(\text{Mod}(\Sigma)) = \begin{cases} \langle \gamma \rangle & g=2 \\ \mathbb{1} & g > 2 \end{cases}$$

Prop: If $N \cap \mathcal{X}(\Sigma)$ has no pA elem., then $N = \mathbb{1}$ or $\langle \gamma \rangle$.

pt:



pA elem. of $\mathcal{X}(\Sigma)$

idea: look mod/d w/ small rk HF

eg. $\text{rk } \hat{H}F(S' \times S^2) = 2$

Thm (Hedden-Ni): If M irred., $b_1 > 0$ and $\text{rk } \hat{H}F(M) = 2$ then $M \cong \pm S_0^3(\Sigma_1)$

Prop: Let $\varphi: \Sigma \rightarrow \Sigma$ be a pA and elem Torelli.

$\exists f: \Sigma \hookrightarrow S' \times S^2$ s.t. $\text{rk } \hat{H}F((S' \times S^2)_f^{\varphi^n}) \neq 2$.

pt idea:

$f: \Sigma \hookrightarrow S' \times S^2$ Heegaard Splitting

$\exists M$ s.t. $(S' \times S^2)_f^{\varphi^n}$ mutant irred

w/ no essential tori. $S_0^3(\Sigma_1)$ genus 1 knot implies

$S_0^3(\Sigma_1)$ has essential torus $\therefore \text{rk } \hat{H}F((S' \times S^2)_f^{\varphi^n}) \neq 2$ □

Cor: $N = \mathbb{I}$ or $\langle \gamma \rangle$.

Mutation by γ :

Thm: Mutation by γ is detected by $\hat{HF}(M, s)$
↑
spin^s str.

idea: \hat{HF} detect Thurston norm mutant knots
w/ diff. genus.

Thm: Mutation along non-sep. surface can be detected
by rk \hat{HF} .

Thm: Mutation by elliptic inv. - $\textcircled{D} \rightarrow \textcircled{D}^{\pi}$

sep. torus w/ one side bounding a knot complement.

$\hat{HF}(M, s)$ is pres.

Thm: (Moore - Starkston) infinite family of mutant

knot pairs where rk \hat{HFK} agree but

\hat{HFK} distinguish.