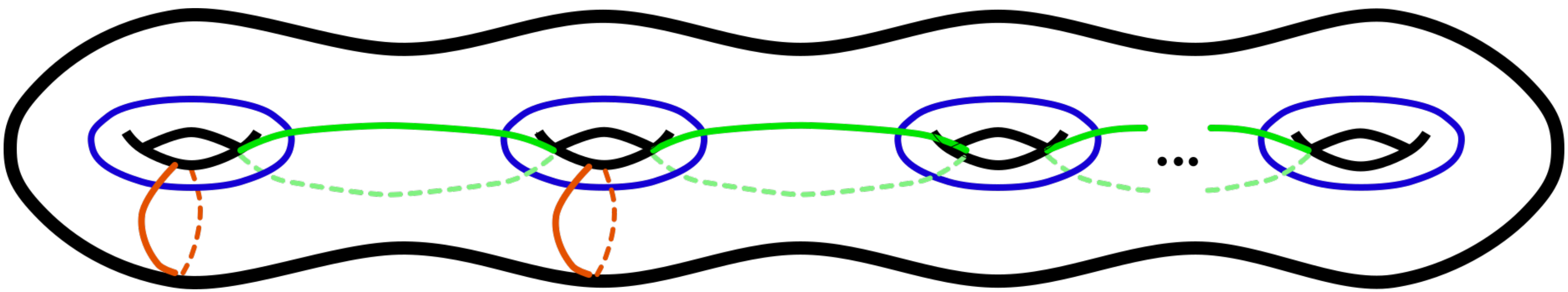
Lightning Talks I
Tech Topology Conference

December 5, 2015

Generating mapping class groups with torsion elements

Justin Lanier
Georgia Tech

Generating Mod(S_g)



$2g+1$ Dehn twists generate.
(Humphries)

Generating Mod(S_g)

	Order of elements	Number of elements	Genus
Luo	2	$6(2g+1)$	$g \geq 3$
Brendle-Farb	2	6	$g \geq 3$
Kassabov	2	5	$g \geq 5$
	2	4	$g \geq 7$
Monden	3	3	$g \geq 3$
	4	4	$g \geq 3$

Obstacle:

When do higher-order elements even exist in $\mathrm{Mod}(S_g)$?

Theorem 1 (Lanier '15)

For $k \geq 5$ and $g \geq (k\text{-}1)(k\text{-}2)$, Mod($S_g$) contains an element of order k.

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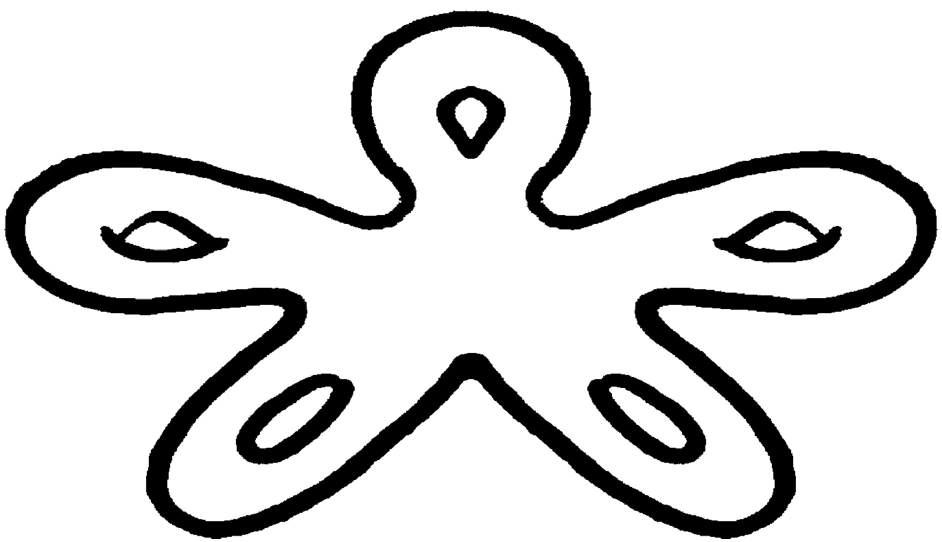
For $k \geq 5$ and $g \geq (k-1)(k-2)$, $\mathrm{Mod}(S_g)$ contains an element of order k.

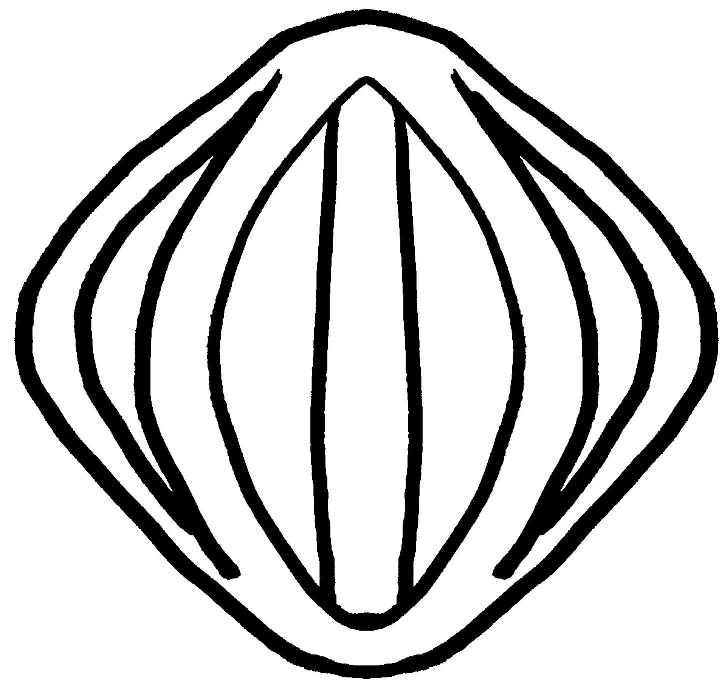
Theorem 2 (Lanier '15)

For $k \geq 5$ and $g \geq (k-1)(k-2)$, $\mathrm{Mod}(S_g)$ is generated by **4** elements of order k.

Theorem 1



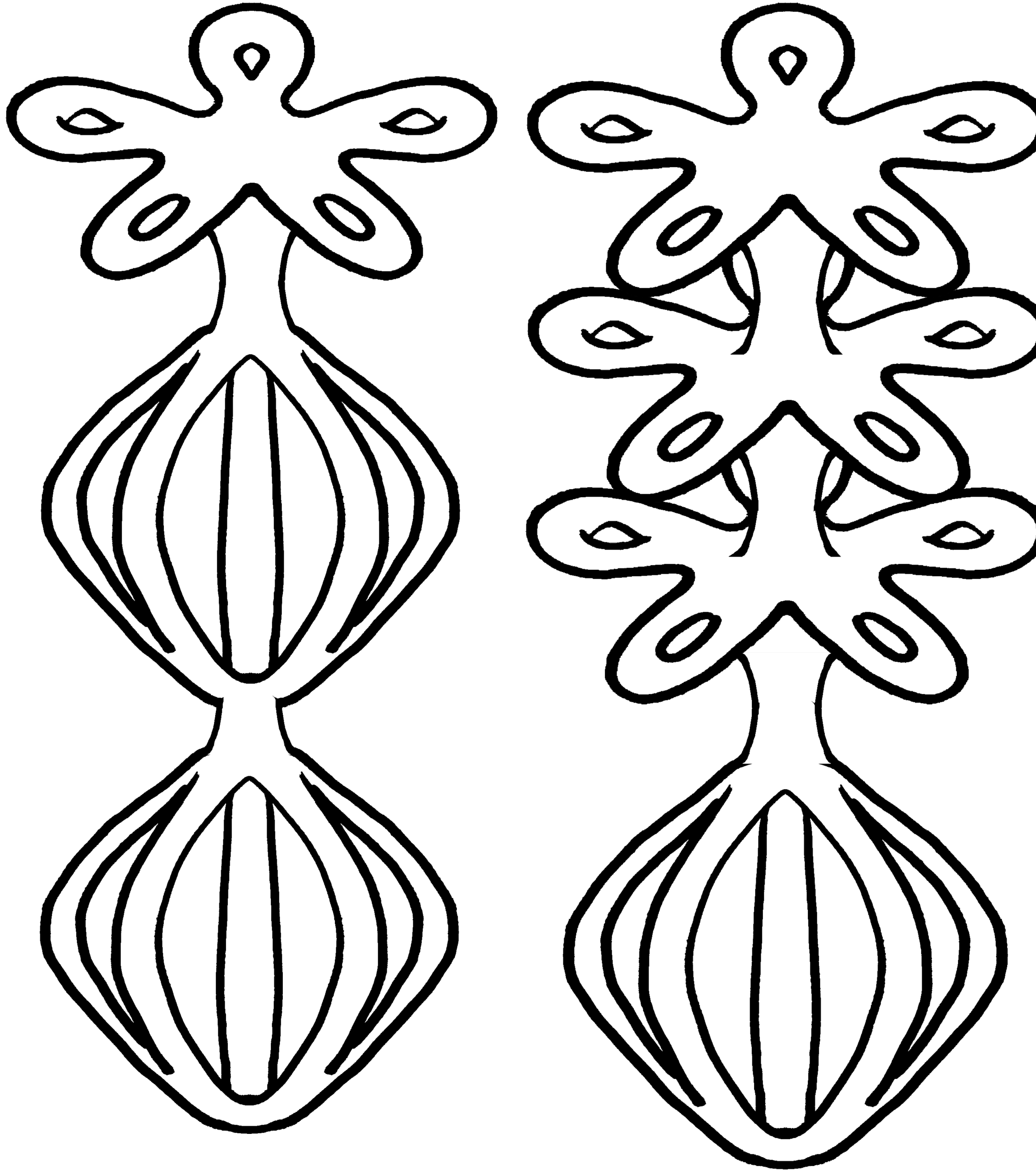
genus k



genus k-1

Theorem 1

Frobenius coin problem

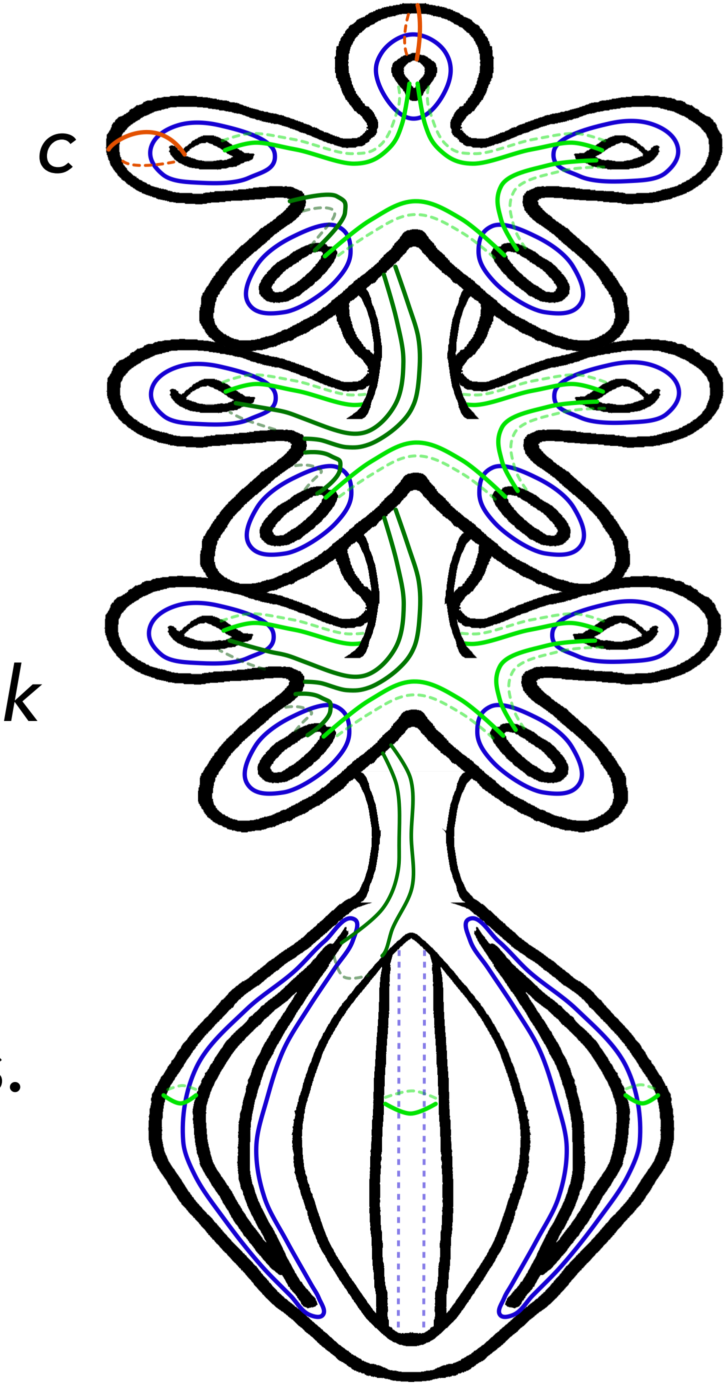


Theorem 2

Step 1: Write T_c as a product of elements of order k.

Step 2: Find elements of order k taking c to the other curves.

Step 3: Optimize to 4 elements.



Further Questions

- Can 4 be further optimized?
- What is the last g for which an element of order k fails to exist?
- Can similar results be obtained for finite index subgroups of $\mathrm{Mod}(S_g)$?

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Thank you!

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Georgia Tech

Knots in $S^1 \times S^2$ with L-space surgeries

Faramarz Vafaee
California Institute of Technology

December, 2015
joint with Yi Ni

Knots in $S^1 \times S^2$ admitting L-space fillings

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- Example:
  - Start with a solid torus $V = S^1 \times D^2$ with meridian μ.
  - Let $K \subset V$ be a *Berge-Gabai* knot, i.e. K has a non-trivial solid torus filling.
  - There is a slope λ such that $V' = V_\lambda(K)$ is another solid torus, with meridian μ'.
  - Dehn filling V along μ' will give us a lens space L.
  - K, when viewed as a knot in the lens space L, has an $S^1 \times S^2$ surgery; namely, $L_\lambda(K)$ has a genus one Heegaard splitting with the property that the meridians of the two solid tori coincide (this common meridian is μ').

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  - There is a slope λ such that $V' = V_\lambda(K)$ is another solid torus, with meridian $\mu^{'}$.
  - Dehn filling V along $\mu^{'}$ will give us a lens space L.
  - K, when viewed as a knot in the lens space L, has an $S^1 \times S^2$ surgery; namely, $L_\lambda(K)$ has a genus one Heegaard splitting with the property that the meridians of the two solid tori coincide (this common meridian is $\mu^{'}$).
- Any lens space obtainable by longitudinal surgery on some knots in $S^1 \times S^2$ may be obtained this way. (Rasmussen)

Knots in S^3 with L-space surgeries

- $K \subset S^3$ with some L-space surgery fibered. (Ni)
- K induces the tight contact structure on S^3.
- K is strongly quasi positive. (Hedden)

Knots in L-spaces admitting $S^1 \times S^2$ fillings

Theorem (Ni-V.)

Suppose $L \subset S^1 \times S^2$ is a knot with an L-space surgery. Then the complement of L in $S^1 \times S^2$ fibers over S^1.

Proposition (Ni-V.)

If K is a knot in an L-space Y with some $S^1 \times S^2$ surgery, then K is Floer simple.

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- Recall: A knot K in a $\mathbb{Q}HS^3$ Y is *Floer simple* if rk $\widehat{HFK}(Y, K) = |H_1(Y; \mathbb{Z})|$.

A rationally fibered, Floer simple knot induces a tight contact structure

Proposition (Ni-V.)

Let K be a rationally fibered, Floer simple knot in a $\mathbb{Q}HS^3$ Y. The contact structure induced by the open book decomposition corresponding to the fibration of (Y, K) is tight.


Thank you

Semigroups of L-space Cable Knots and the Upsilon Function

Shida Wang
Indiana University

December 2015

Tech Topology Conference
Georgia Institute of Technology

Outline

Algebraic knots and semigroups

L-space knots and a generalization

The Upsilon function and an application

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An isolated plane curve singular point z is a point on a complex curve in $C \subset \mathbb{C}^2$, such that C is smooth at all points sufficiently close to z, with the exception of z itself.
For a sufficiently small $r > 0$, C intersects the ball $B(z, r) \subset \mathbb{C}^2$ transversally along a link L, which is called an algebraic link.
If L is connected, it is called an algebraic knot.

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For a singular point (C, z), let $\varphi(t) = (x(t), y(t))$ be a local analytic parametrization of C with $\varphi(0) = z = (z_1, z_2)$.
Then φ induces a map $\varphi^* : \mathbb{C}[[x, y]] \to \mathbb{C}[[t]]$ by $f(x, y) \mapsto f(x(t) - z_1, y(t) - z_2)$.
The map $\operatorname{ord}\colon \mathbb{C}[[t]] \to \mathbb{Z}_{\geqslant 0}$ maps a power series in one variable to its order at 0.
The image $S \subset \mathbb{Z}_{\geqslant 0}$ of the composition $\operatorname{ord} \circ \varphi^*$ is closed under addition.
S is defined to be the semigroup of the singular point / algebraic knot.
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The semigroup and the Alexander polynomial determines each other.

Let S_K be the semigroup of an algebraic knot K. Then $\Delta_K(t) = (1 - t)(\sum_{s \in S_K} t^s)$ in $\mathbb{Z}[[t]]$.

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Example of the torus knot $T_{3,7}$

Let $K = T_{3,7}$. Its semigroup is $S_K = \langle 3, 7 \rangle = \{0, 3, 6, 7, 9, 10, 12\} \cup \mathbb{Z}_{>12}$.
$\Delta_K(t) = 1 - t + t^3 - t^4 + t + 6 - t^8 + t^9 - t^{11} + t^{12} = (1 - t)(1 + t^3 + t^6 + t^7 + t^9 + t^{10} + t^{12} + \sum_{s>12} t^s)$.

Algebraic knots and semigroups

L-space knots and a generalization

The Upsilon function and an application

L-Space Knot and a Generalization

Definition(Ozsváth-Szabó 2005)

The knot K is called an L-space knot if some positive surgery on K gives a 3-manifold that is an L-space.

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There is an increasing sequence of integers $0 = \alpha_0 < \alpha_1 < \cdots < \alpha_{2n} = 2g(K)$ such that the Alexander polynomial of K is $\Delta_K(t) = \sum_{i=0}^{2n} (-1)^i t^{\alpha_i}$.

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That is, $S_K = \{\alpha_0, \cdots, \alpha_1 - 1, \alpha_2, \cdots, \alpha_3 - 1, \cdots, \alpha_{2n-2}, \cdots, \alpha_{2n-1} - 1, \alpha_{2n}\} \cup \mathbb{Z}_{>\alpha_{2n}}$.
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An counterexample: the pretzel knot $P(-2, 3, 7)$

It is an L-space knot. Its $S_K = \{0, 3, 5, 7, 8, 10\} \cup \mathbb{Z}_{>10}$, which is not a semigroup.

Main Results

Theorem (Hedden 2009)

Let K be a nontrivial L-space knot and $q \geqslant p(2g(K) - 1)$. Then $K_{p,q}$ is an L-space knot.

Theorem (Hom 2011)

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Corollary

If an L-space knot K is an iterated torus knot, then S_K is a semigroup.

Algebraic knots and semigroups

L-space knots and a generalization

The Upsilon function and an application

The Υ Function (Ozsváth-Stipsicz-Szabó 2014)

Properties

- $\Upsilon_K(t)$ is a piecewise linear function of t on $[0, 2]$.
- $\Upsilon_K(t) = \Upsilon_K(2 - t)$.
- $\Upsilon_{-K}(t) = -\Upsilon_K(t)$ and $\Upsilon_{K_1 \# K_2}(t) = \Upsilon_{K_1}(t) + \Upsilon_{K_2}(t)$.
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For L-space knots: computable by the Alexander polynomial

The invariant $\Upsilon_K(t)$ for an L-space knot is computed by the formula
$\Upsilon_K(t) = \max\limits_{0 \leqslant 2i \leqslant n} \{m_{2i} - t(g - \alpha_{2i})\}$, where

$$
\begin{aligned}
m_0 &= 0 \\
m_2 &= -2(\alpha_1 - \alpha_0) \\
&\cdots \\
m_{2n} &= -2(\alpha_1 - \alpha_0) - \cdots - 2(\alpha_{2n-1} - \alpha_{2n-2}).
\end{aligned}
$$

An Application

The Υ function

$+$

properties of the Alexander polynomial for algebraic knots

$\Downarrow$

nonexistence of cobordism of minimal genus between some pairs of algebraic knots (Feller-Krcatovich / W. 2015)

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Corollary

Similar results for iterated torus L-space knots.

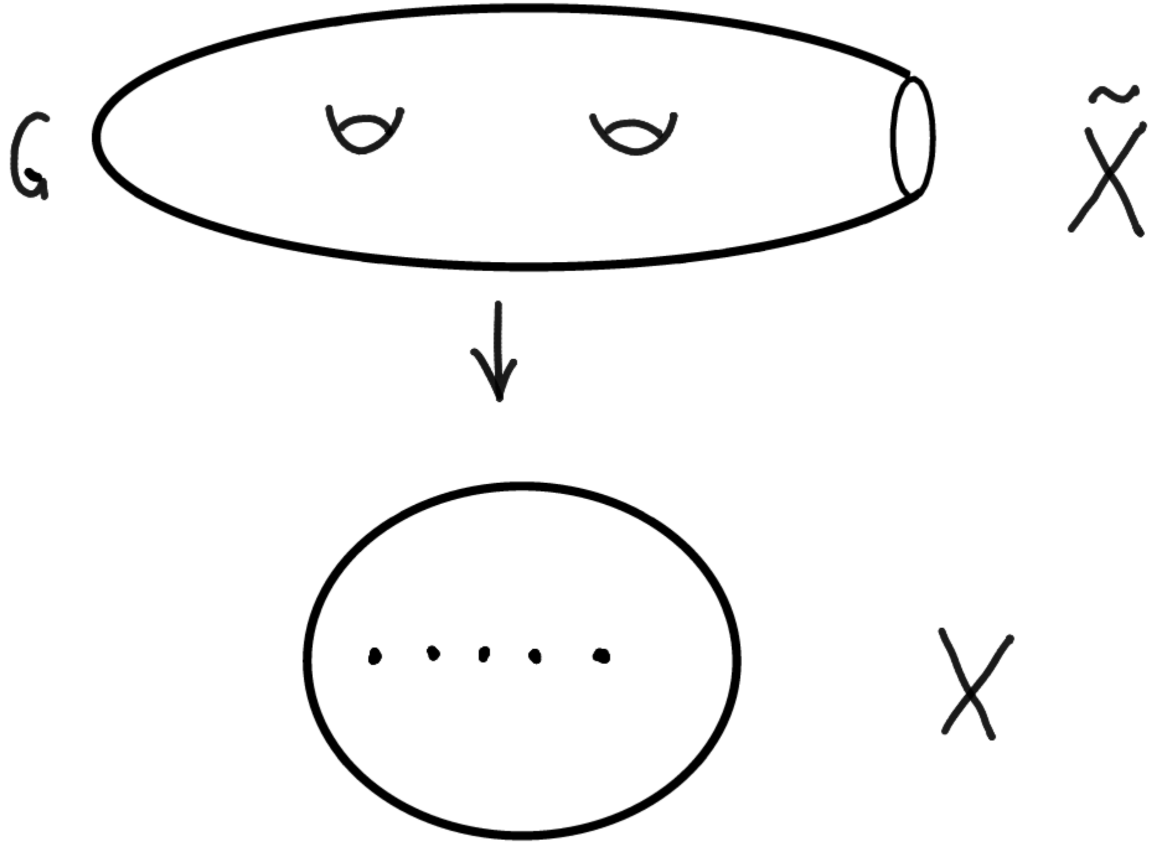
Thank you!

The Lifting Mapping Class Group of a Superelliptic Cover

Becca Winarski
University of Wisconsin-Milwaukee

Joint work with Ty Ghaswala

The Birman-Hilden Theorem



Then

$$\mathrm{MCG}(\tilde{X}) \cong \mathrm{MCG}(X)$$

$\mathrm{MCG}(X)$ = braid group

In general, we have subgroups

$$\mathrm{LMCG}(X) \overset{\text{f.i.}}{<} \mathrm{MCG}(X)$$

$$\mathrm{SMCG}(\tilde{X}) < \mathrm{MCG}(\tilde{X})$$

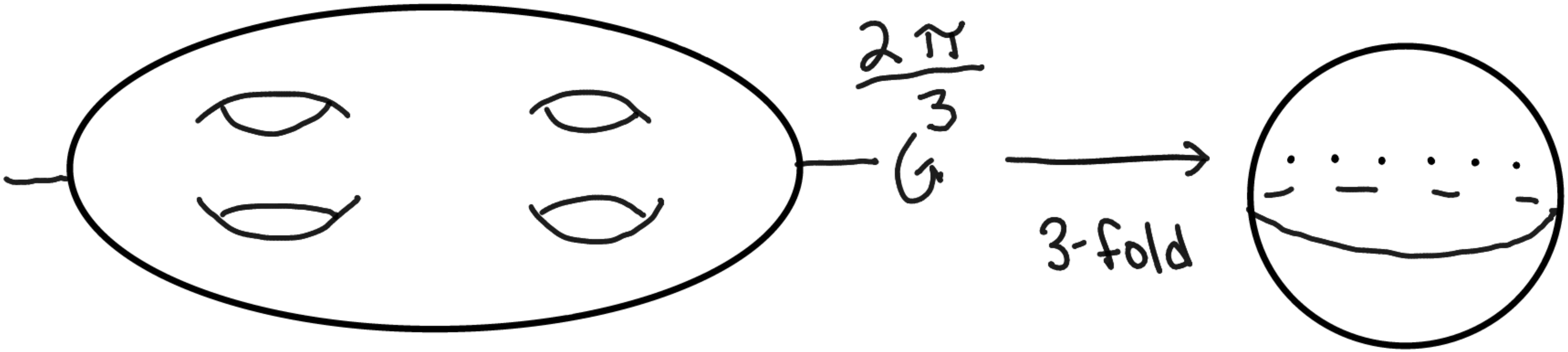
s.t. $$\mathrm{LMCG}(X) \cong \mathrm{SMCG}(\tilde{X}) / \mathrm{Deck}$$

In the hyperelliptic case,

$$\mathrm{LMCG}(X) = \mathrm{MCG}(X)$$

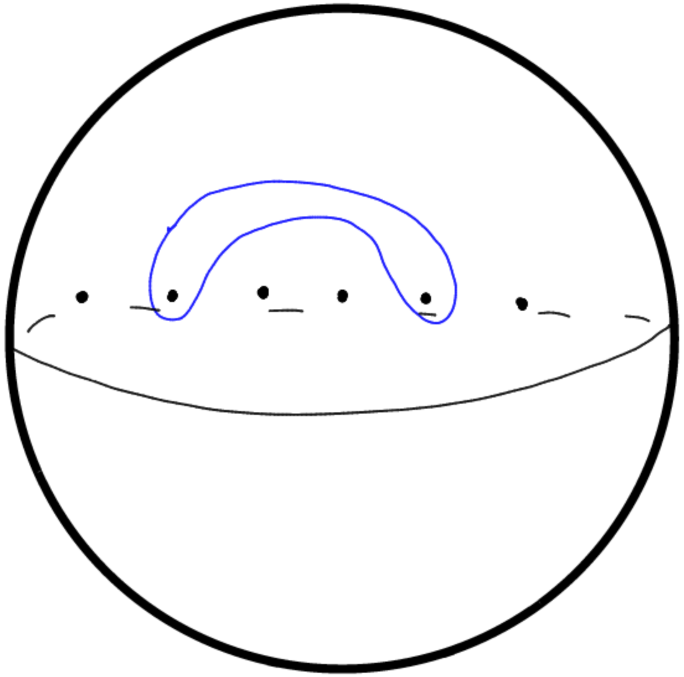
Our work:

Find a presentation for LMCG(X) for <u>superelliptic covers</u>

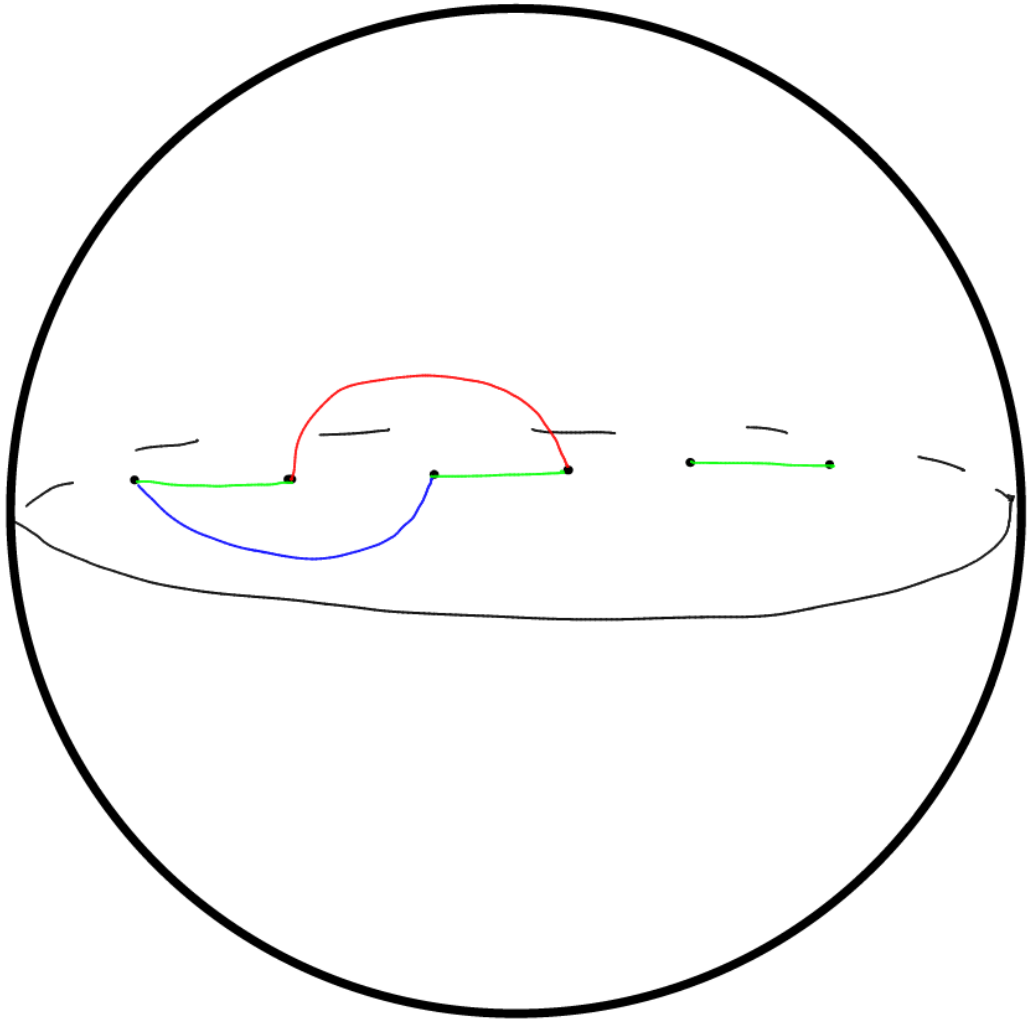


Generators:

Pure braid group generators



odd half twists
even half twists
parity flips



Relations

- Braid relations
- Commutator relations
- Odd permutations $\leftrightarrow$ Even permutations
- Half twists squared are Dehn twists
- Conjugation relations

Thank you!

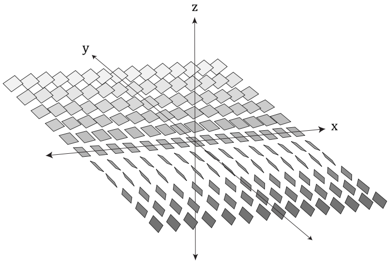
An A_∞ Structure for Legendrians from Generating Families

Ziva Myer

Bryn Mawr College
Advisor: Lisa Traynor

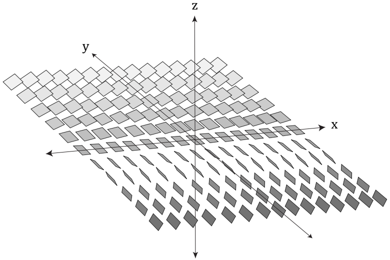
December 5, 2015

Contact Manifold $(J^1 M, \xi)$



The standard contact structure on $\mathbb{R}^3$: $\xi = \ker(dz - ydx)$.

Contact Manifold (J^1M, ξ)



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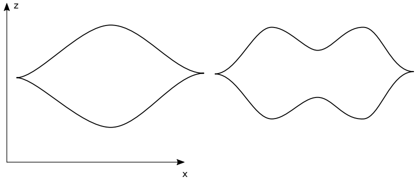

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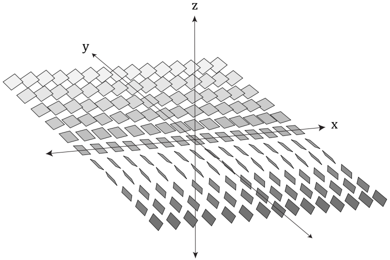

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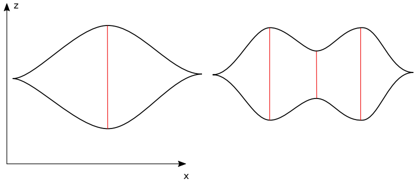

Important feature: Reeb Chords

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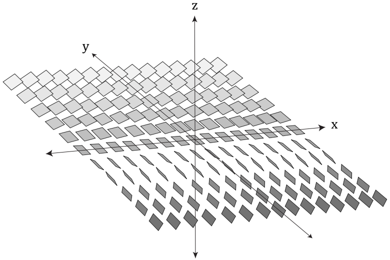

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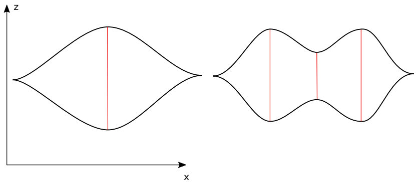

Important feature: Reeb Chords

Goal: Define algebraic invariants for Legendrians from Reeb chords.

Techniques for Invariants

Pseudoholomorphic Curves

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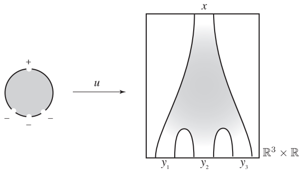

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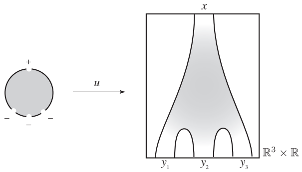

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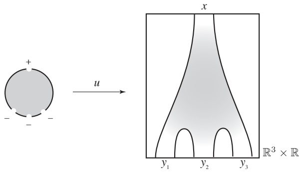

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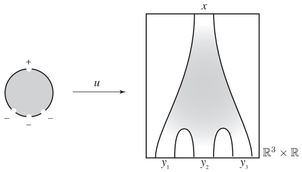

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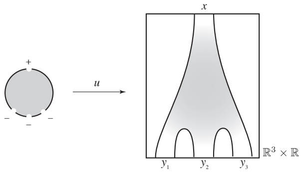

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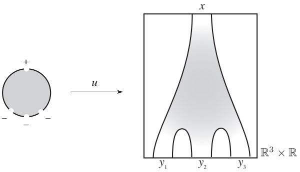

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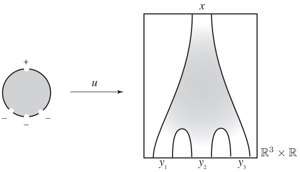

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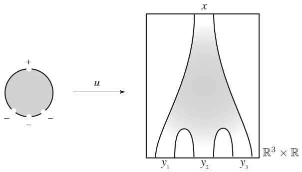

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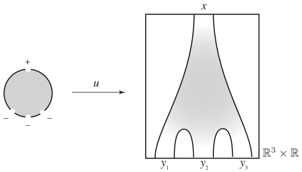

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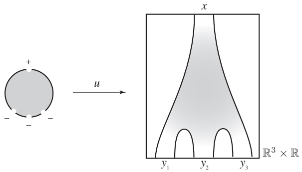

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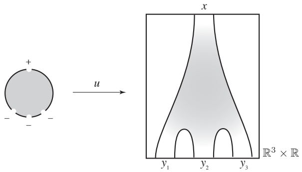

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  Yes! (My thesis work)

A_∞ Structure from Generating Families

Technique: Morse Flow Trees

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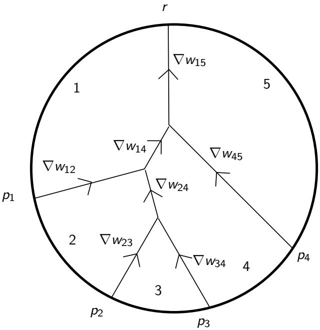
$m_k : C_+^{\otimes k}(w_F) \longrightarrow C_+(w_F)$ counts isolated trees:



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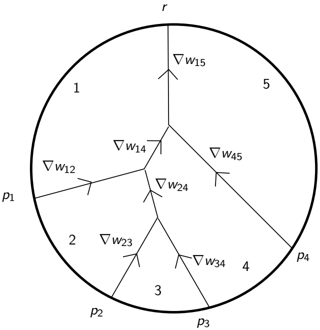
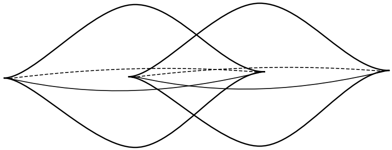


A_∞ relations come from compactifying 1-dimensional spaces of trees.

$$\sum_{i+j+k=l} m_{i+1+k} \circ (1^{\otimes i} \otimes m_j \otimes 1^{\otimes k}) = 0$$

Future Directions

- Generalize to (higher dimensional) links



Future Directions

- Generalize to (higher dimensional) links
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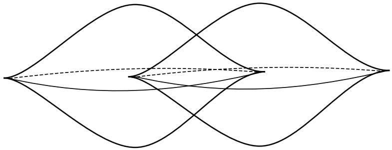

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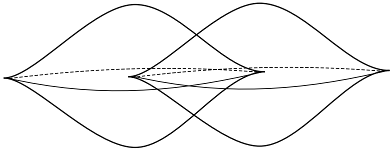

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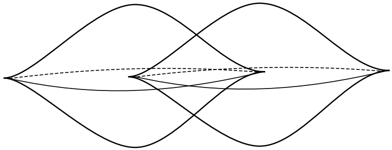


Thank you!

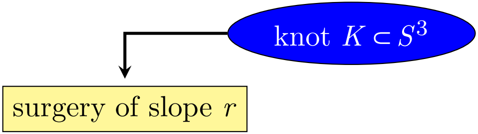
Exceptional Cosmetic Surgeries on S^3

Huygens C. Ravelomanana

University of Georgia

December 05, 2015

knot $K \subset S^3$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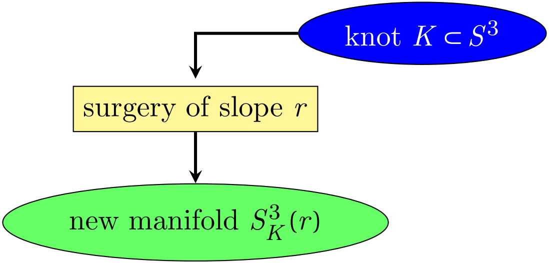
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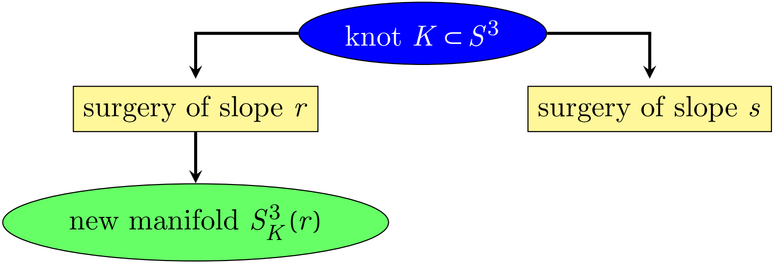
new manifold $S^3_K(r)$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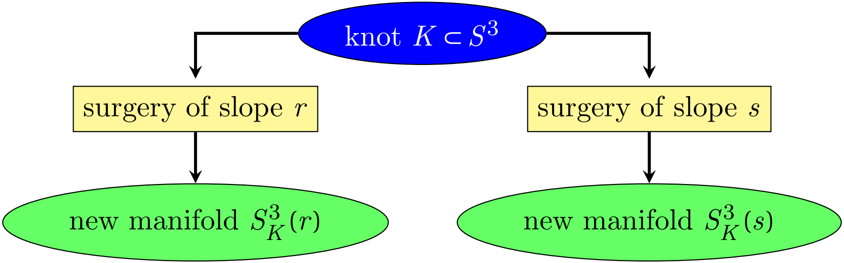
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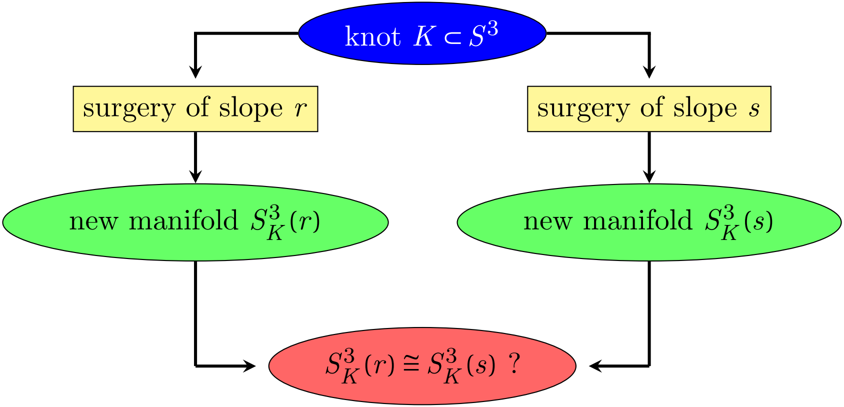
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$S^3_K(r) \cong S^3_K(s)$?

Definition

- Two Dehn surgeries $S^3_K(r)$ and $S^3_K(s)$ are called cosmetic if there is a homeomorphism $h : S^3_K(r) \to S^3_K(s)$.

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- They are called truly cosmetic if h is orientation-preserving.

Some examples

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- If K is the unknot, then $S^3_K(p/q) = L(p,q)$ so

$$S^3_K\left(p/q_1\right) \cong S^3_K\left(p/q_2\right) \quad \text{iff} \quad \pm q_1 \equiv q_2^{\pm 1} \ [\text{mod } p],$$

for relatively prime pairs of integers (p, q_1) and (p, q_2).

Fact

Apart from these examples there are no known knots in S^3 which admit cosmetic surgeries.

The conjecture

Conjecture (A) in problem 1.81 of “Kirby list of problem in low-dimensional topology”. Assume K is a non-trivial knot.

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Conjecture (Cosmetic surgery conjecture)

Two surgeries with inequivalent slopes are never truly cosmetic.

Main result

Let K be a hyperbolic knot in S^3, and $r, s \in \mathbb{Q} \cup \{\infty\}$ two distinct exceptional slopes on $\partial \mathcal{N}(K)$.

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Theorem (R.)

If $S^3_K(r) \cong S^3_K(s)$ as oriented manifolds, then the surgery must be irreducible, toroidal and non-Seifert fibred, moreover

$$\{r, s\} = \{+1, -1\}.$$

Consequences

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There are no exceptional truly cosmetic surgeries on

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- If a hyperbolic knot $K \subset S^3$ admits an exceptional truly cosmetic surgery then the Heegaard Floer correction term of any $1/n$ $(n \in \mathbb{Z})$ surgery on K satisfies $d(S^3_K(1/n)) = 0$.

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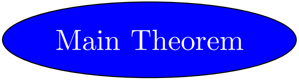
- If Y is the result of this surgery then:

$$|t_0(K)| + 2\sum_{i=1}^{n} |t_i(K)| \leq \mathrm{rank} HF_{\mathrm{red}}(Y).$$

The Proof



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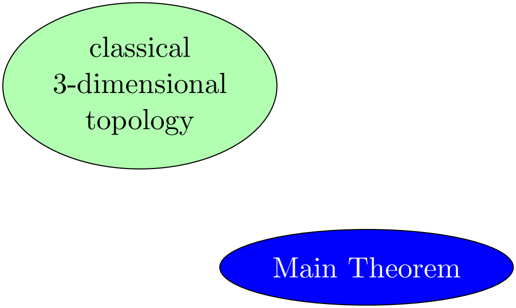

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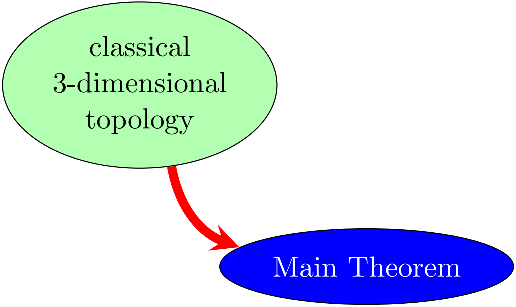

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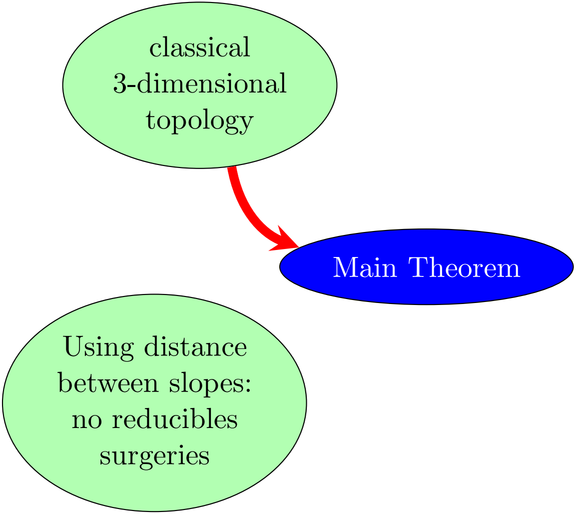
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Main Theorem
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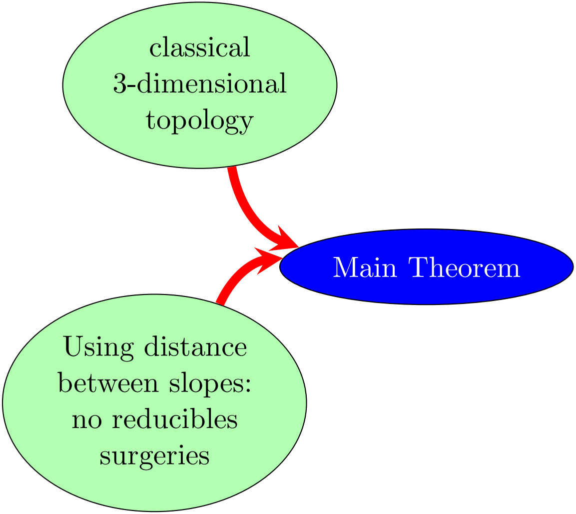
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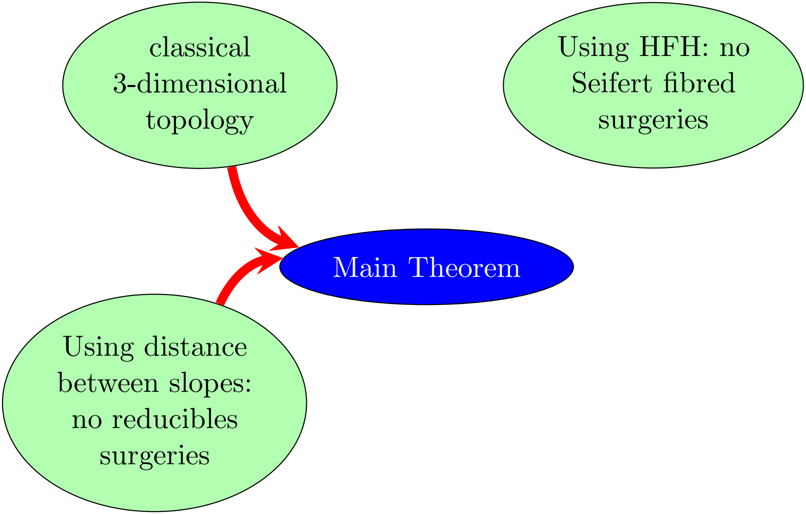
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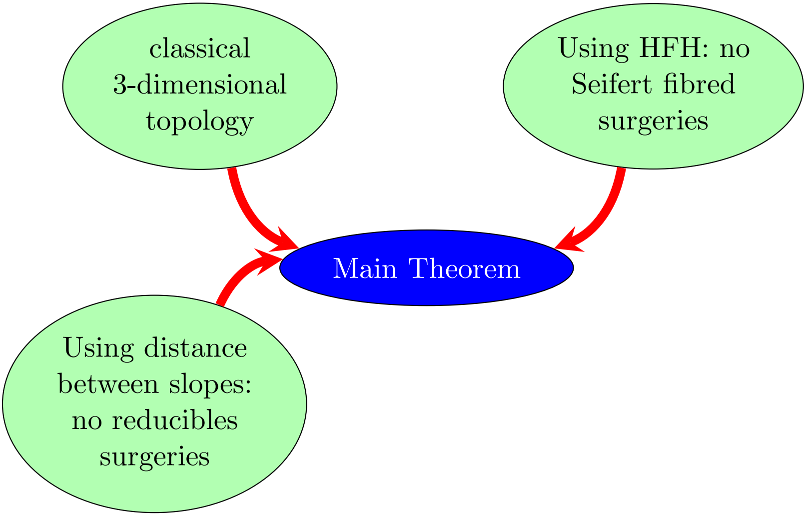
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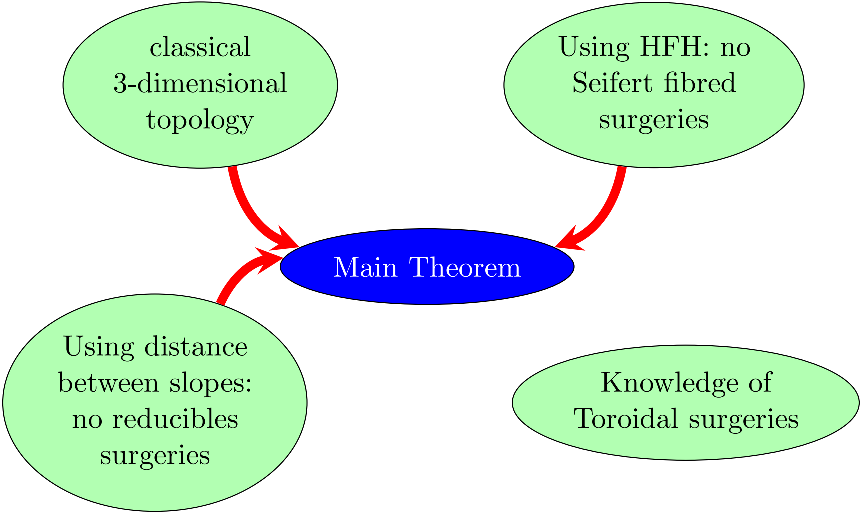
classical 3-dimensional topology
Using HFH: no Seifert fibred surgeries
Main Theorem
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Knowledge of Toroidal surgeries

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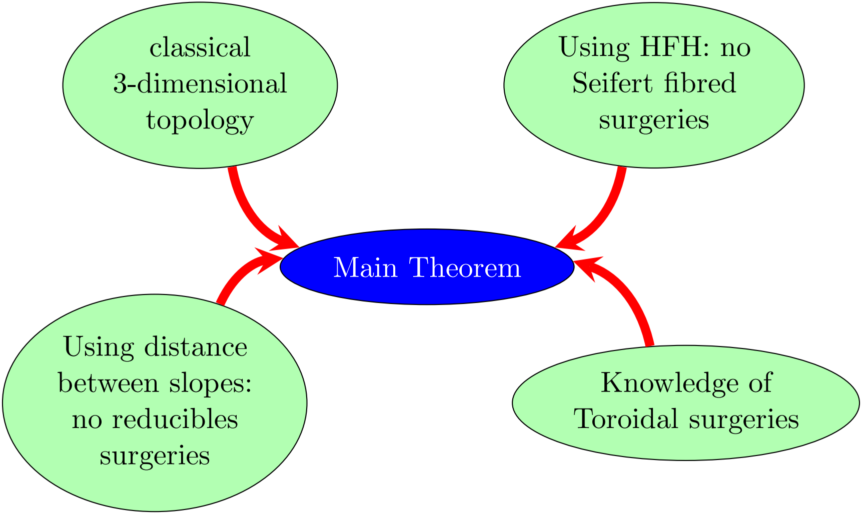
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Spheres, tori, and outer automorphisms of the free group

Funda Gultepe

University of Illinois at Urbana-Champaign