Lightning Talks II
Tech Topology Conference

December 9, 2017

Fillings of Iterated Planar Contact Manifolds

Bahar Acu

Northwestern University

Lightning Talks Session I
Tech Topology Conference
December 8, 2017

Main objects of study

Contact manifolds

$(M^{2n+1}, \xi = \ker \lambda)$ where ξ=maximally nonintegrable hyperplane field satisfying $\lambda \wedge (d\lambda)^n \neq 0$.

$\lambda :=$ contact form
$\xi :=$ contact structure

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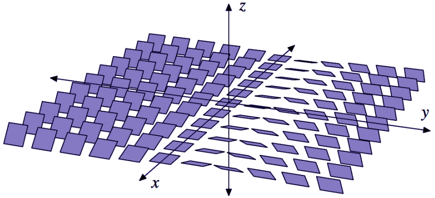

$(\mathbb{R}^3, \xi = \ker(dz - ydx))$

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Symplectic manifolds

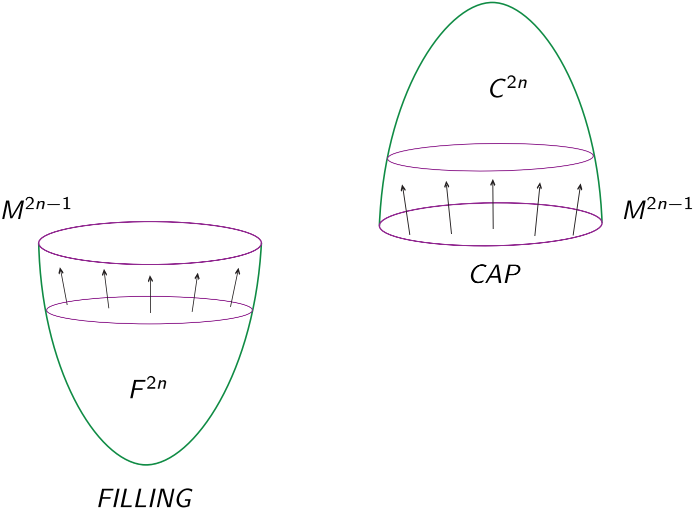
(W^{2n}, ω) where ω is a closed nondegenerate ($\omega^n \neq 0$) 2-form on W.

$\omega :=$ symplectic structure

Goal

To study **symplectic fillings** of certain higher-dimensional contact manifolds and, by using this result, prove a higher-dimensional **symplectic capping** result for that class.

Fillings vs. Caps



Types of fillings of contact manifolds

In any given dimension,

Fact

$\{Stein\} = \{Weinstein\} \subset \{Exact\} \subset \{Strong\} \subset \{Weak\} \subset \{Tight\}$

Motivating questions

Question

Does every contact manifold M admit symplectic caps?

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Does every contact manifold M admit symplectic caps?

Answer

Yes,

- *if M^{2n+1} has a Stein filling (Lisca-Matić).*
- *if dim $M = 3$ then M has infinitely many distinct symplectic caps (Etnyre-Honda).*

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Does every symplectic manifold with boundary M embed as a domain into a closed symplectic manifold? i.e. can M be symplectically capped off?

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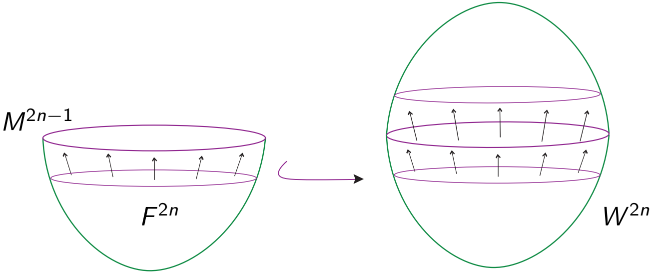

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Does every symplectic manifold with boundary M embed as a domain into a closed symplectic manifold? i.e. can M be symplectically capped off?

Answer

Yes,

- *if M is Stein fillable (Lisca-Matić).*
- *if $\dim M = 3$ and M is weakly fillable (Eliashberg, Etnyre).*

Generalization attempt

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Can we do the same thing in higher dimensions?

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Not easy!

One needs to know symplectic mapping class group of the capped page.

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Potential Remedy

Iterated planar Lefschetz fibrations/open books.

Idea: carry 3-dimensional symplectic capping result by Eliashberg-Etnyre to higher dimensions inductively!

The fruit of the attempt

M^{2n+1}: contact manifold
F^{2n}: page of the supporting open book of M
B^{2n-1}: binding of the supporting open book of M

Conjeorem (Acu-Etnyre-Ozbagci)

If B has an exact symplectic cobordism to B', call X where $\partial X = -B \cup B'$, then there exists an **exact** *symplectic cobordism*

$$Y = M \times [0,1] \bigcup_{\partial X \times D^2 = B \times \{1\} \times D^2} X \times D^2$$

from M to a $(2n+1)$-dimensional contact manifold M' supported by an open book whose binding is B' and page is $F \cup X$.

Theorem (Acu-Etnyre-Ozbagci)

Y is a **strong** *symplectic cobordism.*

Symplectic caps of iterated contact 5-manifolds

Iterated planar contact 5-manifold $:=$ contact manifold with planar contact binding

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If the Conjeorem is true, then we have:

Corollary

Every iterated planar contact 5-manifold can be symplectically capped off.

Idea: come up with an exact cobordism from M to S^5 and then cap off S^5 since planar open books have exact cobordisms to $S^3 (=$ binding of $S^5)$.

Thank you!

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Generalized Alexander's Theorem

Sudipta Kolay

School of Mathematics
Georgia Institute of Technology

December 9, 2017

Introduction

Closing up the ends of a braid gives a link, called a *closed braid*.

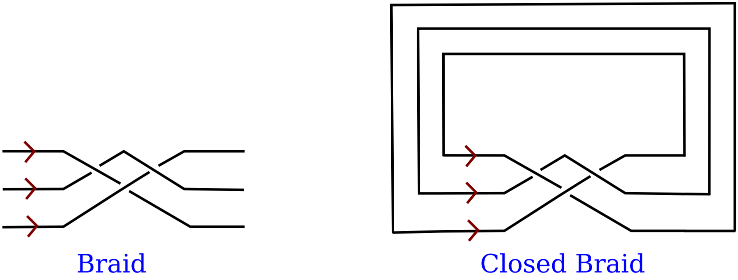

Figure : Closure of a braid

Question

Is every link a closed braid?

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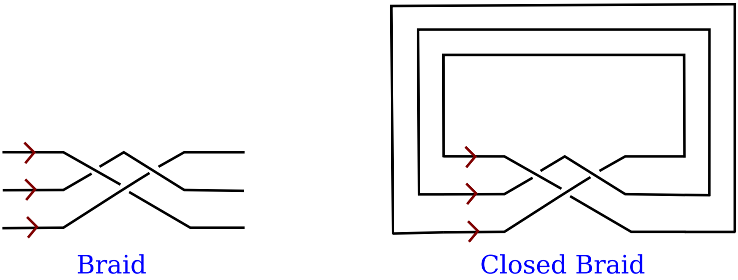


Figure : Closure of a braid

Alexander's Theorem (1923)

Every oriented link in $\mathbb{R}^3$ is isotopic to a closed braid.

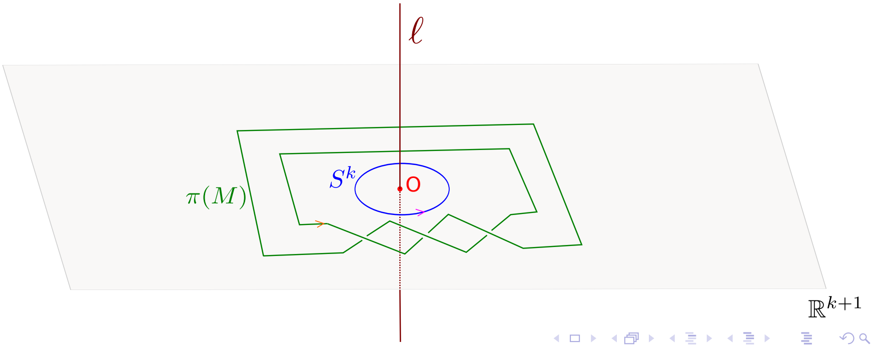
Closed Braids

Definition

We say $f(M)$ is a *closed braid* if it misses ℓ and the composition

$$M^k \xrightarrow[\text{embedding}]{f} \mathbb{R}^{k+2} \setminus \ell \xrightarrow[\text{orth. proj.}]{\pi} \mathbb{R}^{k+1} \setminus O \xrightarrow[\text{rad. proj.}]{p} S^k$$

is an oriented branched covering map.



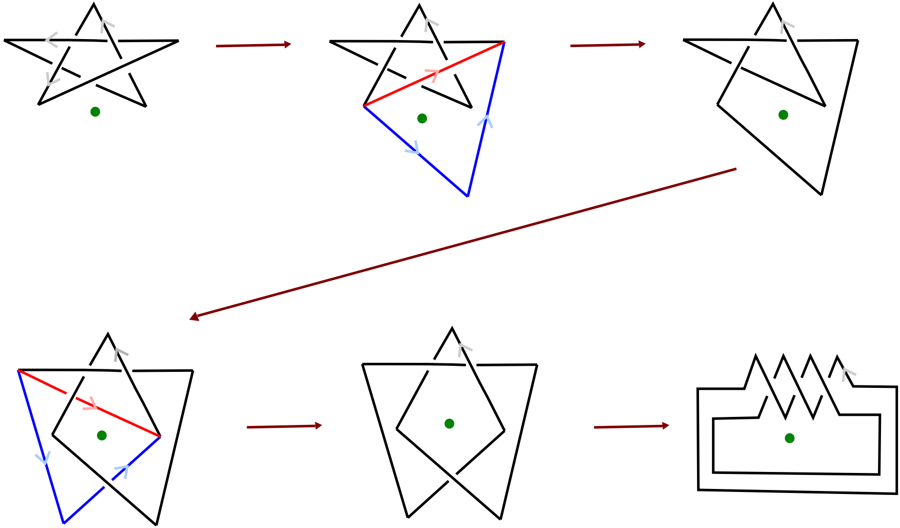
Isotoping to a closed braid

P.L. Generalized Alexander's Theorem

Any closed oriented p.l. $(n-2)$-link in $\mathbb{R}^n$ can be p.l. isotoped to be a closed braid for $3 \leq n \leq 5$.

- $n = 3$, Alexander (1923).
- smooth ribbon surfaces in $\mathbb{R}^4$, Rudolph (1983).
- $n = 4$, Viro (1990), Kamada (1994).
- $n = 5$, K. (*2017*).

Dimension 3: an example

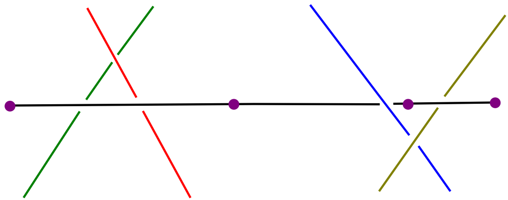


Dimension 3: Proof

Alexander's Theorem

Every oriented link in $\mathbb{R}^3$ is isotopic to a closed braid.

- *Claim 1.* If a clockwise simplex has only over-crossings, then we can find an embedded triangle crossing ℓ by going sufficiently over.
- *Claim 2.* The result of a cellular move along such a triangle is that a clockwise simplex is replaced by counterclockwise simplices.



Questions

1. Can every smooth link in $\mathbb{R}^5$ be isotoped to be a closed braid?

Theorem (Etnyre-Furukawa, 2017)

If "yes", then smooth every embedding $M^3 \hookrightarrow S^5$ can be isotoped to be a transverse contact embedding.

2. What happens in higher dimensions (p.l. and smooth)?

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Thank You!

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An Excursion in Gluing Maps

Ryan Leigon

Joint with Federico Salmoiraghi

Tech Topology Conference 2017

Sutured Floer Homology

- Heegaard Floer theory assigns chain complexes to 3-manifolds:

$$Y^3 \longrightarrow HF^\circ$$

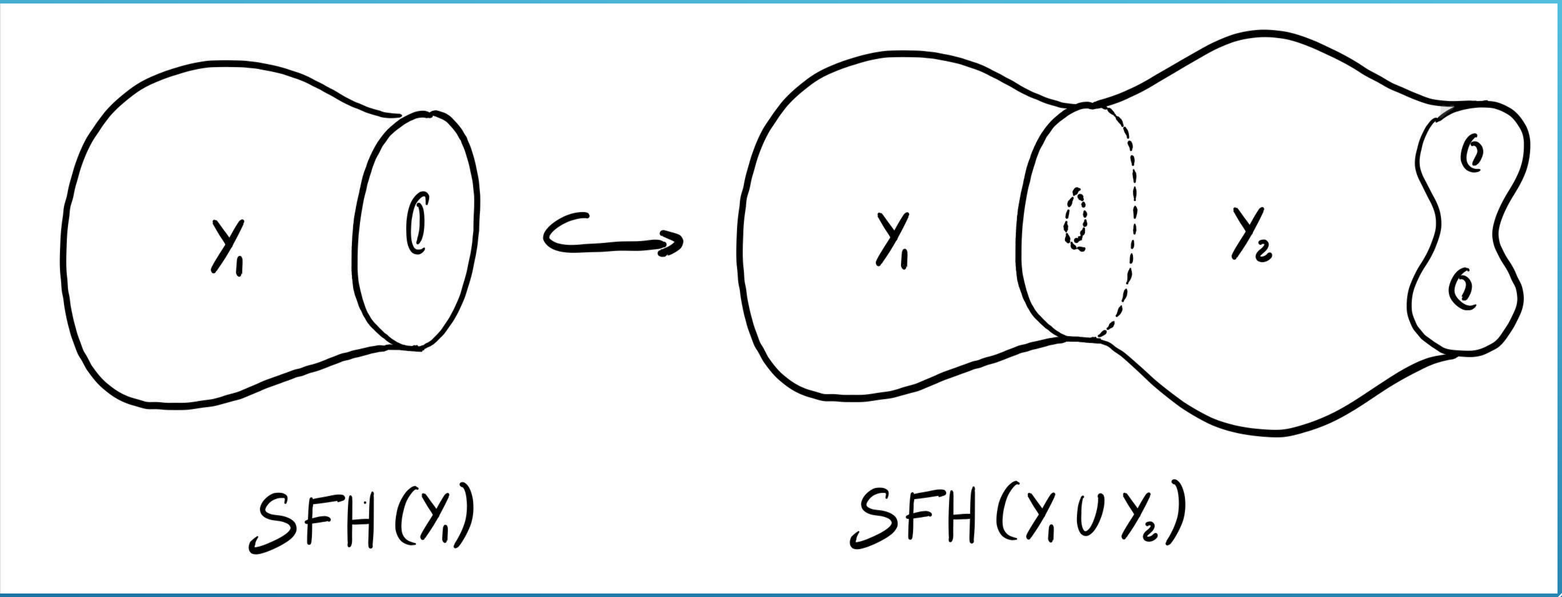
$$K \subset Y^3 \longrightarrow HFK^\circ$$

$$(Y,\Gamma) \longrightarrow SFH$$

- What happens to SFH when we glue two manifolds together?

The Honda-Kazez-Matic Map

- We view the process of gluing as an inclusion:



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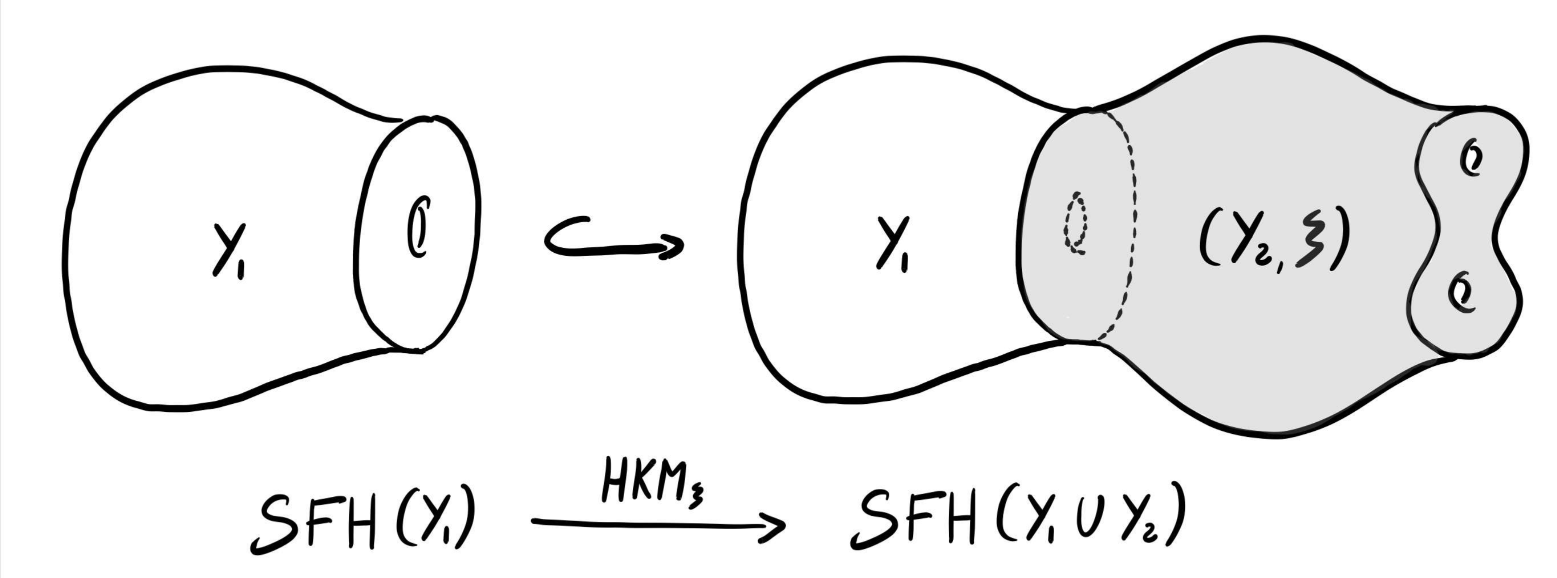
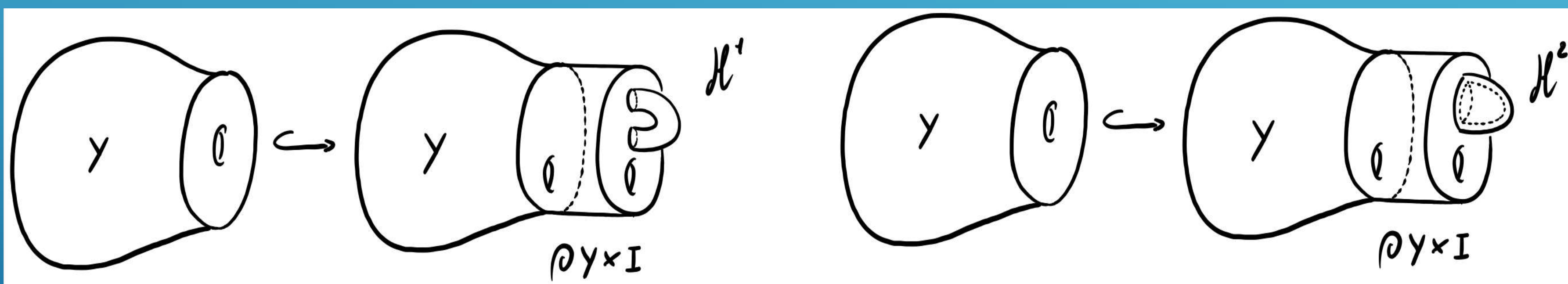


- The HKM map depends on the contact structure ξ

Computing HKM

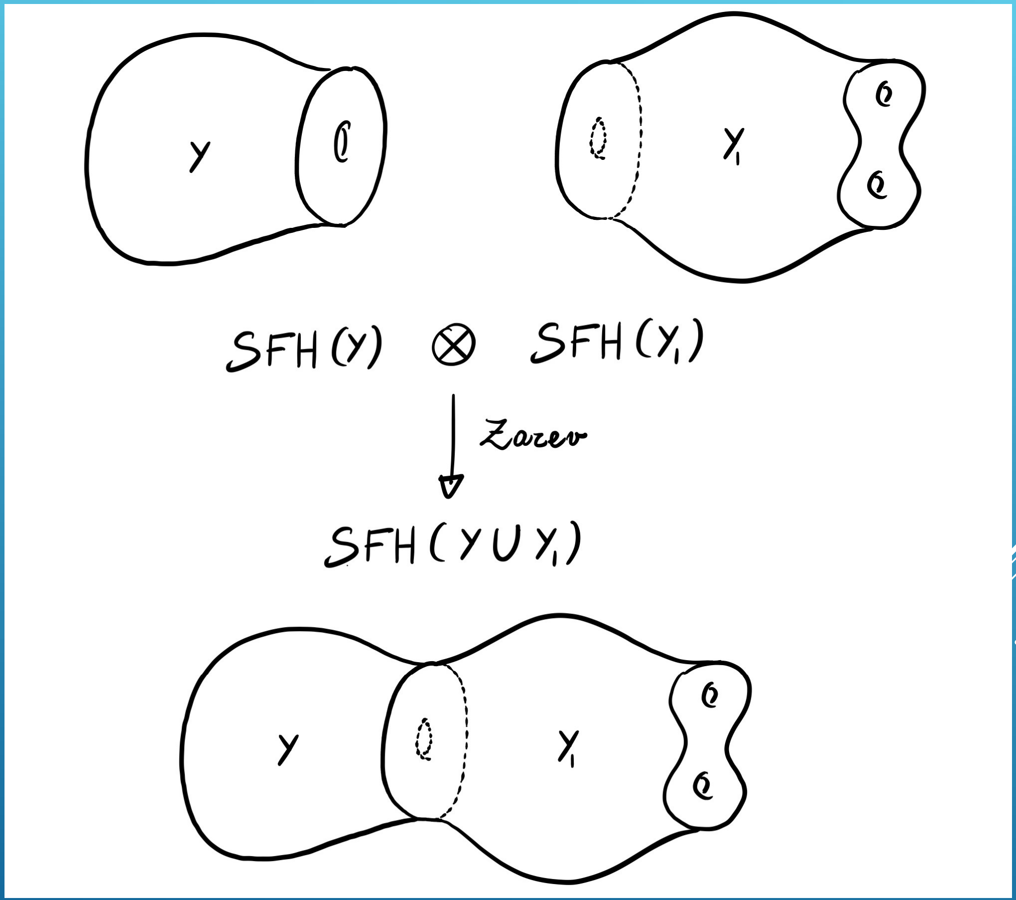
- HKM is impossible to explicitly compute, even in most elementary cases:



- Problem: Constructing HKM requires "padding"

Zarev's Gluing Map

Zarev's map is a pairing:



Features:

- Formal algebraic map
- No padding needed
- No contact geometry involved
- Complexity is captured by the underlying algebra

Theorem(with Salmoiraghi; Zarev):

When properly interpreted, the HKM gluing map is equivalent to Zarev's map gluing.

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Cor: HKM can be redefined without the padding.

Proof Idea:

1. Sufficient to prove for 1-and 2-handle attachments

2. Decompose the HKM construction into simple pieces

3. Show that maps corresponding to the simple pieces are Zarev maps (easy for 1-handle, difficult for 2-handle) □

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Essential embeddings and immersions of surfaces in a 3-manifold.

Aamir Rasheed

Florida State University

December 9, 2017

Outline

In this talk we will describe how:

- Fundamental groups of essential embedded surfaces (surface groups) are in some sense maximal.
- These surfaces can be orientable, non-orientable with or without boundary.
- Further we will show that fundamental groups of immersed surfaces are either maximal or can be realized by a covering map.

Fundamental group of an embedded torus is maximal

The following is a well known fact.

Theorem (Hempel)

Let M be an orientable Haken manifold such that $f : T \to M$ is an embedded essential (π_1- injective and not boundary parallel) torus. Given a subgroup $G = Z \times Z$ of $\pi_1(M)$ such that $f_*(\pi_1 T) \subset G$, then it must be the case that $f_*(\pi_1 T) = G$.

In particular, given two essential embeddings of a torus $f : T \to M$ and $g : T \to M$ with $f_*(\pi_1 T) \subset g_*(\pi_1 T)$ then we must have $f_*(\pi_1 T) = g_*(\pi_1 T)$ Does this generalize to higher genus surfaces? The answer is yes.

Surface groups are maximal

Theorem

Let M be a Haken manifold such that $f_1 : F_1 \to M$ and $f_2 : F_2 \to M$ are essential embeddings of closed surfaces. Further assume that the given embeddings are 2-sided. Suppose that $f_{1*}(\pi_1 F_1) \subset f_{2*}(\pi_1 F_2)$ then it must be the case that $f_{1*}(\pi_1 F_1) = f_{2*}(\pi_1 F_2)$. In fact we can conclude that f_1 and f_2 are isotopic.

Note that the surfaces in the above theorem may be non-orientable.

Surface groups are maximal

Here is an extension of the previous theorem, where we consider non-closed essential embedded surfaces in a 3-manifold.

Theorem

Let M be a Haken manifold such that $f_1 : F_1 \to M$ and $f_2 : F_2 \to M$ are essential proper embeddings of surfaces. Suppose that $f_{1*}(\pi_1 F_1) \subset f_{2*}(\pi_1 F_2)$ and $\partial F_1 \subset \partial F_2$. Further assume that every boundary component of both surfaces lies in the same connected component of the boundary of M, then it must be the case that $f_{1*}(\pi_1 F_1) = f_{2*}(\pi_1 F_2)$. In fact we can conclude that f_1 and f_2 are isotopic.

Fundamental groups of immersed surfaces are maximal

Next, we deal with immersed surfaces and show that here an analogous theorem holds as well.

Theorem

Let M be a compact, connected, orientable, irreducible manifold such that $f_1 : F_1 \to M$ and $f_2 : F_2 \to M$ are essential immersions of closed surfaces. Suppose that $f_{1*}(\pi_1 F_1) \subset f_{2*}(\pi_1 F_2)$ then either f_1 and f_2 are homotopic and hence $f_{1*}(\pi_1 F_1) = f_{2*}(\pi_1 F_2)$ or f_1 is homotopic to a covering map onto $f_2(F_2)$.

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Unoriented Cobordism Maps on Link Floer Homology

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Tech Topology Conference 2017

Preliminaries

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Link Floer Homology (Ozsváth and Szabó)

HFL^* is an invariant for oriented links in three-manifolds. It is a Maslov graded, Alexander filtered $\mathbb{Z}_2[U]$-module.

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Unoriented Link Floer Homology (Ozsváth, Stipsciz and Szabó)

HFL' is an invariant for unoriented links in three-manifold. It is an δ-graded $\mathbb{Z}_2[U]$-module.

- We treated all w, z-basepoints the same type (a single variable U for each basepoint).

Link Cobordism Maps on Link Floer Homology

Question

Whether an oriented (or unoriented resp.) link cobordism (W, F) from L_0 to L_1 induces a map on link Floer homology (or unoriented link Floer homology resp.).

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Remark. All the above constructions need extra data on the surface F.

Bipartite Disoriented Links

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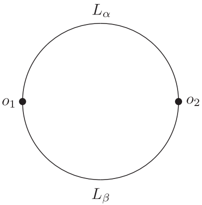
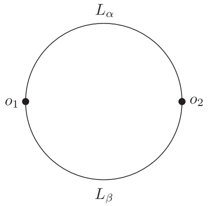


Figure: Bipartite Link

Bipartite Disoriented Links



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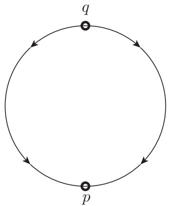

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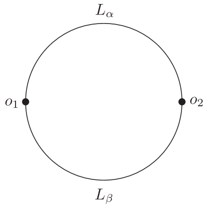

Figure: Bipartite Link

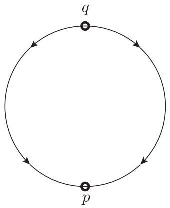

Figure: Disoriented Link

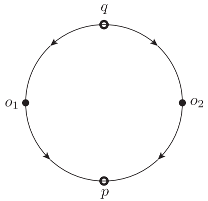

Figure: Bipartite Disoriented Link $(\mathcal{L}, \mathbf{O})$.

Bipartite Disoriented Link Cobordisms

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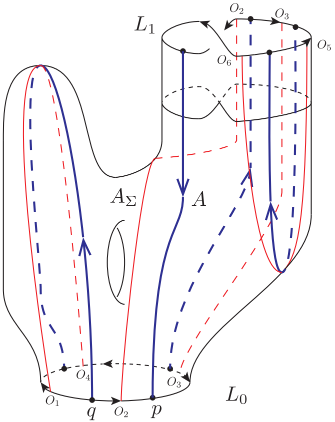

Figure: Bipartite Disoriented Link Cobordism

Bipartite Disoriented Link Cobordisms

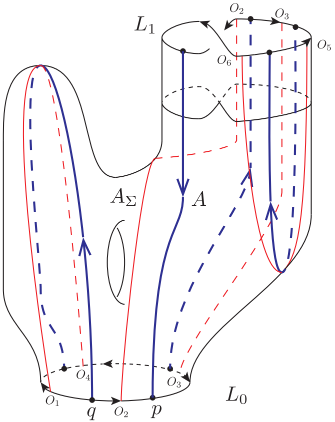

Figure: Bipartite Disoriented Link Cobordism

- **Red curves**: Tracks the motion of basepoints

Bipartite Disoriented Link Cobordisms

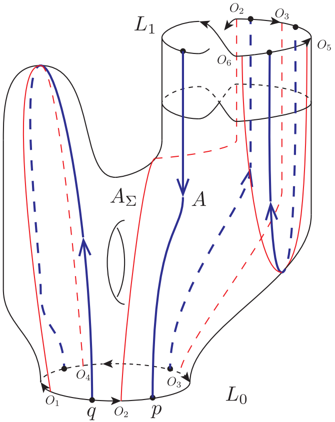

Figure: Bipartite Disoriented Link Cobordism

- **Red curves**: Tracks the motion of basepoints
- **Blue curves** (oriented): Tracks the motion of index zero and three critical points.

Main Theorem

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Theorem (H. Fan)

Let $\mathfrak{W}^1$ be a bipartite disoriented link cobordism from $(\mathcal{L}^0, \mathbf{O}^0)$to $(\mathcal{L}^1, \mathbf{O}^1)$ (For simplicity, we consider F in $S^3 \times I$). Then we can define a $\mathbb{Z}$-filtered chain map:

$$F_{\mathfrak{W}} : HFL'(\mathcal{L}^0, \mathbf{O}^0) \to HFL'(\mathcal{L}^1, \mathbf{O}^1),$$

which is an invariant of $\mathfrak{W}^1$. Furthermore, if $\mathfrak{W}^2$ is a bipartite disoriented link cobordism from $(\mathcal{L}^1, \mathbf{O}^1)$ to $(\mathcal{L}^2, \mathbf{O}^2)$ in S^3, we have:

$$F_{\mathfrak{W}^2} \circ F_{\mathfrak{W}^1} = F_{\mathfrak{W}^2 \circ \mathfrak{W}^1}$$

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- This theorem can be extended to bipartite disoriented link cobordism with the surface F homologically trivial and torsion Spin^c-structure.
- Given a band move for links in S^3, our construction agrees with Ozsváth, Stipsicz and Szabó's construction via grid diagrams.

Applications

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- Hogancamp and Livingston (2017) defined involutive upsilon invariant for knots, which is an knot concordance invariant.

Applications

- Hogancamp and Livingston (2017) defined involutive upsilon invariant for knots, which is an knot concordance invariant.
- We will extend the involutive upsilon invariant from knots to links and study the relation between involutive upsilon invariant and the unoriented four-ball genus in an upcoming paper.

Thank you!

Lightning Talks II
Tech Topology Conference

December 9, 2017

Twisted rabbits and Hubbard trees

Becca Winarski
University of Wisconsin-Milwaukee

joint with Jim Belk, Justin Lanier and Dan Margalit

  

The twisted rabbit problem

$$p(z) = z^2 + c$$

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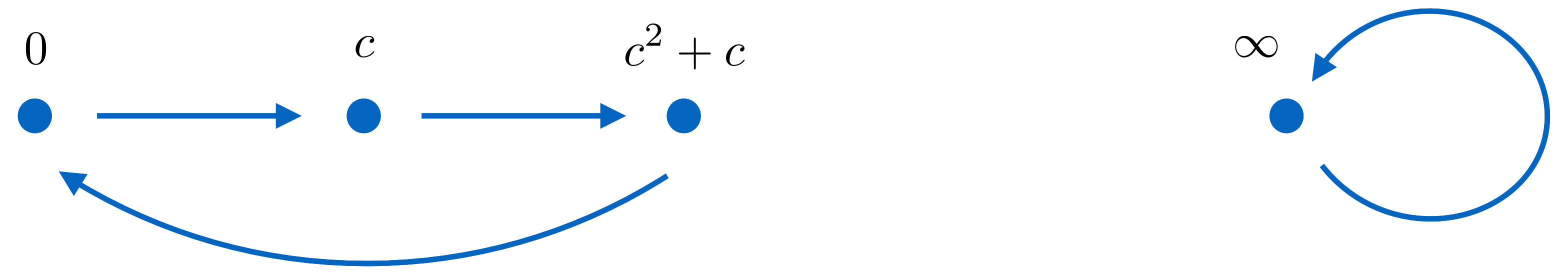
P:



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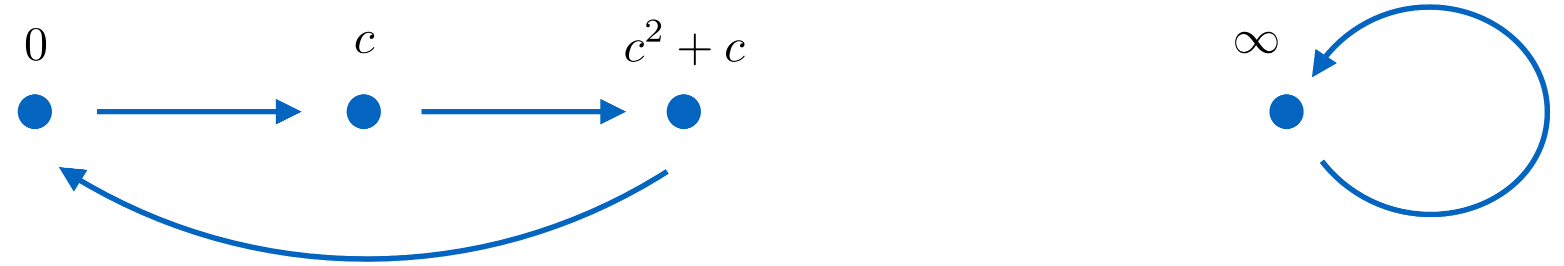
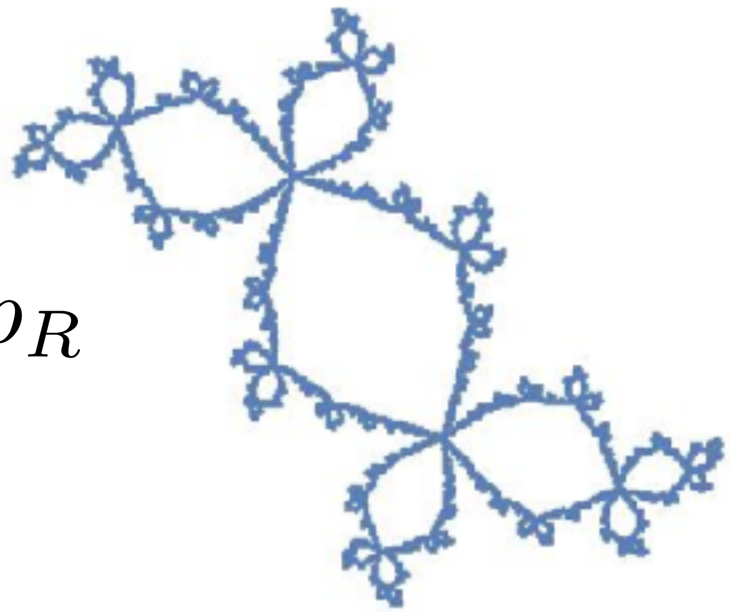


3 values of c $\longrightarrow$ p_R, p_C, p_A

Julia sets

Rabbit
Im(c)>0

p_R



Corabbit
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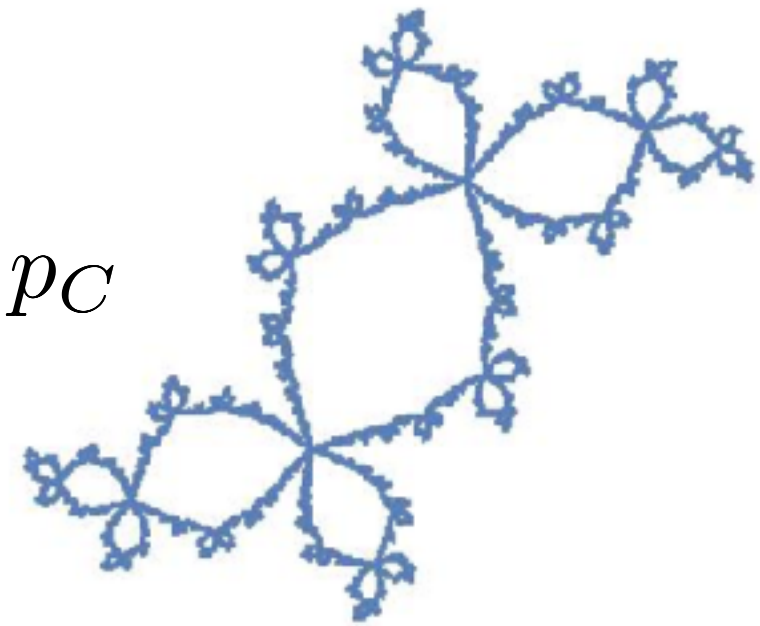
p_C

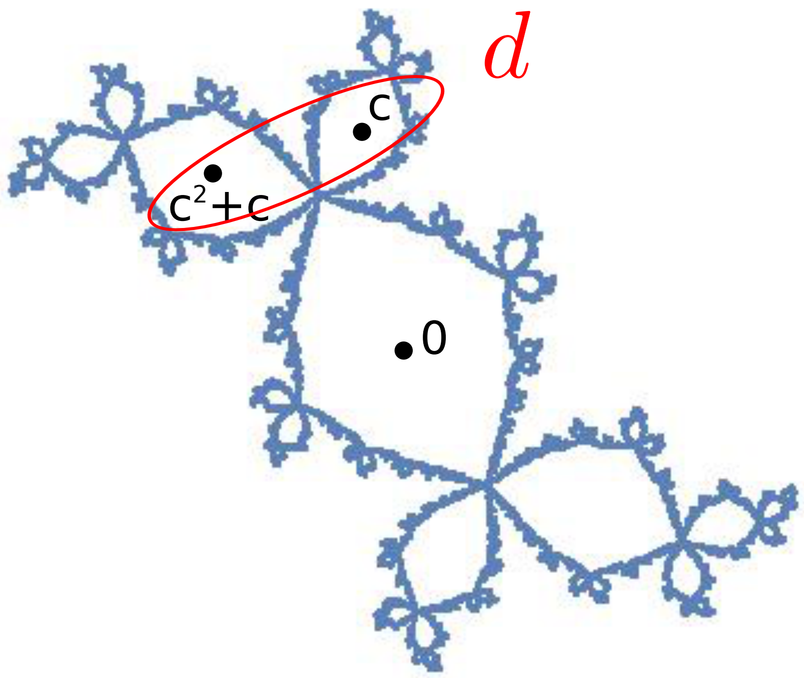


Airplane
Im(c)=0

p_A

Images courtesy of Bill Floyd https://www.math.vt.edu/netmaps/index.php

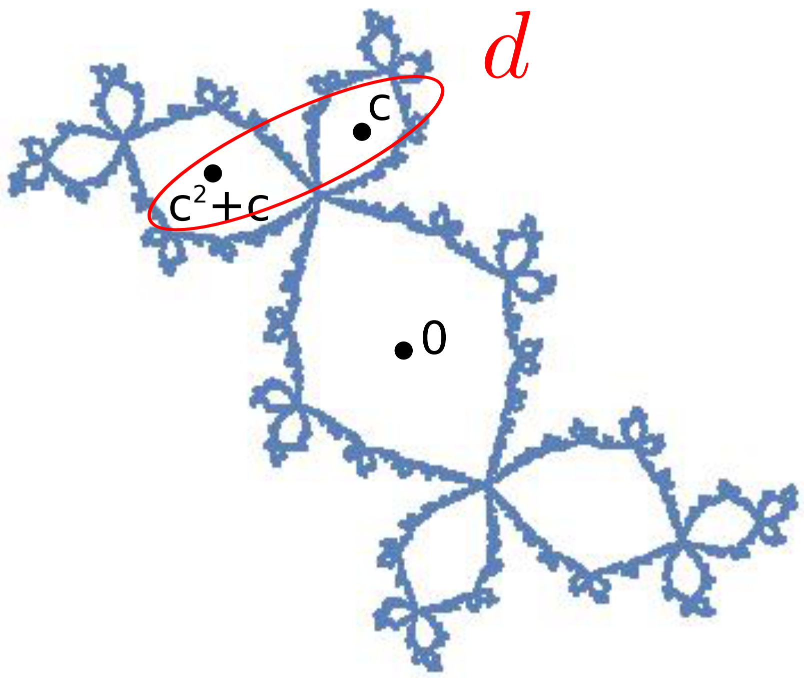
The twisted rabbit problem



p_R

The twisted rabbit problem

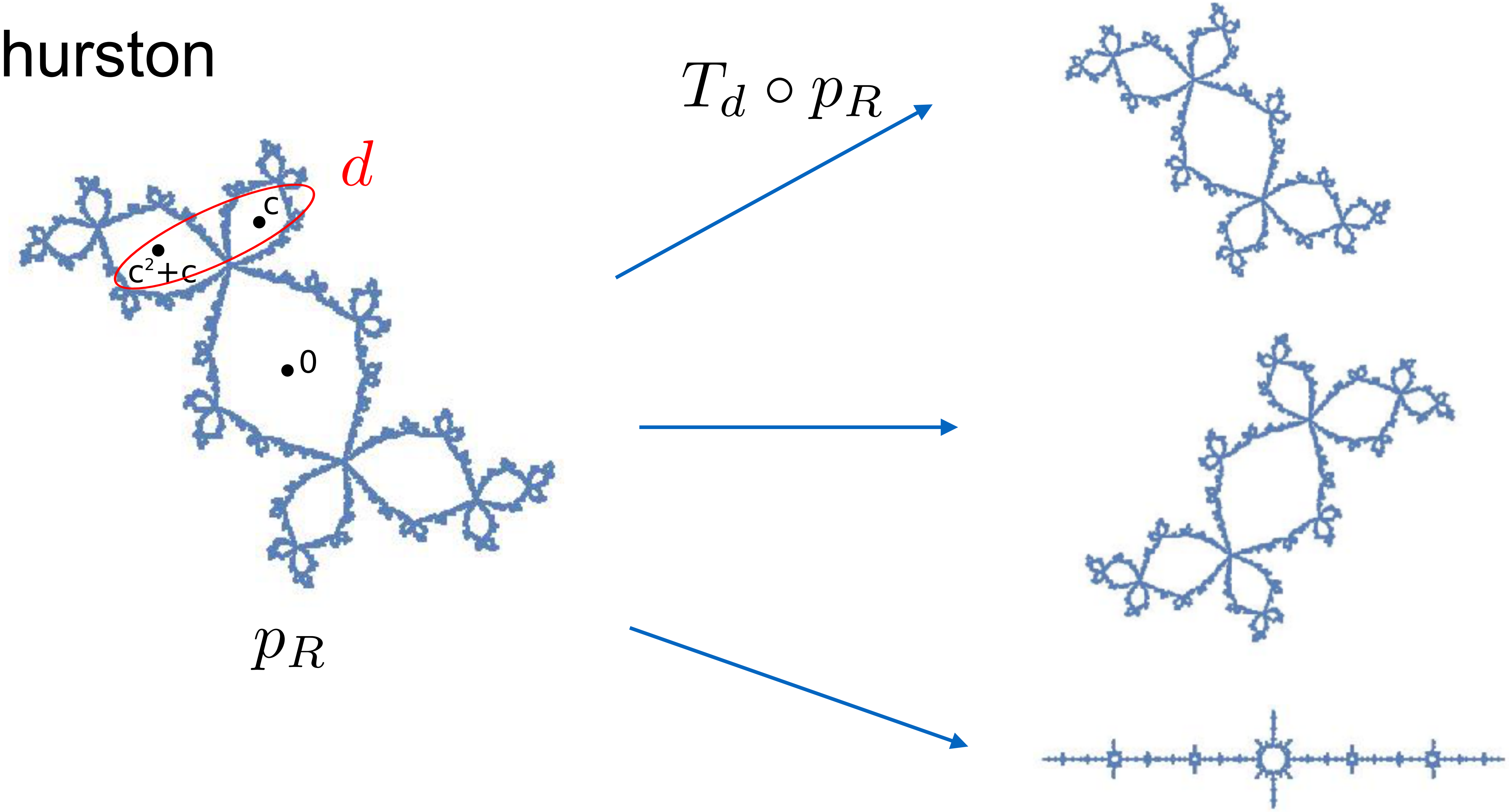
$T_d \circ p_R$



p_R

The twisted rabbit problem

Thurston



The twisted rabbit problem

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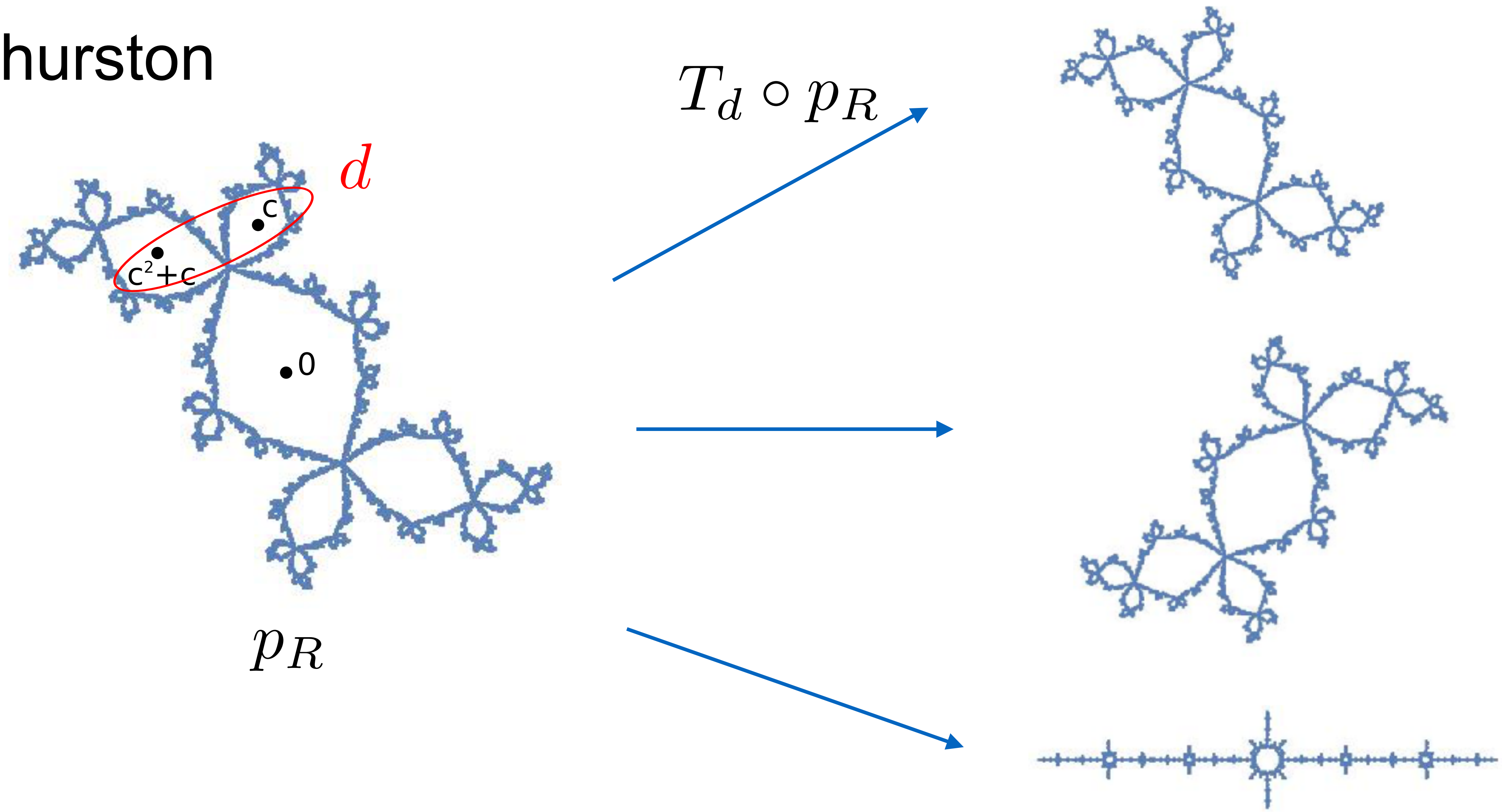

Twisted rabbit problem:

$f \in \mathrm{Mod}(\mathbb{C}, P)$ what is $f \circ p_R$?

Solving TRP

1. Topological description of p_R

Solving TRP

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3. Given f, what is $f \circ p_R$?

Solving TRP

1. Topological description of p_R $\longrightarrow$ branched covers

2. Distinguish p_R, p_C, p_A $\longrightarrow$ Hubbard trees

3. Given f, what is $f \circ p_R$?

$\longrightarrow$ following Bartholdi—Nekyrashevych

Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:

- edges are contained in Julia set
- leaves are in P

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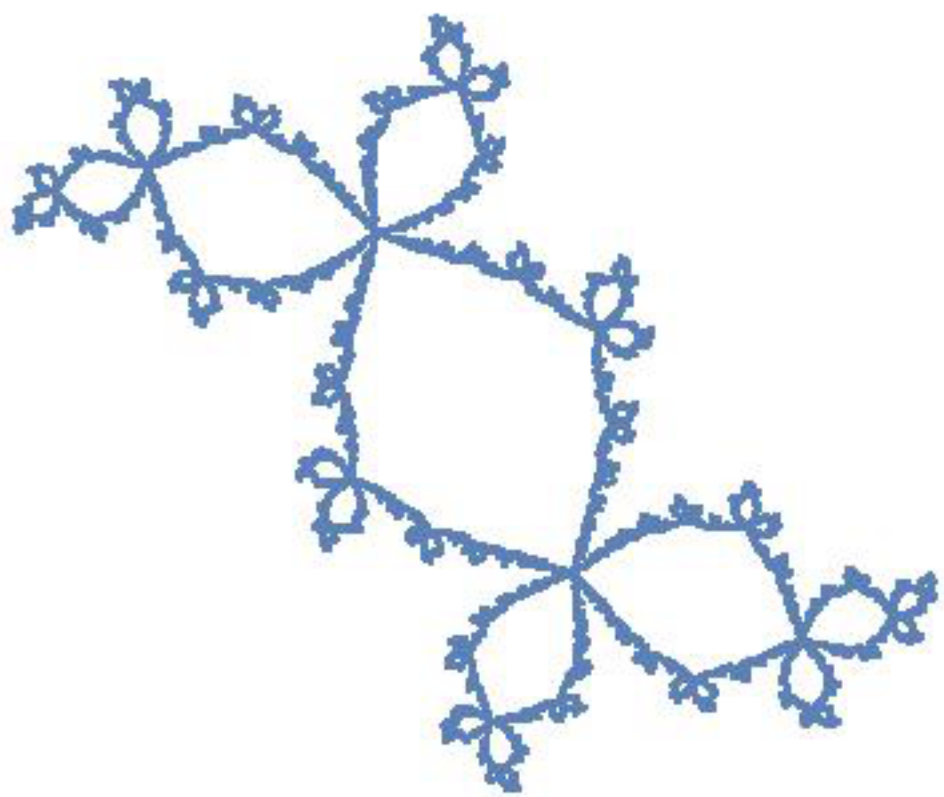

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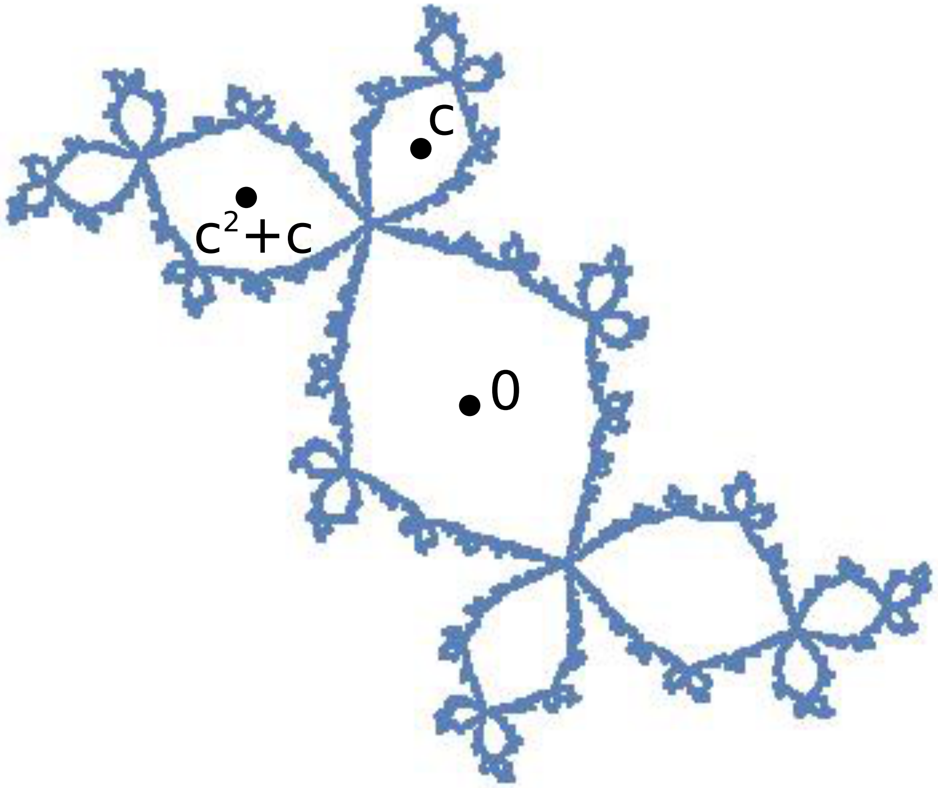

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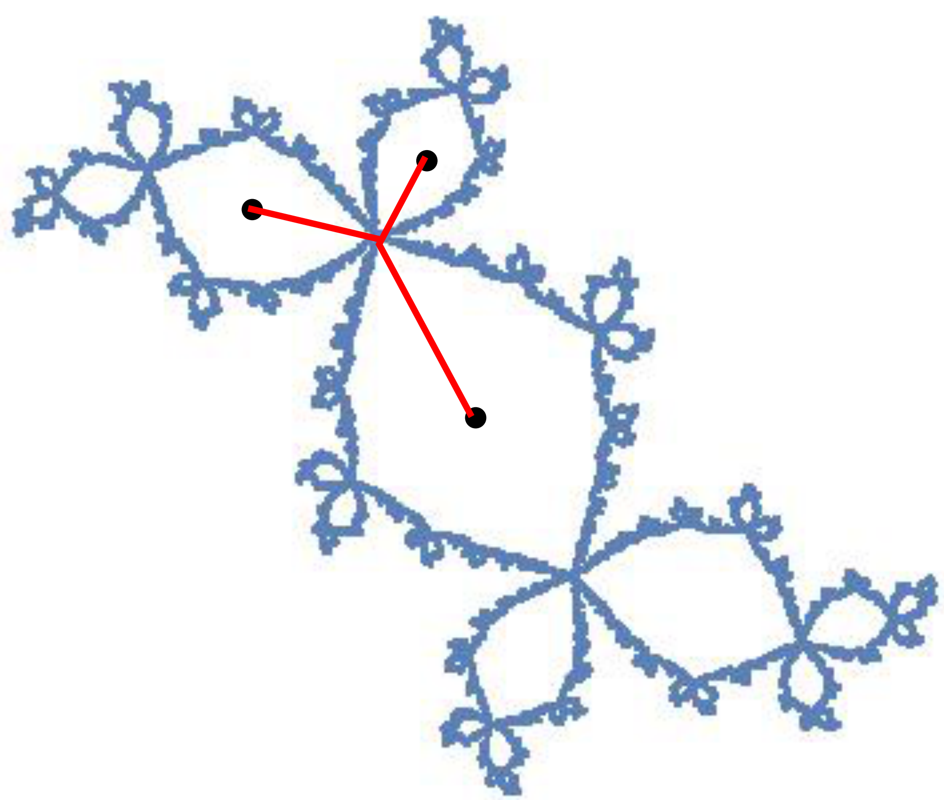

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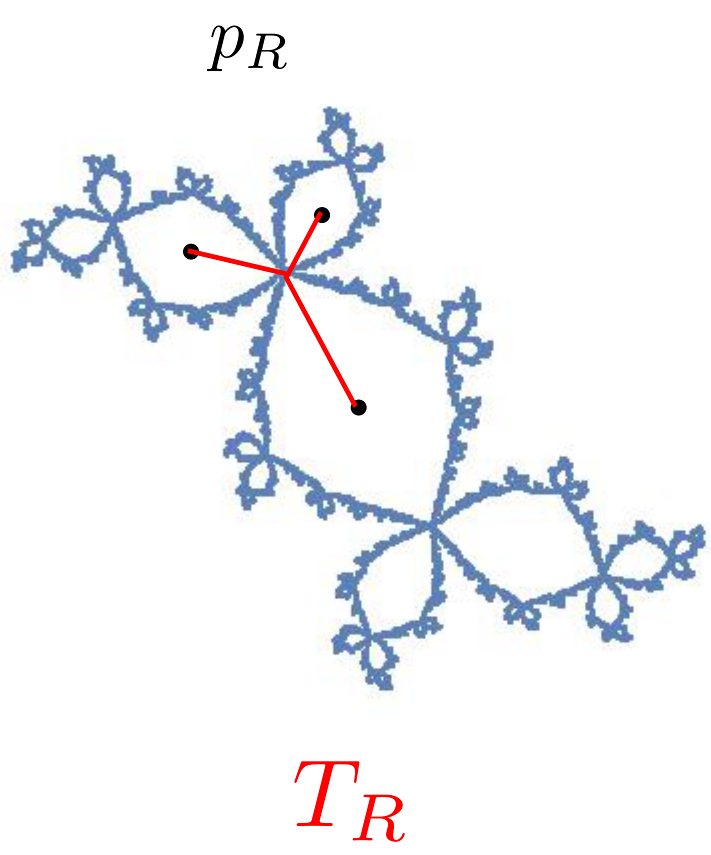

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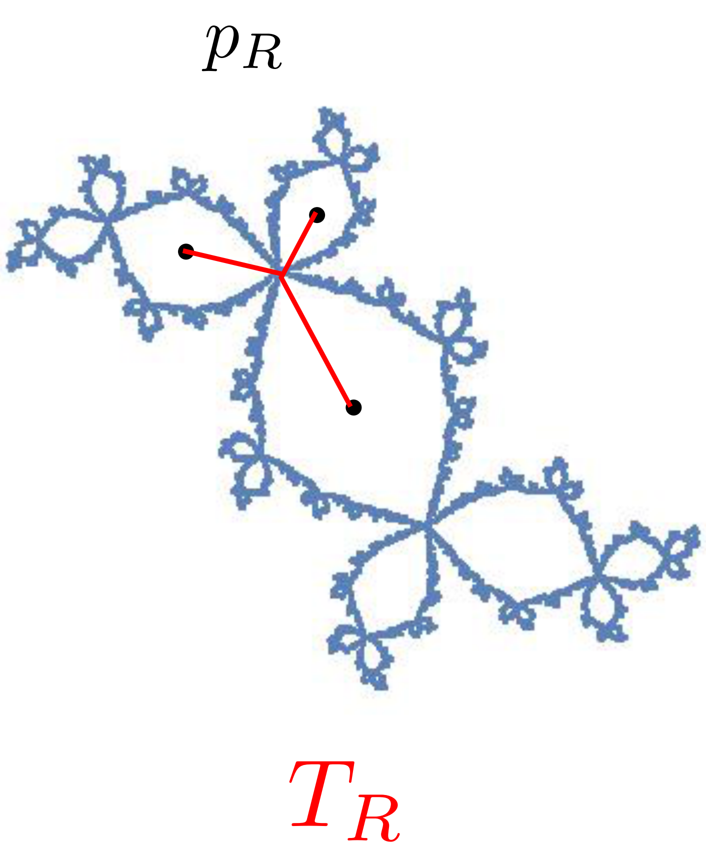
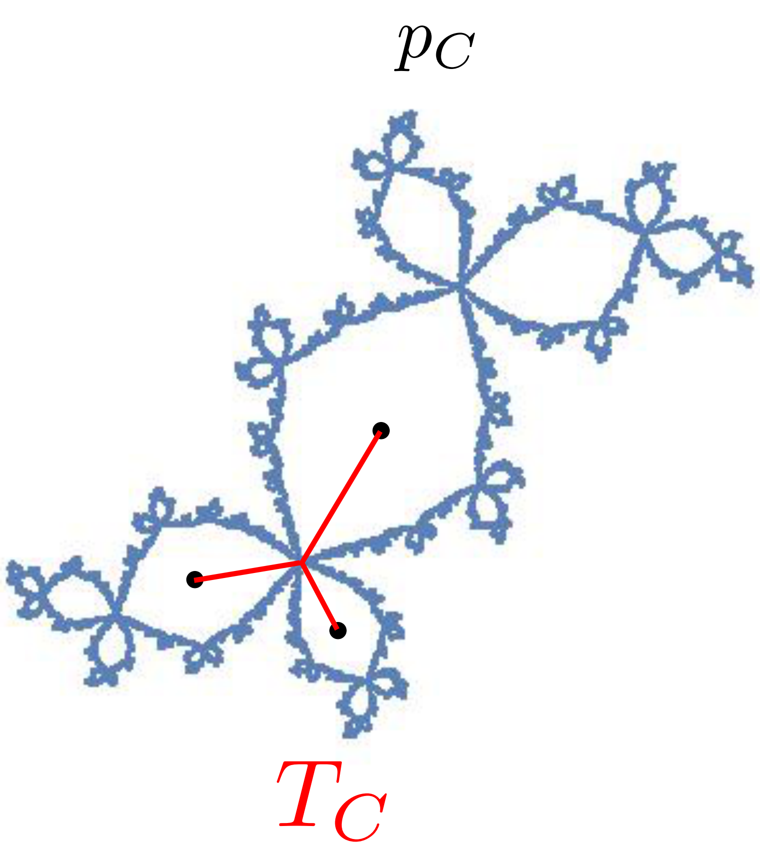


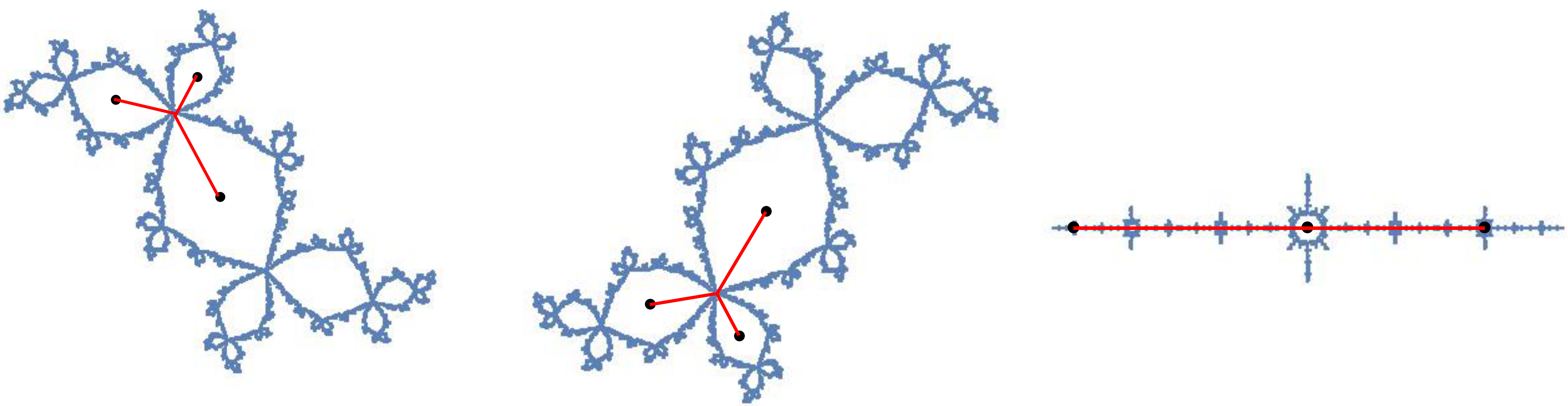


Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:

- edges are contained in (filled) Julia set
- leaves are in P

p_R p_C p_A



T_R T_C T_A

Hubbard trees as an invariant

T_A is combinatorially different from T_R and T_C

Hubbard trees as an invariant

T_A is combinatorially different from T_R and T_C

AND:

- p_R^{-1} rotates the edges of T_R clockwise
- p_C^{-1} rotates the edges of T_C counterclockwise

Hubbard trees as an invariant

T_A is combinatorially different from T_R and T_C

AND:

- p_R^{-1} rotates the edges of T_R clockwise
- p_C^{-1} rotates the edges of T_C counterclockwise

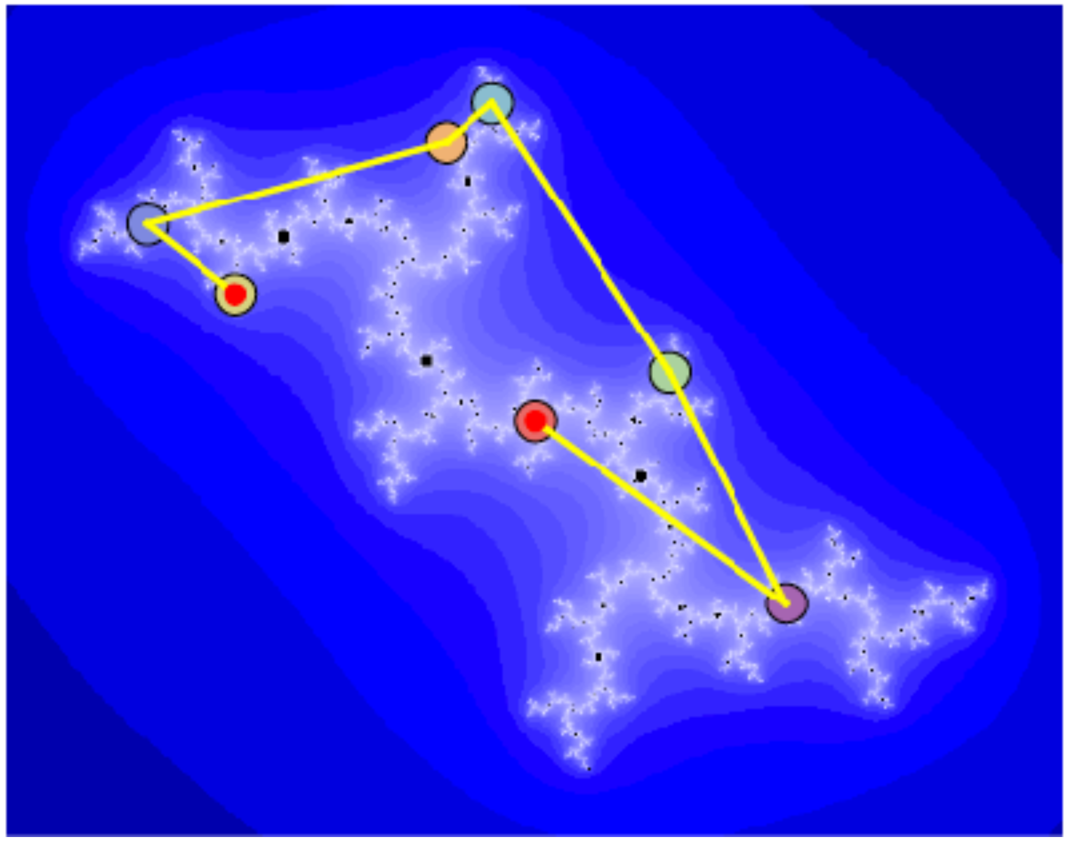
Proposition (Belk, Lanier, Margalit, W)

The Hubbard tree and its direction of rotation under p^{-1} distinguish p_R, p_C, p_A .

The general conjectures

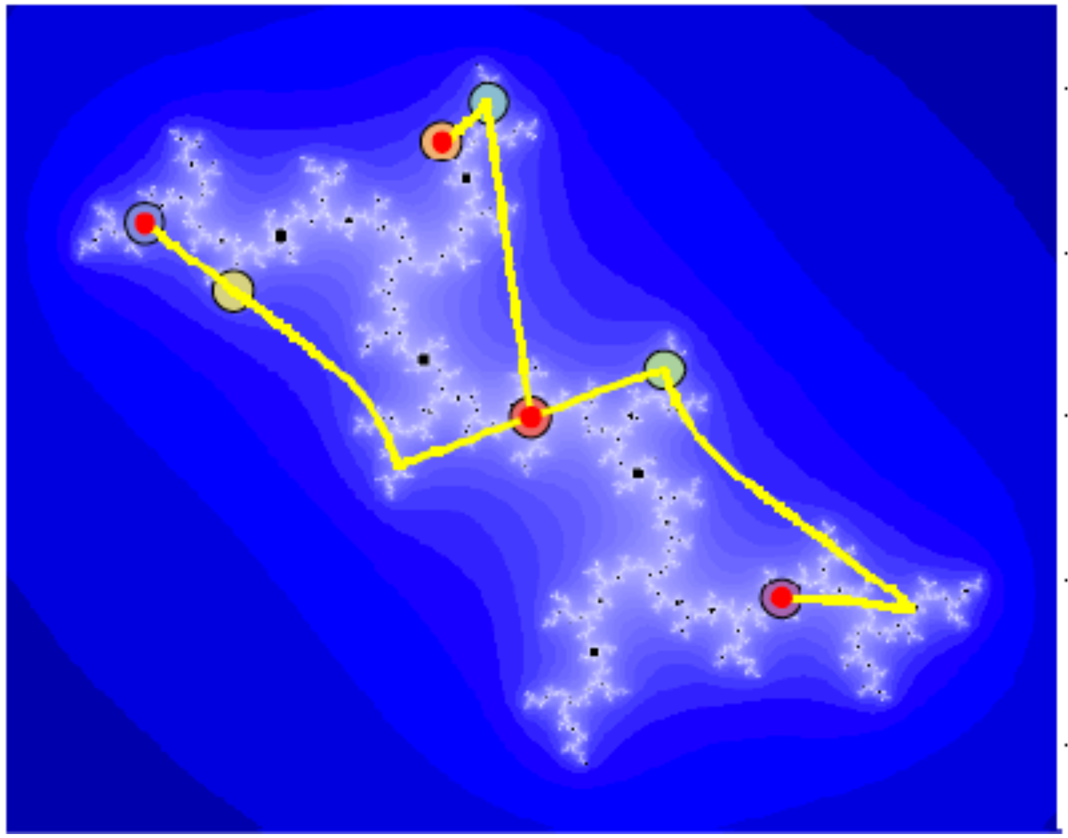
Conjecture 1: Given a polynomial p and a tree T, $\{p^{-n}(T)\}$ will converge to the Hubbard tree for p.

Tree convergence



Images by Jim Belk

Tree convergence



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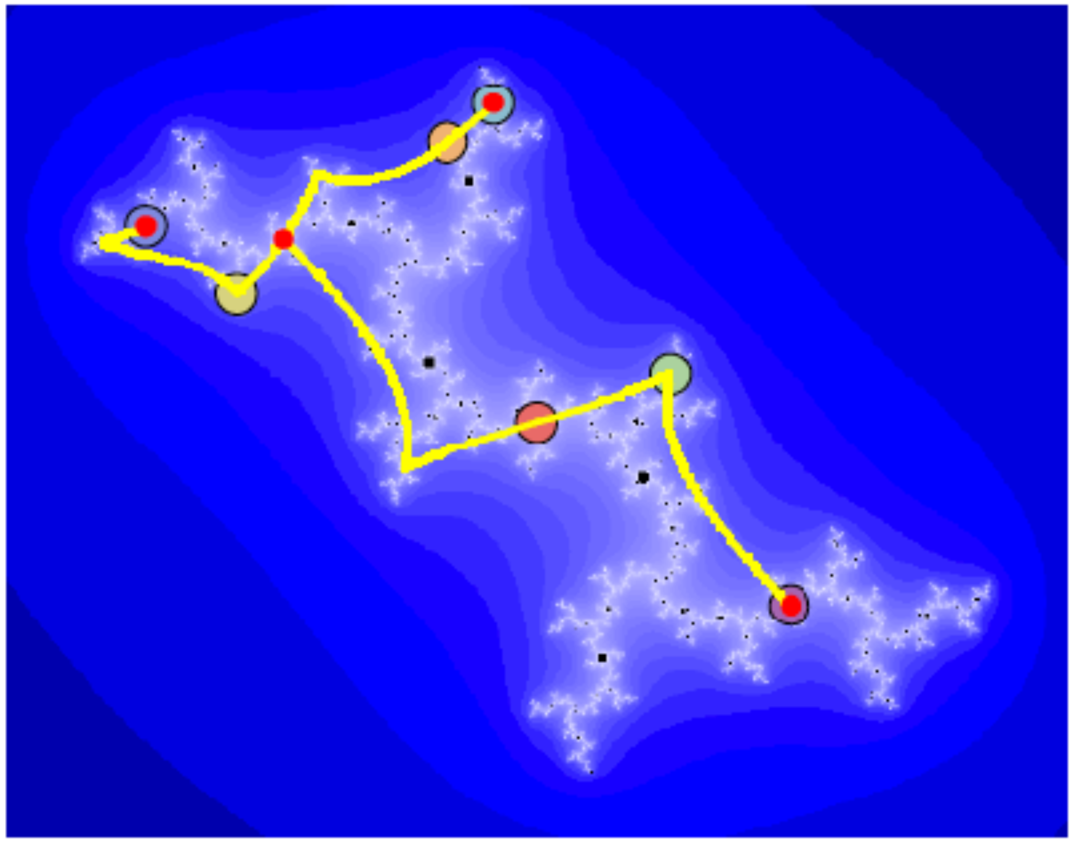

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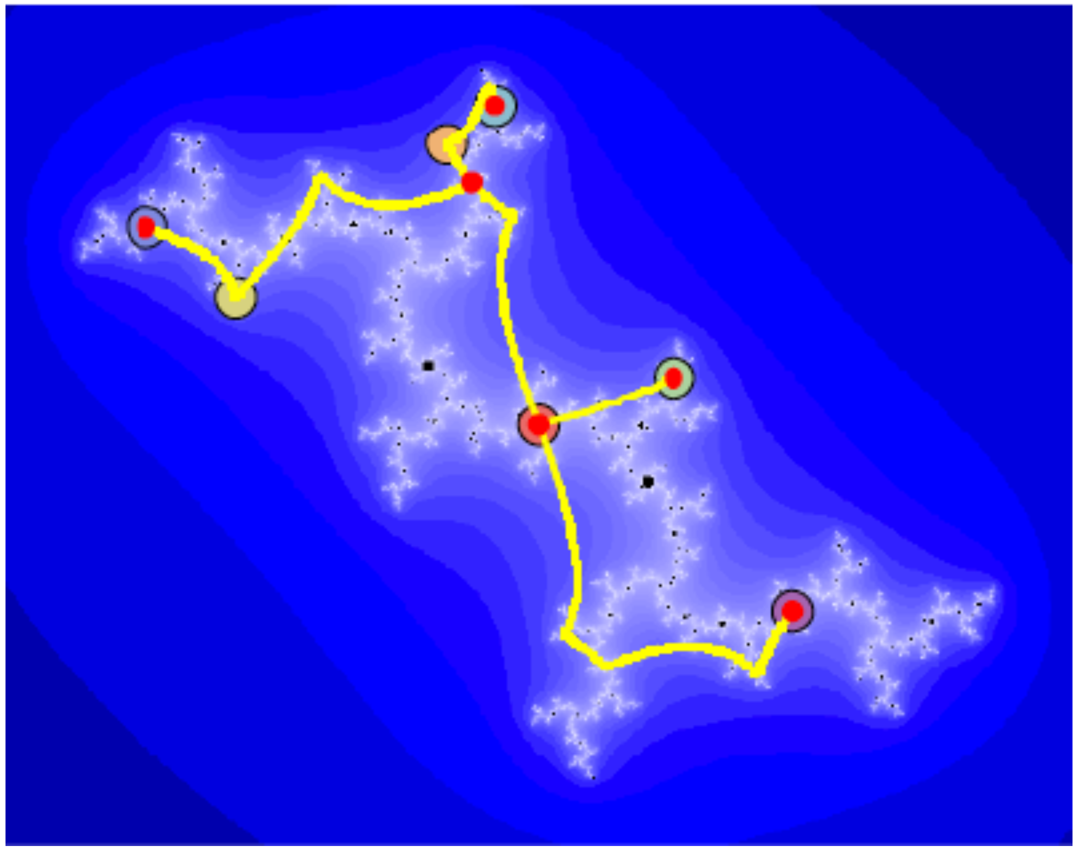

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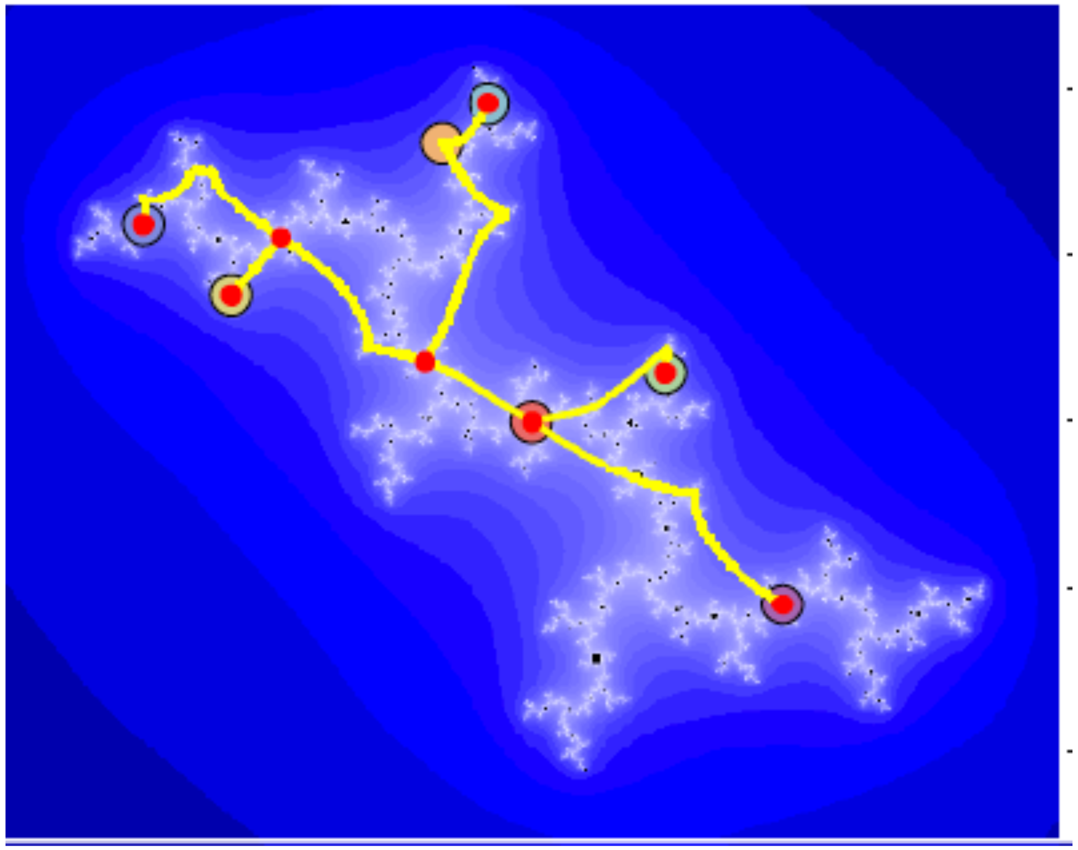

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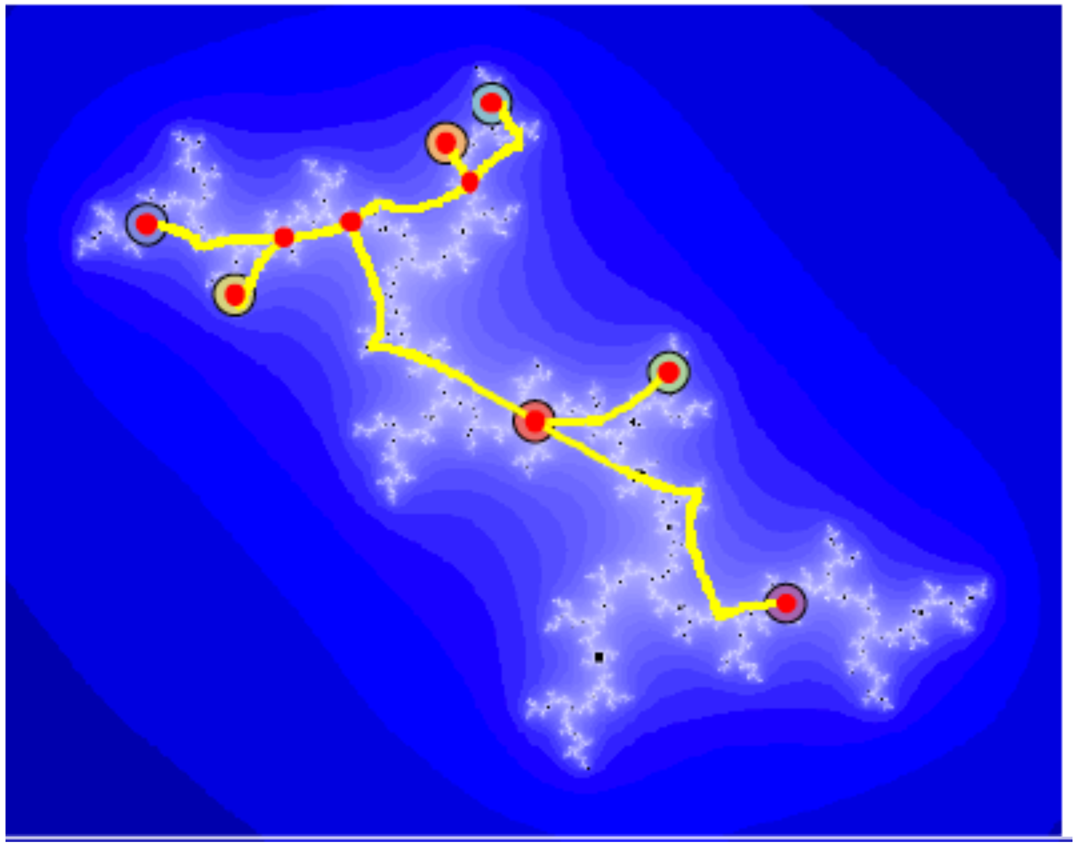

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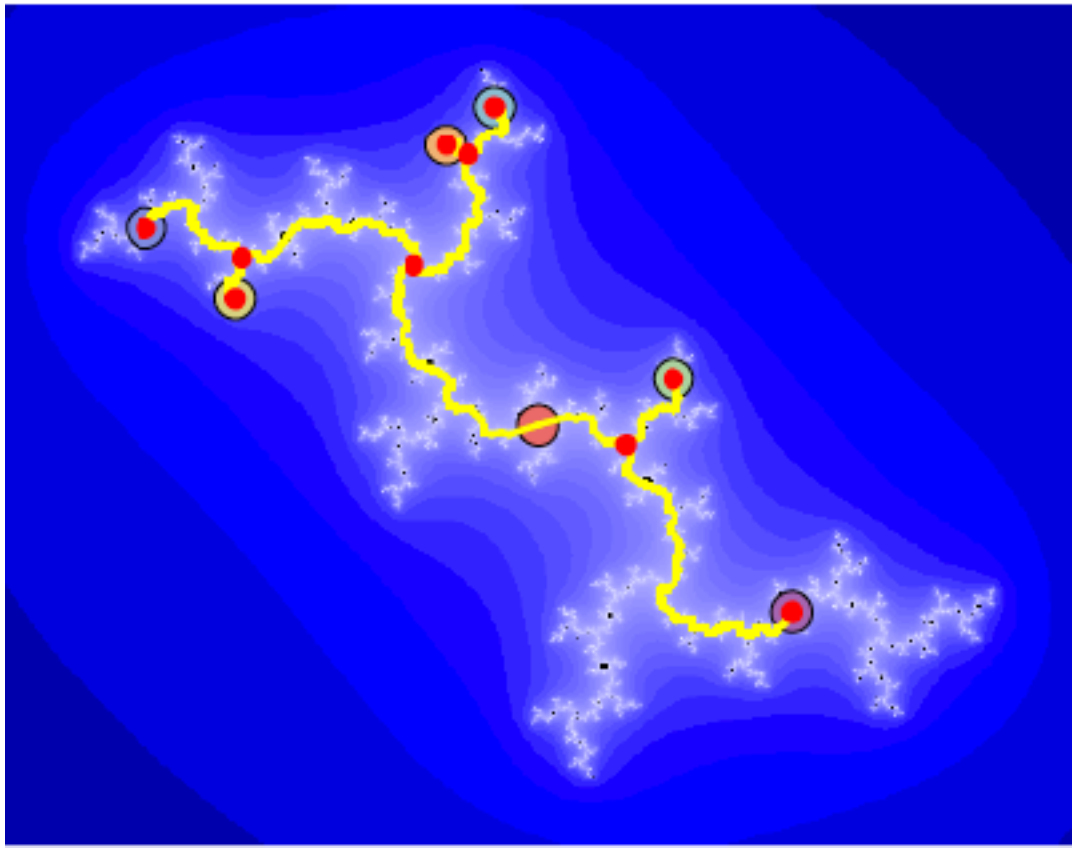

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The general conjectures

Conjecture 1: Given a polynomial p and a tree T, $\{p^{-n}(T)\}$ will converge to the Hubbard tree for p.

Conjecture 2: Given polynomials p_1, p_2, the Hubbard trees and direction of rotation under p_1^{-1}, p_2^{-1} are different.

Lightning Talks II
Tech Topology Conference

December 9, 2017

Towards a new construction of exotic 4-manifolds

Jonathan Simone
University of Virginia

Tech Topology Conference
December 9, 2017

Goal

Cut a nonsimply connected plumbing P with a single cycle out from a 4-manifold X and replace it with a manifold B with small homology (à la the rational blow-down).

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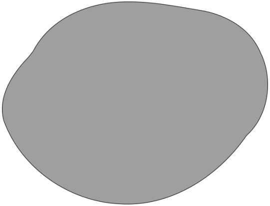

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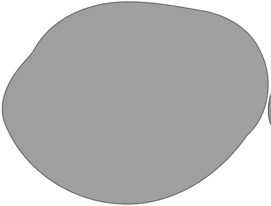
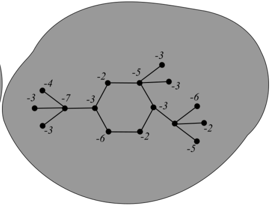


X



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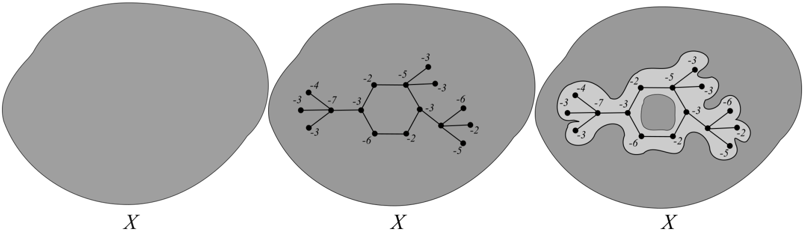

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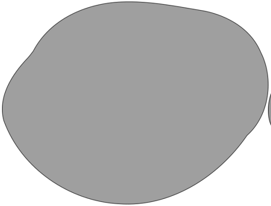

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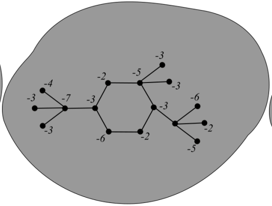

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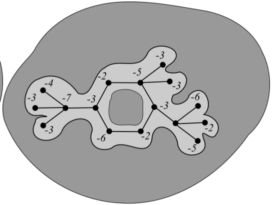

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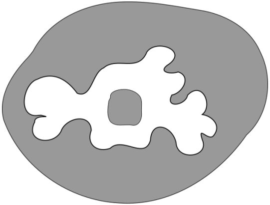

X-P

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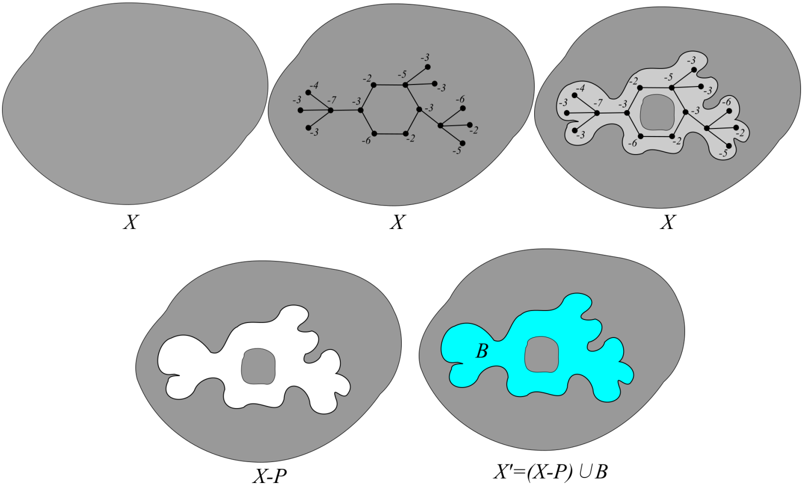

Nonsimply connected plumbings

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Yes, there exists a way to construct many such plumbings.

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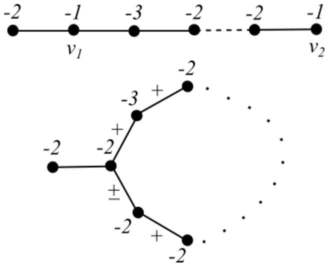

Potentially exotic 4-manifolds

Using this cut-and-paste procedure, there are examples of manifolds $X'_{m,n}$ homeomorphic to $(2m-1)\mathbb{C}P^2 \# n\overline{\mathbb{C}P^2}$ for all $m \geq 1$, $1 \leq n \leq 9m-2$.

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Theorem (S.)

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Under suitable conditions, the Ozsváth-Szabó 4-manifold invariant of X agrees with that of X'.

- If $m \geq 2$, show that $X'_{m,n}$ is symplectic

Symplectic Structures

If $m \geq 2$, then we can circumvent the need for the 4-manifold invariant if $X'_{m,n}$ is symplectic, since $(2m-1)\mathbb{C}P^2 \# n\overline{\mathbb{C}P^2}$ is not symplectic.

Symplectic Structures

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Fact: $X' = (X - C) \cup B$ is symplectic if:

- X is symplectic and C is a symplectic submanifold with convex boundary
- B is symplectic with convex boundary
- The contact structures on $\partial B = \partial C$ induced by the symplectic structures are contactomorphic.

Contact structures

Q: How many tight contact structures does ∂P admit?

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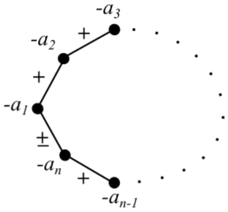
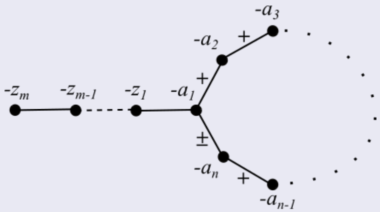
Honda classified the tight contact structures on the boundary of the plumbing depicted below, where $a_i \geq 2$ for all i and $a_1 \geq 3$.



Theorem (S.)

Let $Y_\pm$ be the boundary of the plumbing below, where $a_i, z_j \geq 2$ for all i, j and $a_1 \geq 3$, then, up to isotopy,

- *Y_+ admits exactly $(a_1 - 1)\cdots(a_n - 1)(z_1 - 1)\cdots(z_m - 1)$ Stein fillable contact structures, and*
- *Y_- admits exactly $(a_1 - 1)\cdots(a_n - 1)(z_1 - 1)\cdots(z_m - 1) + z_1(z_2 - 1)\cdots(z_m - 1)$ tight contact structures with no Giroux torsion.*



Proving this relies on a generalization of an important result of Lisca-Matic:

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Theorem (Lisca-Matic)

Let J_1 and J_2 be two Stein structures on a 4-manifold X. If the associated spin^c structures are not isomorphic, then the induced contact structures on ∂X are not isotopic.

Contact structures

Proving this relies on a generalization of an important result of Lisca-Matic:

Theorem (S.)

Suppose (Y, ξ) *is a contact manifold and* $[\omega] \in H^2(Y; \mathbb{R})$ *is an element such that* $c(\xi, [\omega])$ *is nontrivial. Let* (W, J_i) *be a Stein cobordism from* (Y, ξ) *to* (Y', ξ_i) *for* $i = 1, 2$*. If the spin*^{c} *structures induced by* J_1 *and* J_2 *are not isomorphic, then* ξ_1 *and* ξ_2 *are nonisotopic tight contact structures.*

Thank you

Lightning Talks II
Tech Topology Conference

December 9, 2017

A large abelian quotient of the level 4 braid group

Kevin Kordek
joint w/ Dan Margalit
Georgia Tech

Background

- The braid group:

$$B_n = \mathrm{Mod}(\mathbb{D}, p_1, \cdots, p_n)$$

- Integral Burau representation (Burau representation at $t = -1$):

$$\rho_n : B_n \to \mathrm{GL}_n(\mathbb{Z})$$

Definition (The level m braid group)

$$B_n[m] = \ker\left(B_n \xrightarrow{\rho_n} \mathrm{GL}_n(\mathbb{Z}) \to \mathrm{GL}_n(\mathbb{Z}/m\mathbb{Z})\right)$$

- $B_n[m] < B_n$ finite-index.
- Most structure is mysterious.

Background

Some places where these groups pop up:

- Topology/group theory:
  - braid Torelli groups
  - hyperelliptic Torelli groups

- Algebraic geometry: Fundamental groups of
  - finite ("Kummer") covers of $\mathrm{Conf}_n(\mathbb{C})$
  - finite covers of the hyperelliptic loci in M_g

Background

$m = 1$

$B_n[1] = B_n$ *(from definition)*

$m = 2$ (Arnol'd, 1968)

$B_n[2] = PB_n$

- Finite presentations known
- $H^*(-, \mathbb{Q})$ completely determined

$m = 4$ (Brendle-Margalit, 2014)

$B_n[4] = \langle \textit{squares of Dehn twists} \rangle$

Problem

Compute the homology of $B_n[m]$ (especially $B_n[4]$).

Transfer argument: $H < G$ finite-index

$$H_*(H, \mathbb{Q}) \twoheadrightarrow H_*(G, \mathbb{Q})$$

Question

Does $B_n[m]$ have strictly more rational cohomology than B_n?

Theorem (K.-Margalit)

$$\mathit{dim}\ H_1(B_n[4], \mathbb{Q}) = \mathit{dim}\ B_n[4]^{ab} \otimes \mathbb{Q} = 3\binom{n}{4} + 3\binom{n}{3} + \binom{n}{2}$$

for all $n \geq 2$

Compare:

$$\dim\ H_1(B_n, \mathbb{Q}) = 1$$

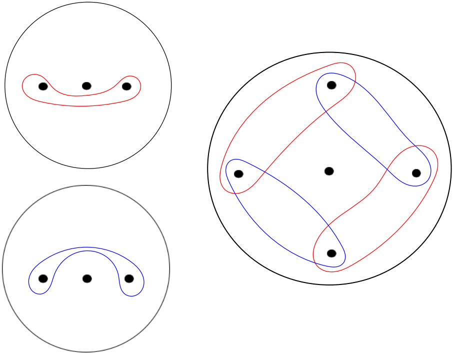
$$\dim\ H_1(B_n[2], \mathbb{Q}) = \binom{n}{2}$$

Question

How does dim $H_1(B_n[m], \mathbb{Q})$ *behave as* $m \to \infty$*?*

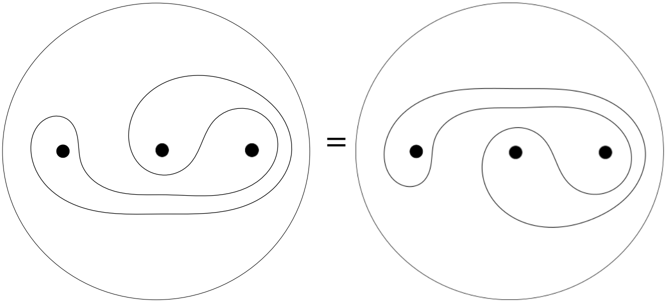
Idea of proof:

- Lower bound: abelian quotients of $B_n[4]$ via covering spaces

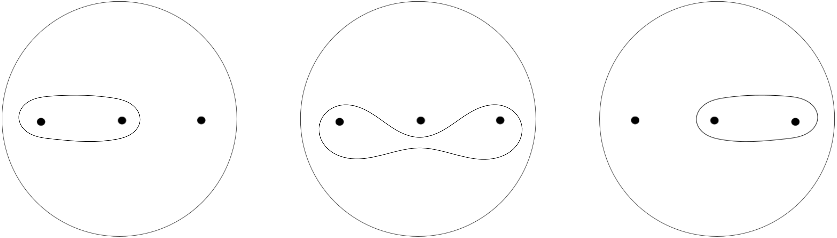


$$B_3[4] \hookrightarrow PB_5 \to H_1(PB_5, \mathbb{Q})$$

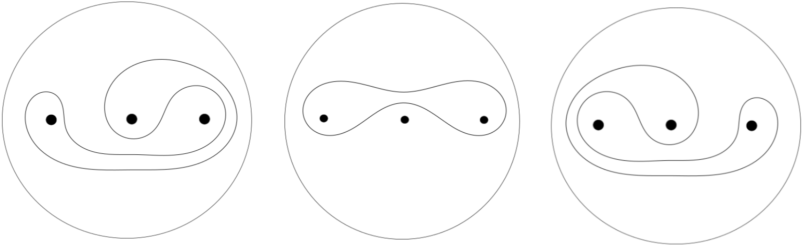
- Upper bound: relations in $B_n[4]$



- Upper and lower bounds agree!



Thank you for your attention!



Lightning Talks II
Tech Topology Conference

December 9, 2017