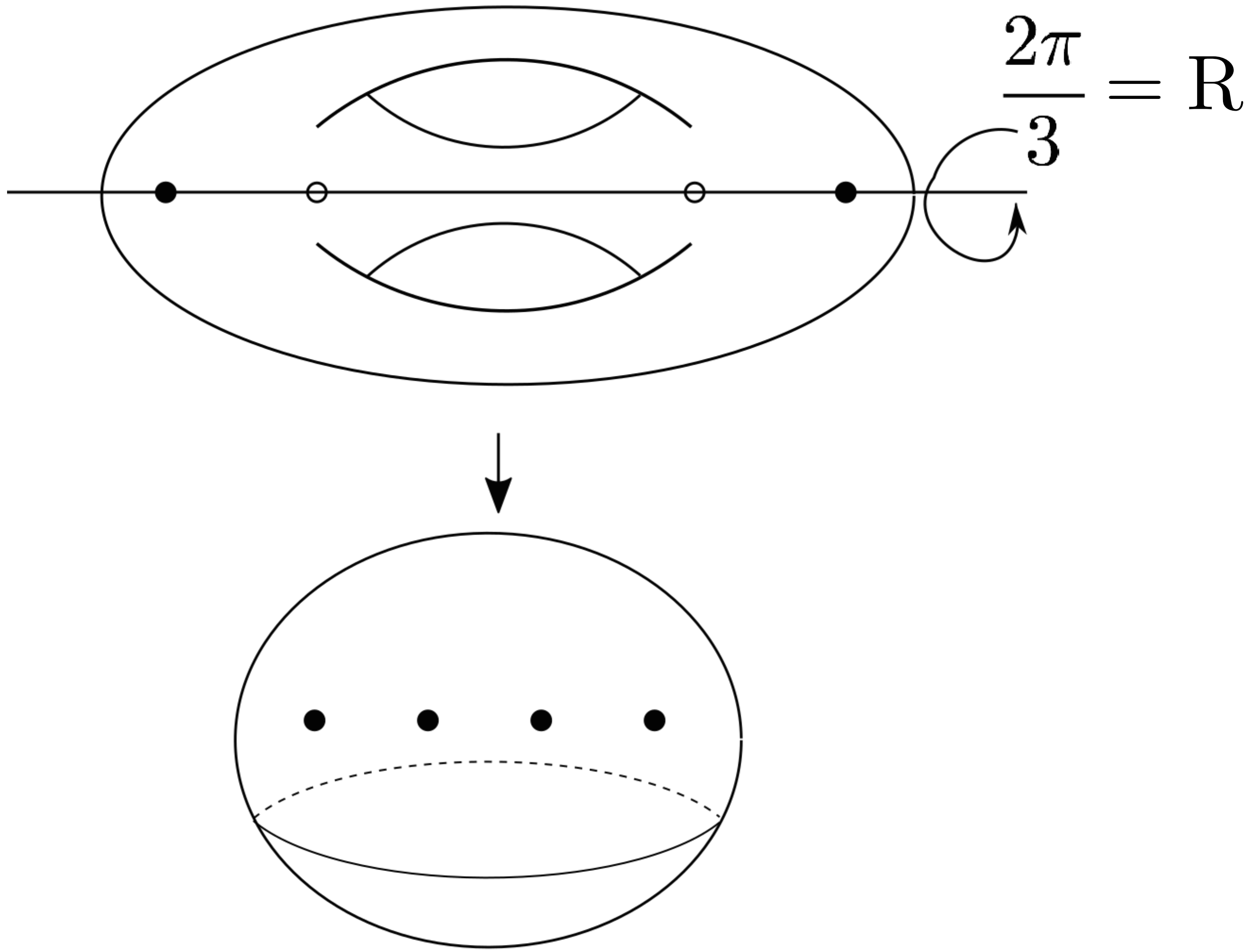
Lightning Talks III
Tech Topology Conference

December 10, 2017

COVERING SPACES, MAPPING CLASS GROUPS, AND THE SYMPLECTIC REPRESENTATION

Sarah Davis
With Laura Stordy, Becca Winarski, Ziyi Zhou
Georgia Institute of Technology
Tech Topology Conference

3-Fold Branched Cover



Symmetric Mapping Class Group

$$\mathrm{SMod}(S_2) = N_{\mathrm{Mod}(S_2)}(\langle R \rangle)$$

Symplectic Representation

$$\Phi : \mathrm{Mod}(S_2) \to \mathrm{Sp}(4, \mathbb{Z})$$

<u>Example:</u>

$$\Phi : R \mapsto E = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

McMullen's Question

Question: Is $\Phi(\mathrm{SMod}(S_g))$ finite index in $N_{\mathrm{Sp}(2g,\mathbb{Z})}(\Phi(\langle R_d \rangle))$?

Venkataramana: Yes, if # branch points ≥ 2*degree

Main Theorem:

$$\Phi(\mathrm{SMod}(S_2)) \qquad N_{\mathrm{Sp}(4,\mathbb{Z})}(\langle E \rangle)$$

Main Theorem:

$$\Phi(\mathrm{SMod}(S_2)) = N_{\mathrm{Sp}(4,\mathbb{Z})}(\langle E \rangle)$$

$$\Phi(\mathrm{SMod}(S_2)) = N_{\mathrm{Sp}(4,\mathbb{Z})}(\langle E \rangle)$$

Find $M \in \mathrm{GL}(4, \mathbb{Z})$ such that:

$$MEM^{-1} = E^{\pm 1}$$

Lemma: It is enough to find M such that:

$$MEM^{-1} = E^{-1}$$

MATLAB Output

$$M = \begin{bmatrix} -z_0 & z_1 - z_2 & z_0 + z_3 & z_1 \\ -z_4 - z_5 & -z_6 & z_4 & z_7 - z_6 \\ z_3 & z_1 & z_0 & z_2 \\ z_4 & z_7 & z_5 & z_6 \end{bmatrix}$$

Refine using the symplectic condition

$$M \Omega M^T = \Omega$$

MATLAB Output

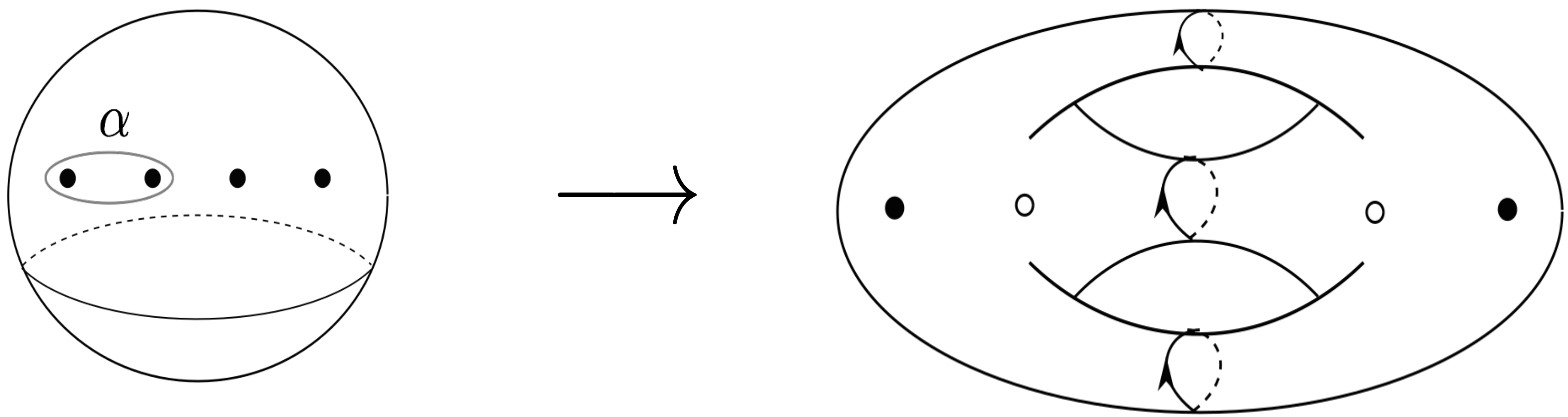
$$\begin{bmatrix} 2x & 1 & -x & 0 \\ -1 & 0 & 0 & 0 \\ x & 0 & -2x & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 2x & -1 & -x & 0 \\ 1 & 0 & 0 & 0 \\ x & 0 & -2x & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -2x & 0 & -x \\ 0 & 0 & 0 & -1 \\ 0 & x & 1 & 2x \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -2x & 0 & -x \\ 0 & 0 & 0 & 1 \\ 0 & x & -1 & 2x \end{bmatrix}$$

$$\begin{bmatrix} x & 1 & x & 1 \\ 0 & 0 & -1 & 0 \\ 2x & 1 & -x & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} x & -1 & x & -1 \\ 0 & 0 & 1 & 0 \\ 2x & -1 & -x & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & x & -1 & 2x \\ 0 & 1 & 0 & 0 \\ -1 & x & 1 & -x \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 & -1 \\ 0 & x & 1 & 2x \\ 0 & -1 & 0 & 0 \\ 1 & x & -1 & -x \end{bmatrix}$$

$$\begin{bmatrix} 0 & x & 1 & 2x \\ 0 & 0 & 0 & 1 \\ 1 & 2x & 0 & x \\ 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & x & -1 & 2x \\ 0 & 0 & 0 & -1 \\ -1 & 2x & 0 & x \\ 0 & -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ x & 0 & -2x & 1 \\ 1 & 0 & 0 & 0 \\ -2x & 1 & x & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 & 0 \\ x & 0 & -2x & -1 \\ -1 & 0 & 0 & 0 \\ -2x & -1 & x & 0 \end{bmatrix}$$

$$\boxed{\Phi(\mathrm{SMod}(S_2))} = N_{\mathrm{Sp}(4,\mathbb{Z})}(\langle E \rangle)$$

Ghaswala-Winarski give generators for $\mathrm{SMod}(S_2)$.



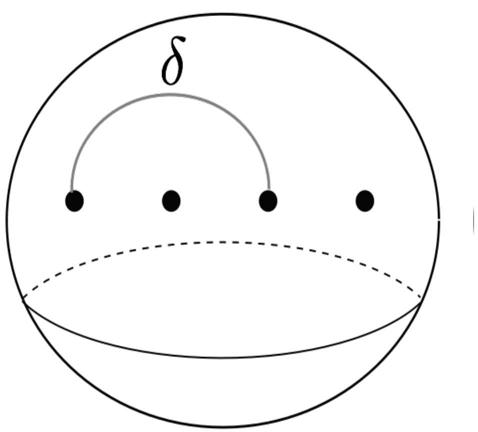
$$\Phi(\widetilde{T}_\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 2 & 1 \end{bmatrix}$$

How can we obtain this matrix from $\Phi(\mathrm{SMod}(S_2))$?

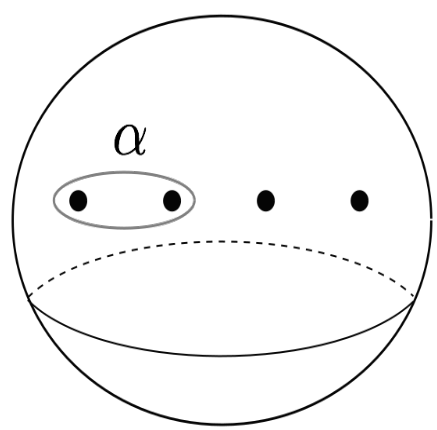
$$\begin{bmatrix} 2x & 1 & -x & 0 \\ -1 & 0 & 0 & 0 \\ x & 0 & -2x & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 1 & -x & 0 \\ -1 & 0 & 0 & 0 \\ x & 0 & -2x & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \Phi\left(\widetilde{H_\delta} \circ \widetilde{T_\alpha}^{\,x} \circ \widetilde{H_c}\right)$$

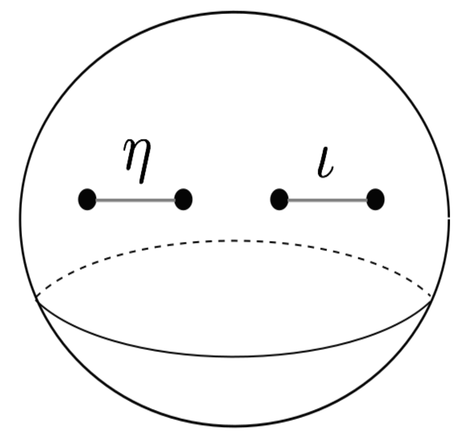
H_δ



T_α



$H_c = H_\eta \circ H_\iota$



Thank you!

Main Theorem:

$$\Phi(\mathrm{SMod}(S_2)) = N_{\mathrm{Sp}(4,\mathbb{Z})}(\langle E \rangle)$$

Lightning Talks III
Tech Topology Conference

December 10, 2017

Salem Number Stretch Factors

Joshua Pankau

University of California, Santa Barbara
Advisor: Darren Long

12/10/2017

I. Background

Definition of pseudo-Anosov map

A homeomorphism ϕ from a closed, orientable surface S to itself is called pseudo-Anosov if there are two transverse, measured foliations, $\mathcal{F}_u$ and $\mathcal{F}_s$, along with a real number $\lambda > 1$, such that ϕ stretches S along $\mathcal{F}_u$ by a factor of λ and contracts S along $\mathcal{F}_s$ by a factor of λ^{-1}. The number λ is known as the stretch factor of ϕ.

I. Background

Definition of pseudo-Anosov map

A homeomorphism ϕ from a closed, orientable surface S to itself is called pseudo-Anosov if there are two transverse, measured foliations, $\mathcal{F}_u$ and $\mathcal{F}_s$, along with a real number $\lambda > 1$, such that ϕ stretches S along $\mathcal{F}_u$ by a factor of λ and contracts S along $\mathcal{F}_s$ by a factor of λ^{-1}. The number λ is known as the stretch factor of ϕ.

Theorem (Thurston 1974)

If λ is the stretch factor of a pseudo-Anosov homeomorphism of a genus g surface, then λ is an algebraic unit such that $[\mathbb{Q}(\lambda) : \mathbb{Q}] \leq 6g - 6$.

I. Background

Definition of pseudo-Anosov map

A homeomorphism ϕ from a closed, orientable surface S to itself is called pseudo-Anosov if there are two transverse, measured foliations, $\mathcal{F}_u$ and $\mathcal{F}_s$, along with a real number $\lambda > 1$, such that ϕ stretches S along $\mathcal{F}_u$ by a factor of λ and contracts S along $\mathcal{F}_s$ by a factor of λ^{-1}. The number λ is known as the stretch factor of ϕ.

Theorem (Thurston 1974)

If λ is the stretch factor of a pseudo-Anosov homeomorphism of a genus g surface, then λ is an algebraic unit such that $[\mathbb{Q}(\lambda) : \mathbb{Q}] \leq 6g - 6$.

Main Question

Which algebraic units can appear as stretch factors?

II. Constructions

There are several general constructions of pseudo-Anosov maps. The following two consist of taking products of Dehn twists.

II. Constructions

There are several general constructions of pseudo-Anosov maps. The following two consist of taking products of Dehn twists.

- Penner's Construction

II. Constructions

There are several general constructions of pseudo-Anosov maps. The following two consist of taking products of Dehn twists.

- Penner's Construction

- Thurston's Construction

II. Constructions

There are several general constructions of pseudo-Anosov maps. The following two consist of taking products of Dehn twists.

- Penner's Construction
- Restriction: Shin and Strenner showed that stretch factors of pseudo-Anosov maps coming from Penner's construction cannot have Galois conjugates on the unit circle.
- Thurston's Construction

II. Constructions

There are several general constructions of pseudo-Anosov maps. The following two consist of taking products of Dehn twists.

- Penner's Construction
- Restriction: Shin and Strenner showed that stretch factors of pseudo-Anosov maps coming from Penner's construction cannot have Galois conjugates on the unit circle.

- Thurston's Construction
- Restriction: Veech showed that if λ is the stretch factor of a pseudo-Anosov map coming from Thurston's construction then $\lambda + \lambda^{-1}$ is a totally real algebraic integer.

III. Salem numbers

Salem number

A real algebraic unit, $\lambda > 1$, is called a Salem number if λ^{-1} is a Galois conjugate, and all other conjugates lie on the unit circle.

III. Salem numbers

Salem number

A real algebraic unit, $\lambda > 1$, is called a Salem number if λ^{-1} is a Galois conjugate, and all other conjugates lie on the unit circle.

Theorem A (P. 2017)

Given a Salem number λ, there are positive integers k, g such that λ^k is the stretch factor of a pseudo-Anosov homeomorphism $\phi : S_g \to S_g$, where ϕ arises from Thurston's construction. Moreover, g depends only on the degree of λ over $\mathbb{Q}$.

IV. Connecting Salem numbers to Thurston's construction

Thurston's construction requires a collection of curves that cut the surface into disks. The intersection matrix of these curves also plays a crucial role.

IV. Connecting Salem numbers to Thurston's construction

Thurston's construction requires a collection of curves that cut the surface into disks. The intersection matrix of these curves also plays a crucial role.

Theorem (P. 2017)

Every Salem number λ has a power k such that $\lambda^k + \lambda^{-k}$ is the dominating eigenvalue of an invertible, positive, symmetric, integer matrix.

IV. Connecting Salem numbers to Thurston's construction

Thurston's construction requires a collection of curves that cut the surface into disks. The intersection matrix of these curves also plays a crucial role.

Theorem (P. 2017)

Every Salem number λ has a power k such that $\lambda^k + \lambda^{-k}$ is the dominating eigenvalue of an invertible, positive, symmetric, integer matrix.

Theorem (P. 2017)

Given an invertible, positive, integer matrix Q, there is a closed, orientable surface S along with a collection of curves that cut S into disks, such that the intersection matrix of those curves is Q.

V. Totally real number fields

Methods and results used to prove Theorem A can be adapted to prove the following:

Theorem B (P. 2017)

Every totally real number field is of the form $K = \mathbb{Q}(\lambda + \lambda^{-1})$ where λ is the stretch factor of a pseudo-Anosov map coming from Thurston's construction.

V. Totally real number fields

Methods and results used to prove Theorem A can be adapted to prove the following:

Theorem B (P. 2017)

Every totally real number field is of the form $K = \mathbb{Q}(\lambda + \lambda^{-1})$ where λ is the stretch factor of a pseudo-Anosov map coming from Thurston's construction.

Thank you!

Lightning Talks III
Tech Topology Conference

December 10, 2017

Why are there are so many spectral sequences from Khovanov homology?

Adam Saltz (University of Georgia)

December 10, 2017

Georgia Tech
Tech Topology Conference

Spectral sequences galore

Theorem (Ozsváth, Szabó; Bloom; Scaduto; Daemi; Kronhemier, Mrowka)

Let L be a link in S^3. Let $\Sigma(L)$ be the double cover of S^3 branched along L. There are spectral sequences

Khovanov homology → *Heegaard / Monopole / Framed instanton / Plane / Singular instanton* *Floer homology of $\Sigma(L)$*

Khovanov homology → *Lee / Bar-Natan / Szabó* *link homology*

I am missing a few words like "mirror of" and "reduced."

Spectral sequences galore

Theorem (Ozsváth, Szabó; Bloom; Scaduto; Daemi; Kronhemier, Mrowka)

Let L be a link in S^3. Let $\Sigma(L)$ be the double cover of S^3 branched along L. There are spectral sequences

Khovanov homology → *Heegaard / Monopole / Framed instanton / Plane / Singular instanton* *Floer homology of $\Sigma(L)$*

Khovanov homology → *Lee / Bar-Natan / Szabó* *link homology*

I am missing a few words like "mirror of" and "reduced."

Khovanov-Floer theories

Definition (Baldwin, Hedden, and Lobb)

A <u>Khovanov-Floer theory</u> is a gadget:

$$\mathcal{D} \rightsquigarrow E^i(\mathcal{D})$$

link diagram — spectral sequence

$$\mathcal{D} \xrightarrow[\text{attachment}]{\text{one-handle}} \mathcal{D}' \rightsquigarrow F_i \colon E^i(\mathcal{D}) \to E^i(\mathcal{D}')$$

map of spectral sequences

- $E^2(\mathcal{D}) = \mathrm{Kh}(\mathcal{D})$
- F_2 agrees with the standard map $\mathrm{Kh}(\mathcal{D}) \to \mathrm{Kh}(\mathcal{D}')$.
- Künneth formula, etc.

Khovanov-Floer theories: the good

Theorem (Baldwin, Hedden, Lobb)

All of the homology theories from the second slide are Khovanov-Floer theories.

Theorem (Baldwin, Hedden, Lobb)

Khovanov-Floer theories are

- *link invariants.*
- *functorial: they assign maps to isotopy classes of link cobordisms in $S^3 \times I$.*

Everything that works for Khovanov homology works for Khovanov-Floer theories because that's how maps on spectral sequences work.

Two maps on homology!

one-handle attachment

filtered chain map F

$F_i \colon E^i(\mathcal{D}) \to E^i(\mathcal{D}')$

$\mathbf{F}_\infty$

$\mathbf{F}_*$

A priori, $F_* \neq F_\infty$!

A DIFFERENT APPROACH

Definition

A strong Khovanov-Floer theory is a gadget:

$$\underset{\text{link diagram}}{\mathcal{D}} \rightsquigarrow \underset{\text{filtered complex}}{\mathcal{K}(\mathcal{D})}$$

$$\underset{\text{handle attachment}}{\mathcal{D} \longrightarrow \mathcal{D}'} \rightsquigarrow \underset{\text{filtered chain map}}{F\colon \mathcal{K}(\mathcal{D}) \to \mathcal{K}(\mathcal{D}')}$$

so that

- For a crossingless diagrams, $H(\mathcal{K}(\mathcal{D}))$ agrees with $\mathrm{Kh}(\mathcal{D})$ (or another Frobenius algebra).
- Handle attachment maps satisfy some relations (e.g. swapping distant handles, Bar-Natan's *S*, *T*, and *4Tu*)
- Künneth formula, etc.

Strong Khovanov-Floer theories: the good

Definition

A strong Khovanov-Floer theory is conic if, for $\mathcal{D}$ with crossings,

$$\mathcal{K} = \mathrm{cone}(\mathfrak{h}\colon\ \mathcal{D}_0 \to \mathcal{D}_1)$$

where $\mathfrak{h}$ is a one-handle attachment map.

Strong Khovanov-Floer theories: the good

Definition

A strong Khovanov-Floer theory is conic if, for $\mathcal{D}$ with crossings,

$$\mathcal{K} = \text{cone}(\mathfrak{h} \colon \mathcal{D}_0 \to \mathcal{D}_1)$$

where $\mathfrak{h}$ is a one-handle attachment map.

Theorem (S.)

Conic strong Khovanov-Floer theories are

- *link invariants. (chain homotopy type)*
- *functorial: they assign (chain homotopy types of) maps to isotopy classes of link cobordisms in $S^3 \times I$.*

Everything that works for Bar-Natan's cobordism-theoretic construction of link homology works for strong Khovanov-Floer theories.

Strong Khovanov-Floer theories: the good

Theorem (S.)

Heegaard Floer homology, singular instanton homology, Szabó homology, and Lee/Bar-Natan homology all produce conic strong Khovanov-Floer theories. (The rest probably are, too.)

Theorem (S.)

A conic strong Khovanov-Floer theory yields a Khovanov-Floer theory.

Strong Khovanov-Floer theories: what's next

🤔

How does this help us understand invariants of transverse links and contact structures?

🤔

What other link homology theories can we use besides Khovanov homology? (E.g. Lin has constructed a spectral sequence from Bar-Natan-Lee homology to monopole Floer homology)

🙏🤔🙏

Can we understand e.g. Heegaard Floer homology via Morse theory on surfaces?

Lightning Talks III
Tech Topology Conference
December 10, 2017

Least Dilatation of Pure Surface Braids

Marissa Loving
University of Illinois at Urbana-Champaign

What?

Why?

How?

Who?

When?

What?

Why?

How?

Who? Me!

When? Now!

What?

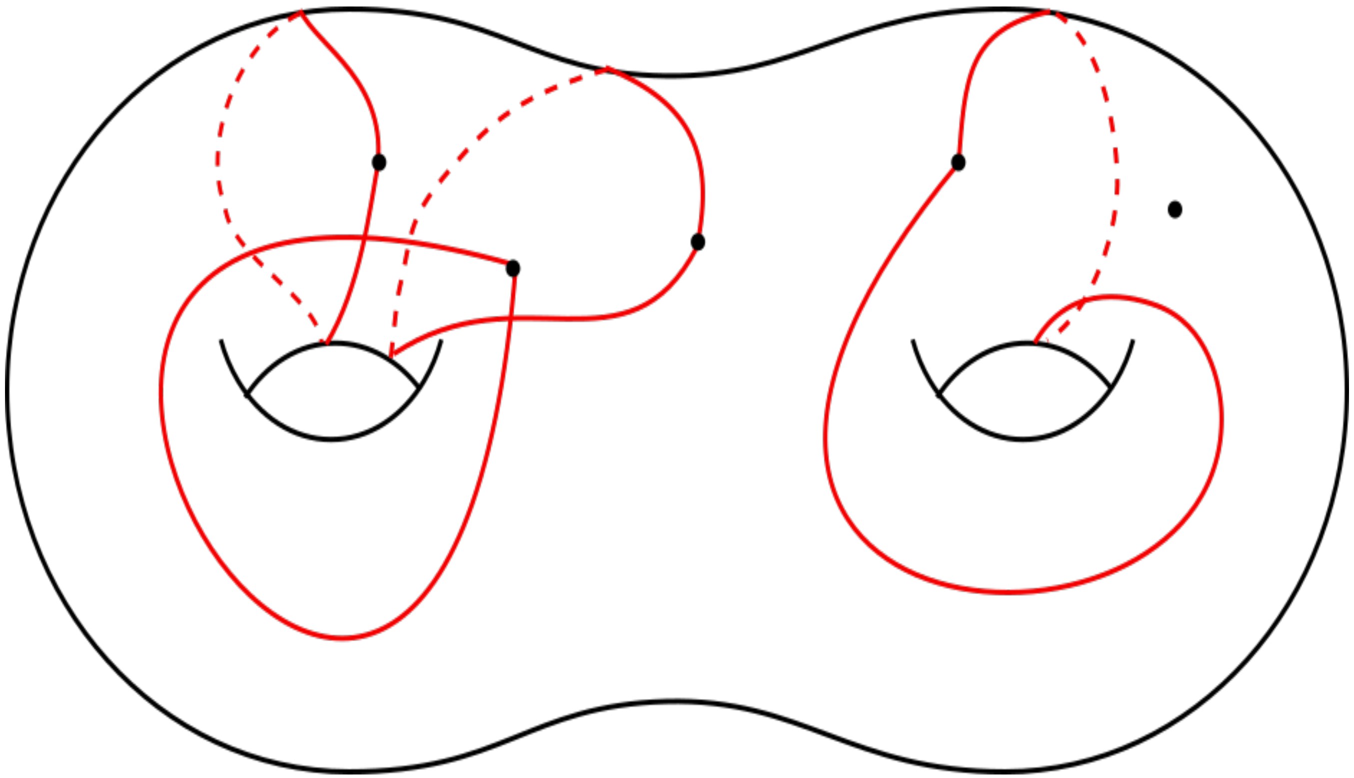
Why?

How?

Who? Me!

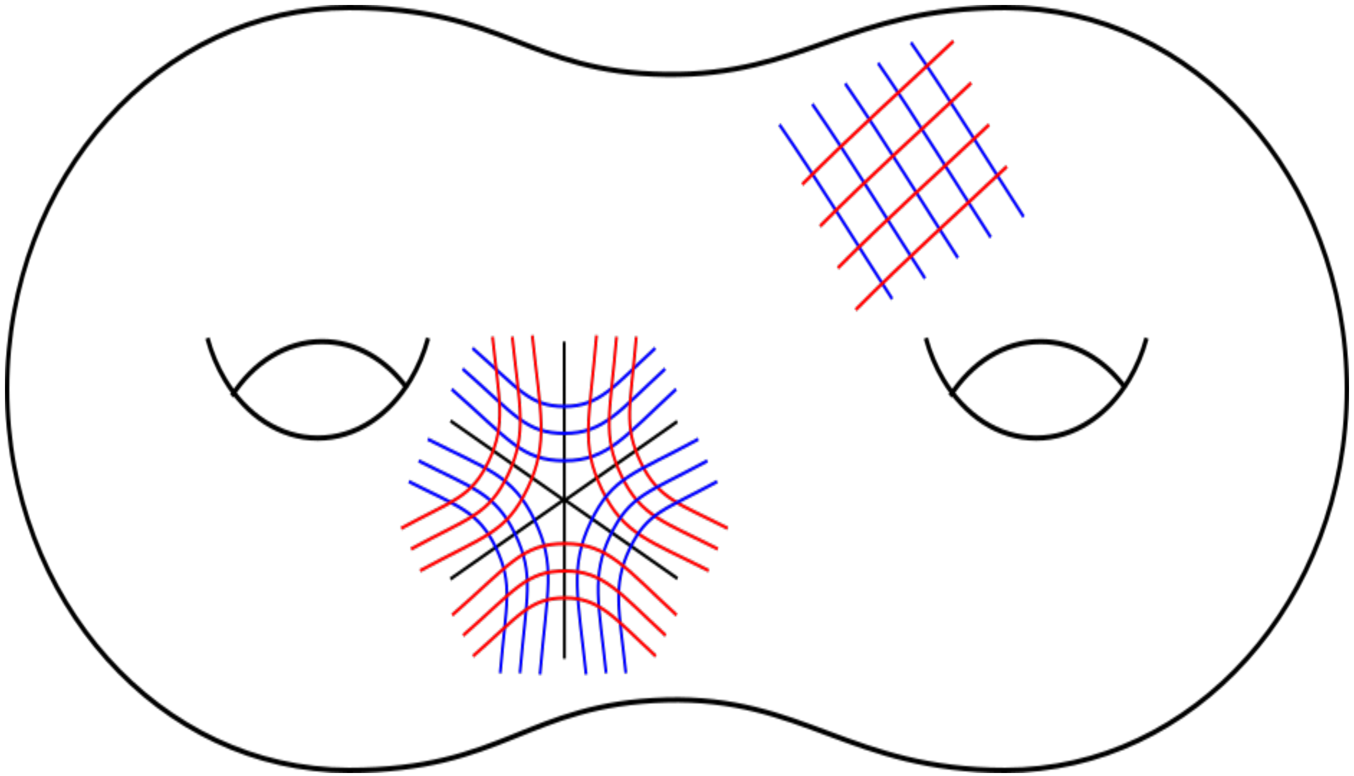
When? Now!

What *are pure surface braids?*



- pure mapping classes
- isotopic to the identity on the closed surface
- denoted $PB_n(S_g)$

What *is the dilatation?*



- a real number > 1
- associated to a mapping class f
- denoted $\lambda(f)$

What *did I prove*?

Theorem (L., 2017)

$$c \log \left\lceil \frac{\log g}{n} \right\rceil + c \ \leq L\left(PB_n(S_g)\right) \leq c' \log \left\lceil \frac{g}{n} \right\rceil + c'$$

Why *should we care?*

Theorem (Penner, 1991)

$L(\mathrm{Mod}(S_g))$ goes to zero as g goes to infinity.

Theorem (Farb-Leininger-Margalit, 2008)

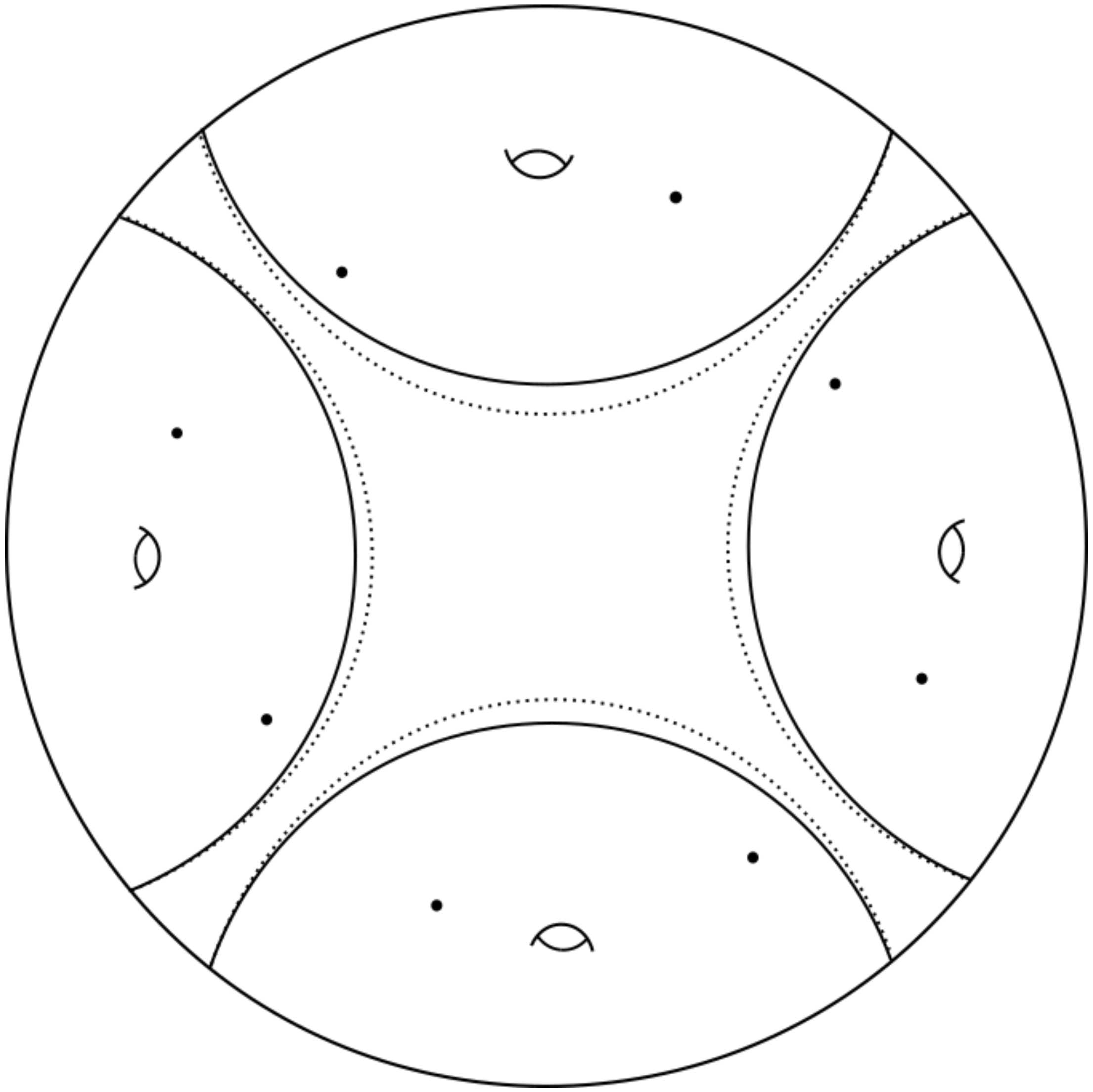
$L(I_g)$ is universally bounded between 0.197 and 4.127.

Theorem (Dowdall, Aougab—Taylor)

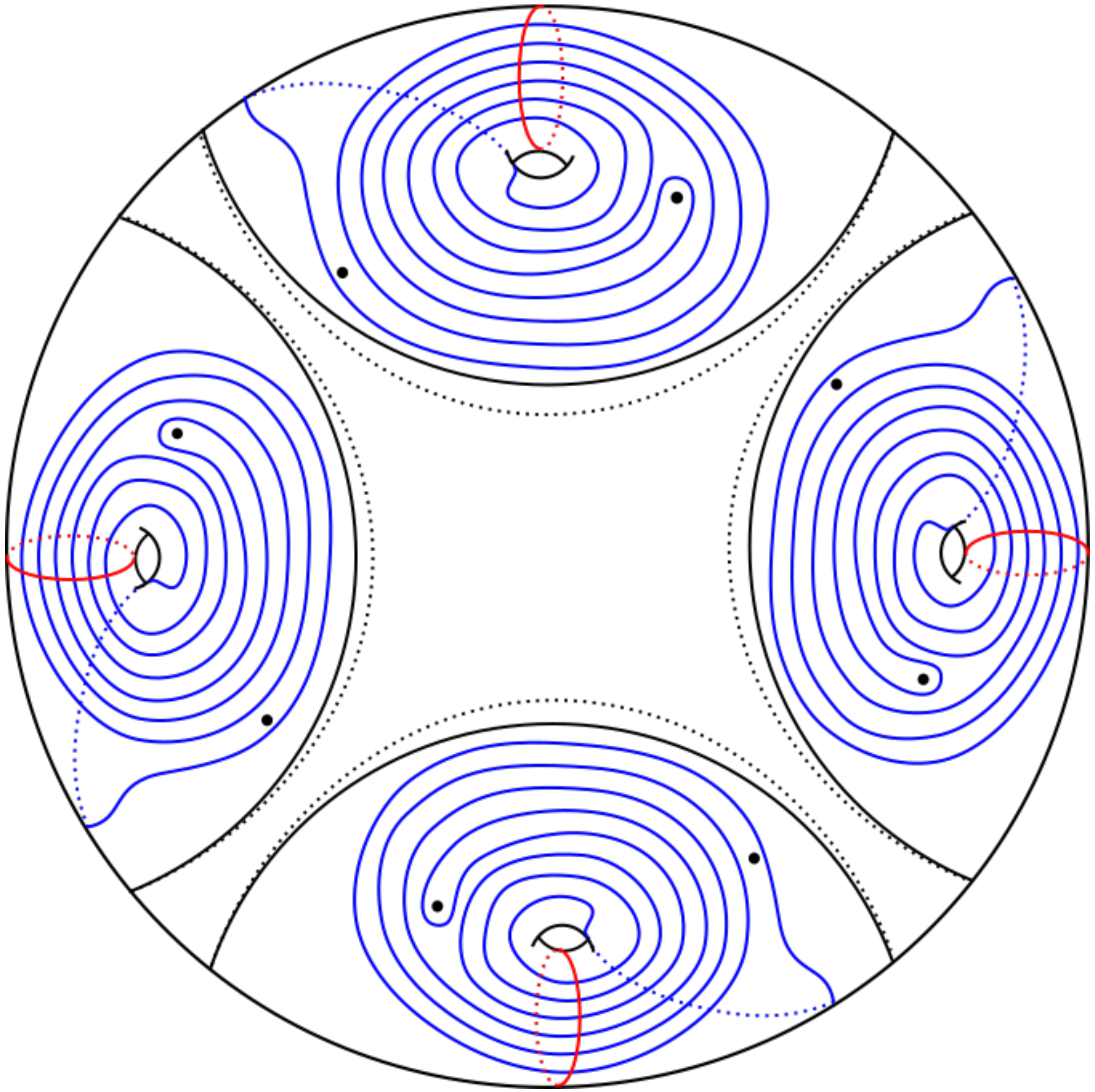
$$\frac{1}{5}\log(2g) \leq L\left(PB_1\left(S_g\right)\right) < 4\log(g) + 2\log(24)$$

How *did I prove it?*

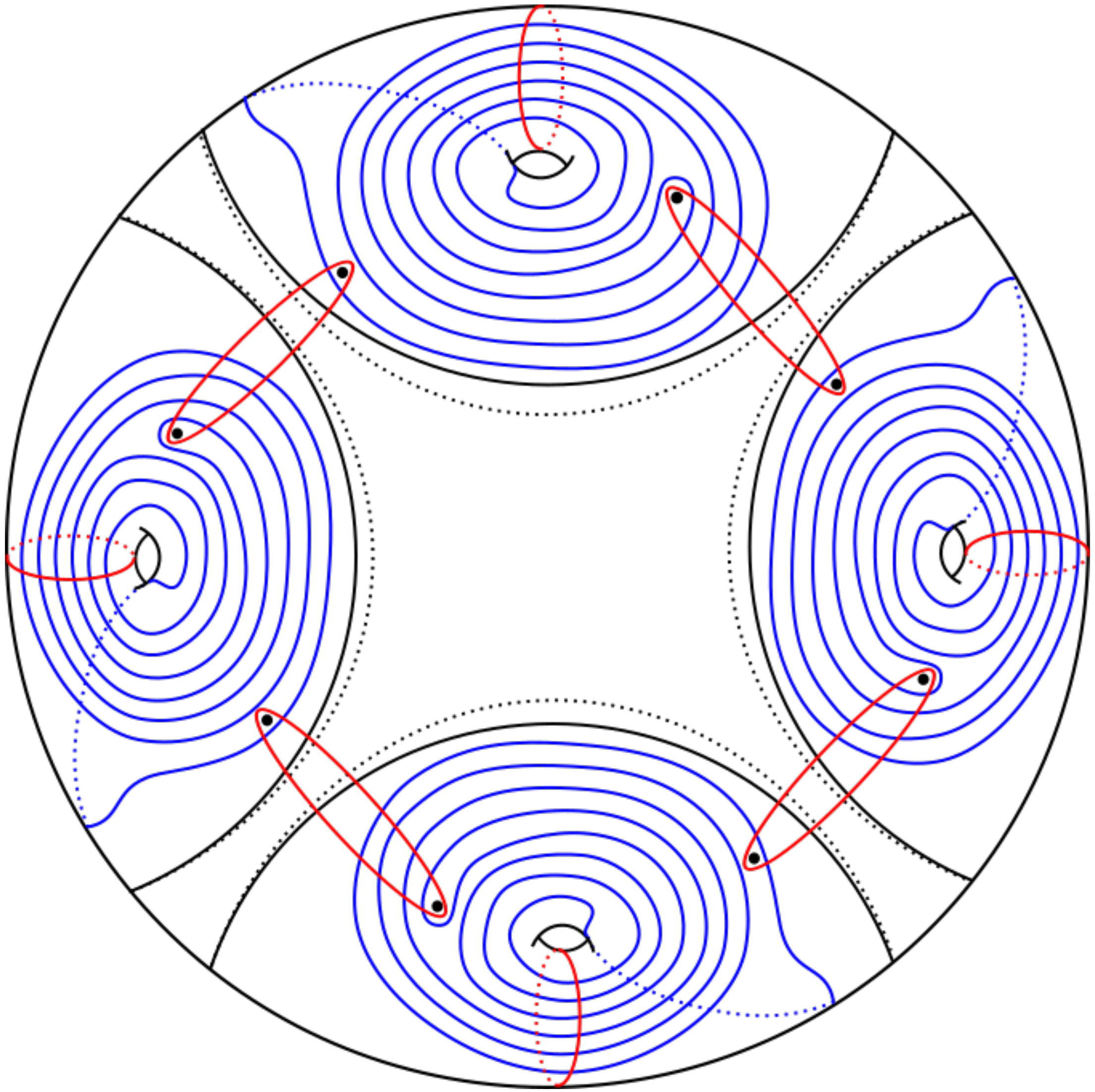
The Upper Bound



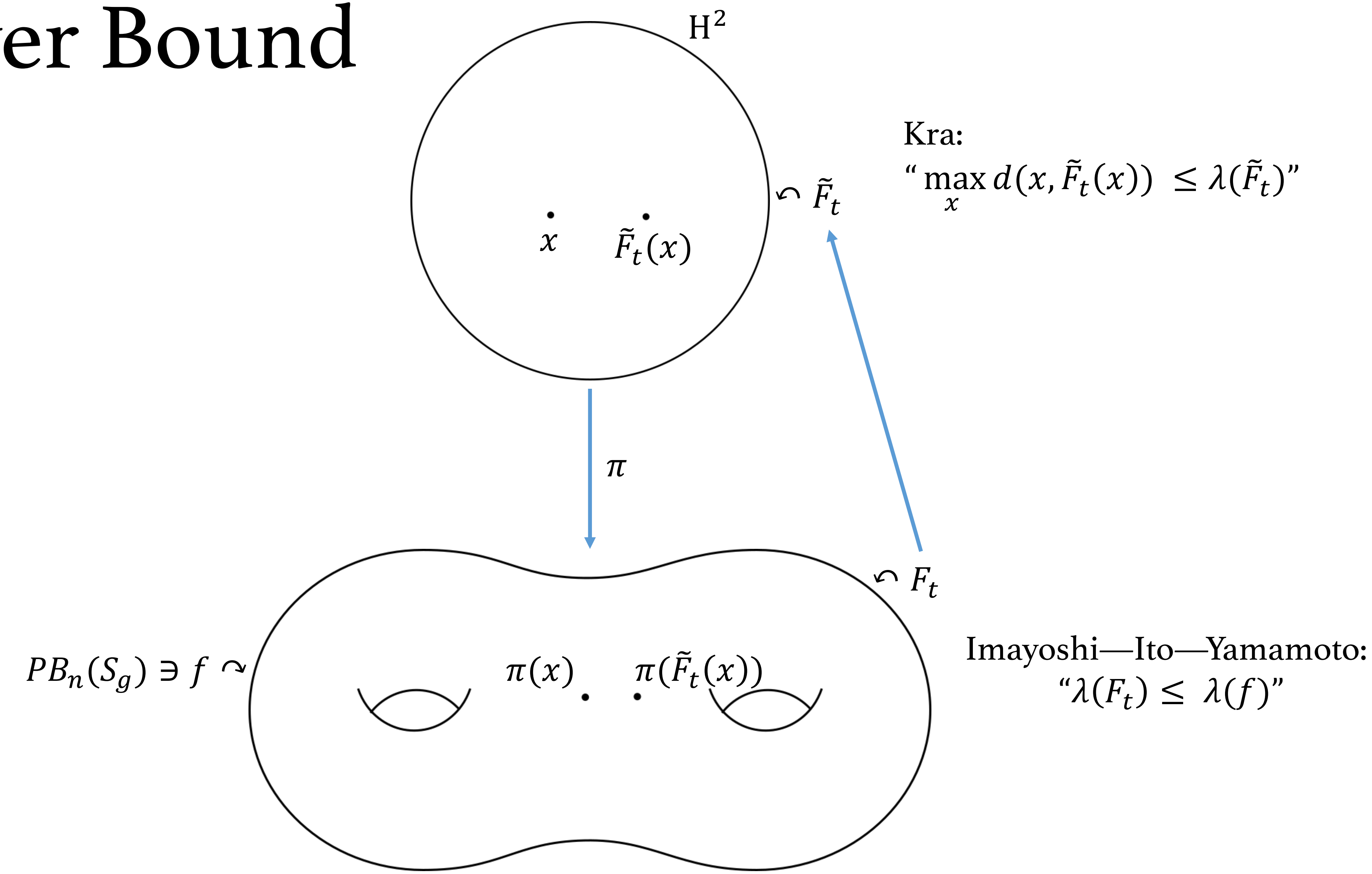
The Upper Bound



The Upper Bound

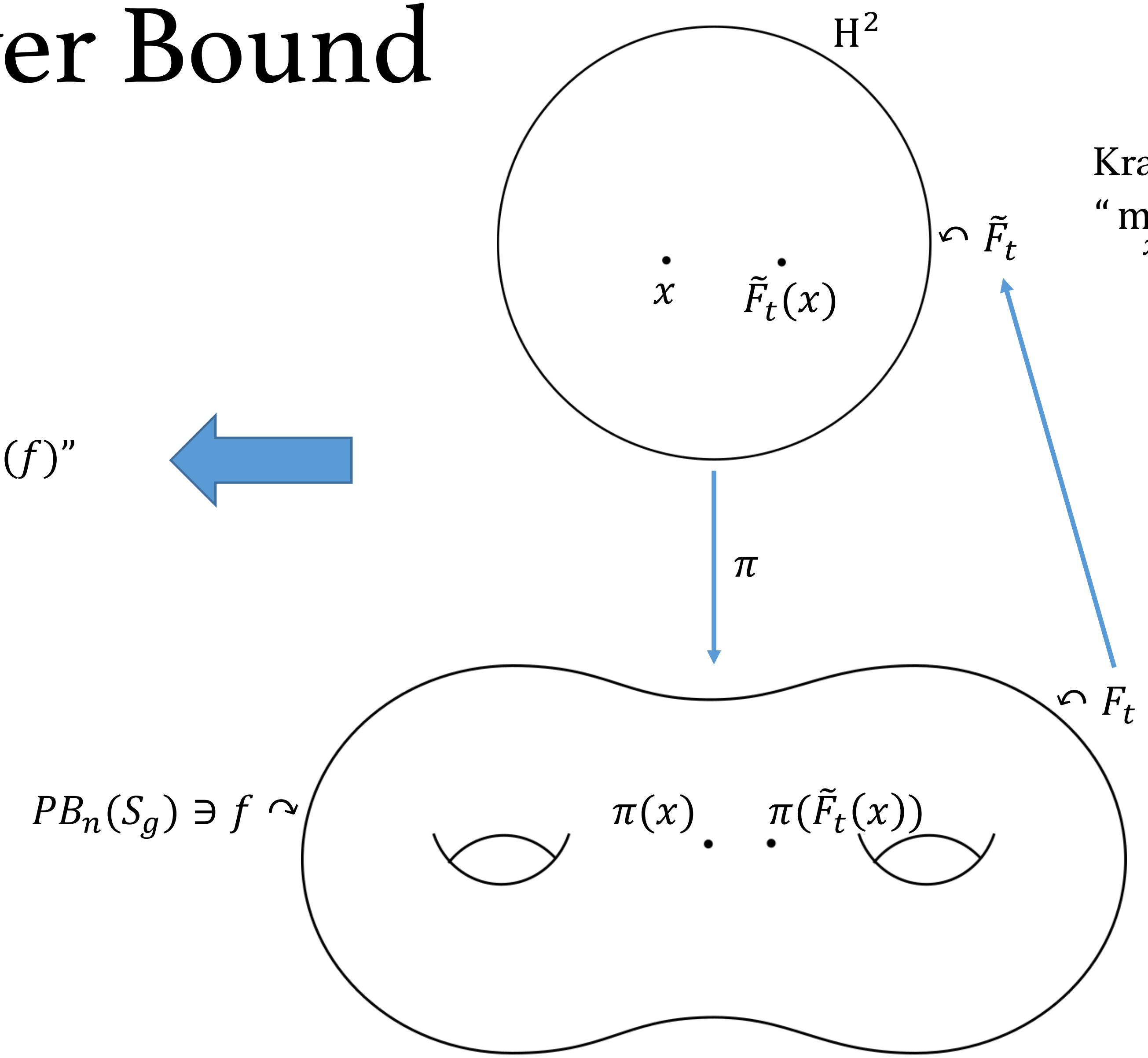


The Lower Bound



The Lower Bound

“ $\max_x \mathrm{d}\big(x, F_t(x)\big) \leq \lambda(f)$”



Kra:
“ $\max_x d(x, \tilde{F}_t(x)) \leq \lambda(\tilde{F}_t)$”

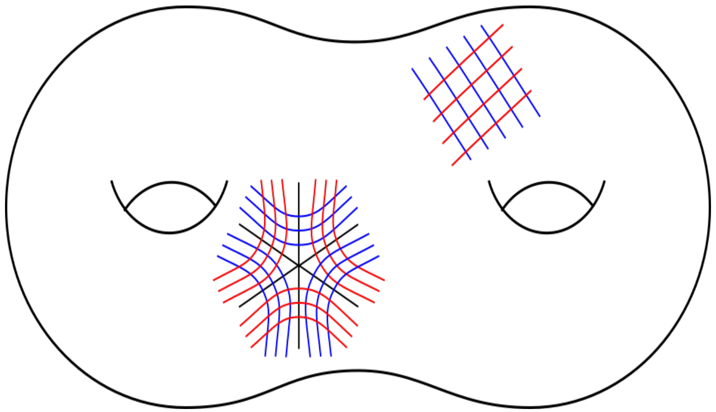
Imayoshi—Ito—Yamamoto:
“$\lambda(F_t) \leq \lambda(f)$”

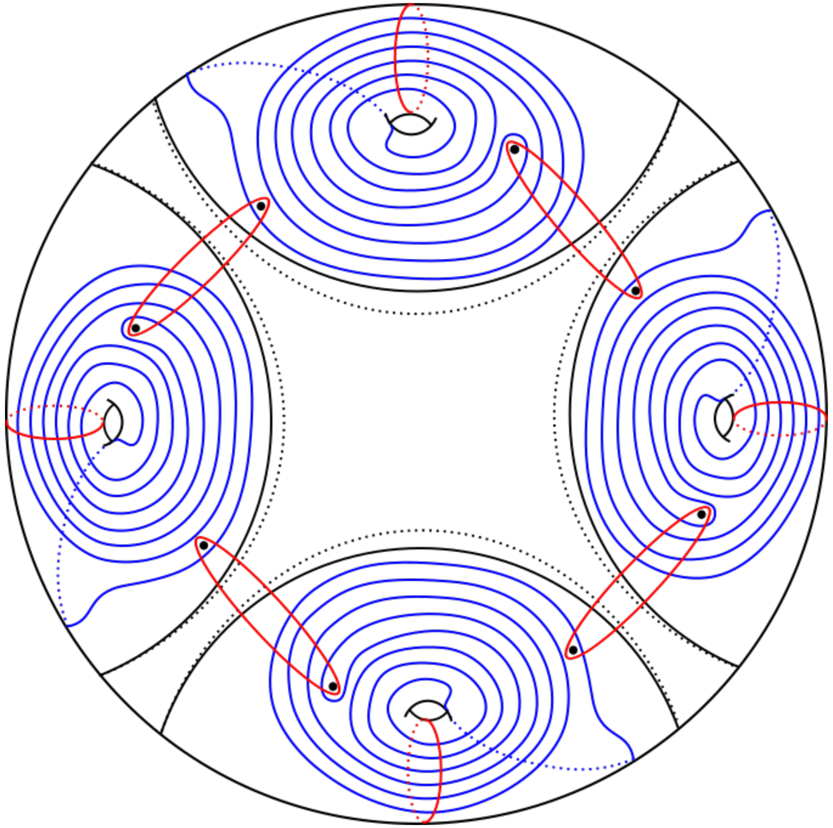
The Lower Bound

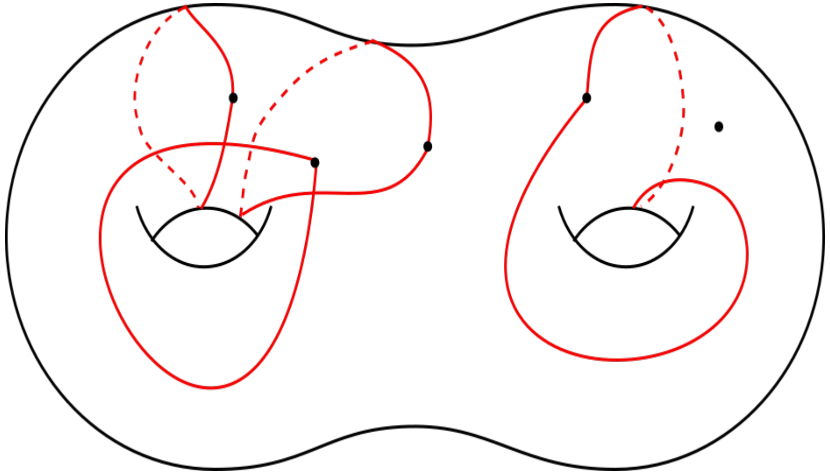
Theorem (L.—Parlier, 2017)

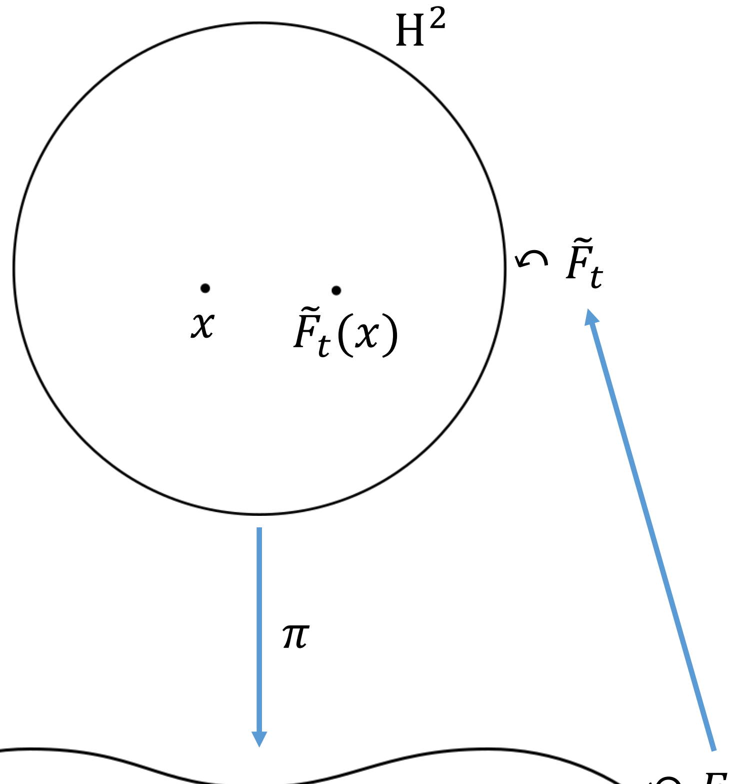
A filling graph Γ embedded in a surface S_g has diameter at least $\frac{\log\left(\frac{g-2}{3}\right)}{40}$.

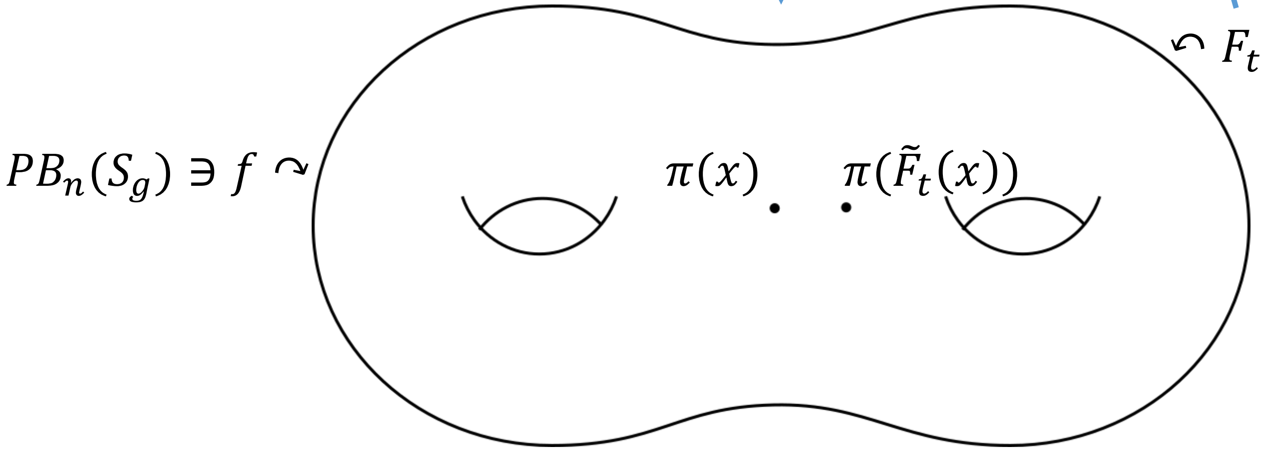
Thank you!








$\mathbb{H}^2$
$\curvearrowleft \tilde{F}_t$
x
$\tilde{F}_t(x)$
π


$\curvearrowleft F_t$
$PB_n(S_g) \ni f \curvearrowright$
$\pi(x)$
$\pi(\tilde{F}_t(x))$

Lightning Talks III
Tech Topology Conference

December 10, 2017

Truncated Heegaard Floer homology and concordance invariants

Linh Truong

Columbia University

Tech Topology Conference, December 2017

Motivation

Our motivation is to better understand knot concordance.

Definition

K_1 and K_2 are **concordant** if they cobound a smooth cylinder in $S^3 \times [0,1]$.

Definition

The **concordance group** is $\mathcal{C} = \{\text{knots in } S^3/\sim, \#\}$, where $K_1 \sim K_2$ if K_1 is concordant to K_2.

Open Questions

There are many open questions about knot concordance.

Question

Is every slice knot a ribbon knot?

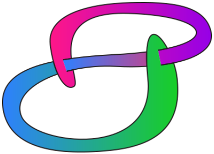


Figure: "Square ribbon knot"; figure by David Eppstein, Wikipedia.

The boundary of a self-intersecting disk with only "ribbon singularities" is called a *ribbon knot*.

Question

Is there any torsion in the concordance group $\mathcal{C}$ besides 2-torsion?

Truncated Heegaard Floer homology

Heegaard Floer homology is an invariant for three-manifolds defined by Ozsváth and Szabó.

Truncated Heegaard Floer homology, denoted $HF^n(Y, \mathfrak{s})$ (Ozsváth-Szabó, Ozsváth-Manolescu), is the homology of the kernel $CF^n(Y, \mathfrak{s})$ of the multiplication map

$$U^n : CF^+(Y, \mathfrak{s}) \to CF^+(Y, \mathfrak{s})$$

where $n \in \mathbb{Z}_+$.

Remark

Note for $n = 1$, truncated Heegaard Floer homology equals $\widehat{HF}(Y, \mathfrak{s})$.

Truncated Concordance Invariants

Motivated by the constructions of the Ozsváth-Szabó $\nu(K)$ and Hom-Wu $\nu^+(K)$, we construct a sequence of knot invariants $\nu_n(K)$, $n \in \mathbb{Z}$:

Definition

For $n > 0$, define

$$\nu_n(K) = \min\{s \in \mathbb{Z} \mid v_s^n : CF^n(S^3_N(K), \mathfrak{s}_s) \to CF^n(S^3) \text{ induces a surjection on homology}\},$$

where N is sufficiently large so that the Ozsváth-Szabó large integer surgery formula holds, and $\mathfrak{s}_s$ denotes the restriction to $S^3_N(K)$ of a Spin^c structure $\mathfrak{t}$ on the corresponding 2-handle cobordism such that

$$\langle c_1(\mathfrak{t}), [\widehat{F}]\rangle + N = 2s,$$

where $\widehat{F}$ is a capped-off Seifert surface for K.

Truncated Concordance Invariants, continued...

Definition

For $n < 0$, define

$$\nu_n(K) = \max\{s \in \mathbb{Z} \mid v_s^n : CF^{-n}(S^3) \to CF^{-n}(S^3_{-N}(K), \mathfrak{s}_s) \text{ induces an injection on homology}\},$$

where N is sufficiently large so that the Ozsváth-Szabó large integer surgery formula holds, and $\mathfrak{s}_s$ denotes the restriction to $S^3_{-N}(K)$ of a Spin^c structure $\mathfrak{t}$ on the corresponding 2-handle cobordism such that

$$\langle c_1(\mathfrak{t}), [\widehat{F}]\rangle - N = 2s,$$

where $\widehat{F}$ is a capped-off Seifert surface for K.
For $n = 0$, we define $\nu_0(K) = \tau(K)$.

Properties of $\nu_n(K)$

The knot invariants $\nu_n(K)$, $n \in \mathbb{Z}$, satisfy the following properties:

- $\nu_n(K)$ is a concordance invariant.
- $\nu_1(K) = \nu(K)$.
- $\nu_n(K) \leq \nu_{n+1}(K)$.
- For sufficiently large n, $\nu_n(K) = \nu^+(K)$.
- $\nu_n(-K) = -\nu_{-n}(K)$, where $-K$ is the mirror of K.
- $\nu_n(K) \leq g_4(K)$.

Homologically thin knots are knots with $\widehat{HFK}$ supported in a single $\delta = A - M$ grading.

Theorem

Let K be a homologically thin knot with $\tau(K) = \tau$.

(i) *If $\tau = 0$, $\nu_n(K) = 0$ for all n.*

(ii) *If $\tau > 0$,*

$$\nu_n(K) = \begin{cases} 0, & \text{for } n \leq -(\tau+1)/2, \\ \tau + 2n + 1, & \text{for } -\tau/2 \leq n \leq -1, \\ \tau, & \text{for } n \geq 0. \end{cases}$$

(iii) *If $\tau < 0$,*

$$\nu_n(K) = \begin{cases} \tau, & \text{for } n \leq 0, \\ \tau + 2n - 1, & \text{for } 1 \leq n \leq -\tau/2, \\ 0, & \text{for } n \geq (-\tau+1)/2. \end{cases}$$

Large Gaps

In fact, the difference between $\nu_n(K)$ and $\nu_{n+1}(K)$ can be arbitrarily big.

Theorem

Let $T_{p,p+1}$ denote the $(p, p+1)$ torus knot. For $p > 3$,

$$\nu_{-1}(T_{p,p+1}) - \nu_{-2}(T_{p,p+1}) = p.$$

Thank you!

Lightning Talks III
Tech Topology Conference

December 10, 2017

Augmentations and Immersed Exact Lagrangian Fillings

Yu Pan

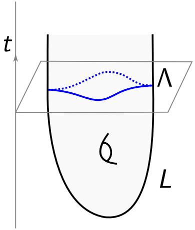
MIT

Tech Topology Conference
Dec. 10th, 2017

Exact Lagrangian fillings

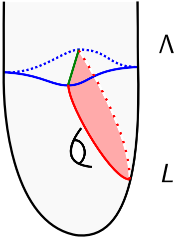
An embedded exact Lagrangian filling of Λ is a 2-dimensional embedded surface L in $(\mathbb{R}_t \times \mathbb{R}^3, \omega = d(e^t\alpha))$ such that

- L is cylindrical over Λ when t is big enough;
- there exists a function $f : L \to \mathbb{R}$ such that $e^t\alpha\big|_{TL} = df$ and f is constant on Λ.



Augmentations

By [Ekholm-Honda-Kálmán, '12],
an exact Lagrangian filling L $\Longrightarrow$ an augmentation ϵ of $\mathcal{A}(\Lambda)$



$$\begin{array}{c} (\mathcal{A}(\Lambda), \partial) \\ \downarrow \epsilon \\ (\mathbb{Z}_2, 0) \end{array}$$

Correspondence

Derived Fukaya Category Augmentation Category

Correspondence

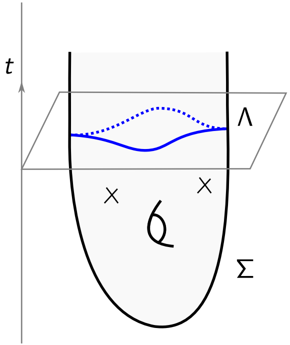
	Derived Fukaya Category	Augmentation Category
Objects:	Exact Lagrangian Fillings	Augmentations

Correspondence

	Derived Fukaya Category	Augmentation Category
Objects:	Exact Lagrangian Fillings	Augmentations

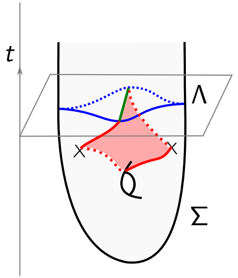
However, not all the augmentations of $\mathcal{A}(\Lambda)$ are induced from embedded exact Lagrangian fillings of Λ.

Immersed Exact Lagrangian fillings



Augmentations induced from immersed exact Lagrangian fillings

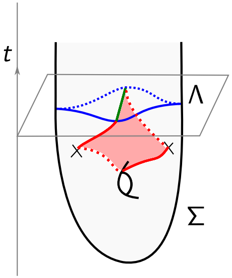
Suppose that Σ can be lifted to an embedded Legendrian surface L in $\mathbb{R} \times \mathbb{R} \times \mathbb{R}^3$ and $\mathcal{A}(L)$ has an augmentation ϵ_L.



$$\begin{array}{c} (\mathcal{A}(\Lambda), \partial) \\ \downarrow f \\ (\mathcal{A}(L), \partial) \end{array}$$

Augmentations induced from immersed exact Lagrangian fillings

Suppose that Σ can be lifted to an embedded Legendrian surface L in $\mathbb{R} \times \mathbb{R} \times \mathbb{R}^3$ and $\mathcal{A}(L)$ has an augmentation ϵ_L.



$$(\mathcal{A}(\Lambda), \partial) \xrightarrow{f} (\mathcal{A}(L), \partial) \xrightarrow{\epsilon_L} (\mathbb{Z}_2, 0)$$

Thus $\epsilon = \epsilon_L \circ f$ is an augmentation of $\mathcal{A}(\Lambda)$.

Result

Theorem (P.-D. Rutherford)

All the augmentations of $\mathcal{A}(\Lambda)$ are induced from possibly immersed exact Lagrangian fillings of Λ.

Lightning Talks III
Tech Topology Conference

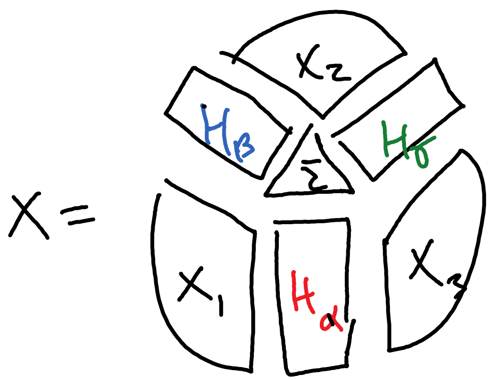
December 10, 2017

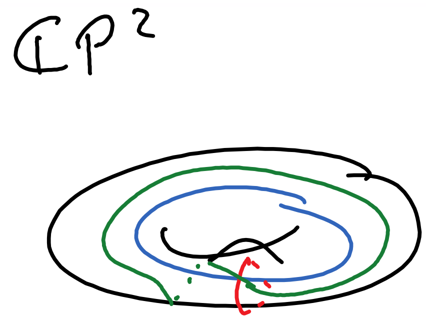
Trisections of Complex Surfaces

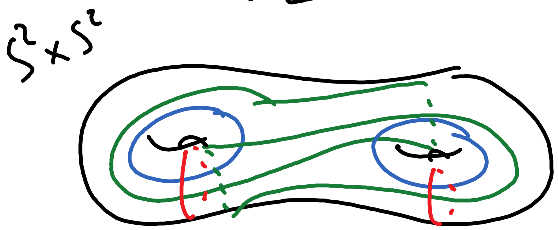
with Jeffrey Meier and Alex Zupan

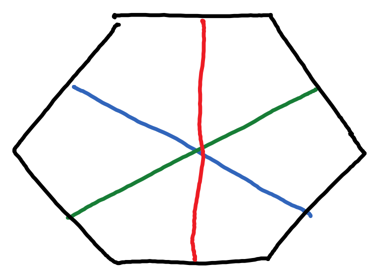
Tech Topology 2017

Trisections of 4-manifolds



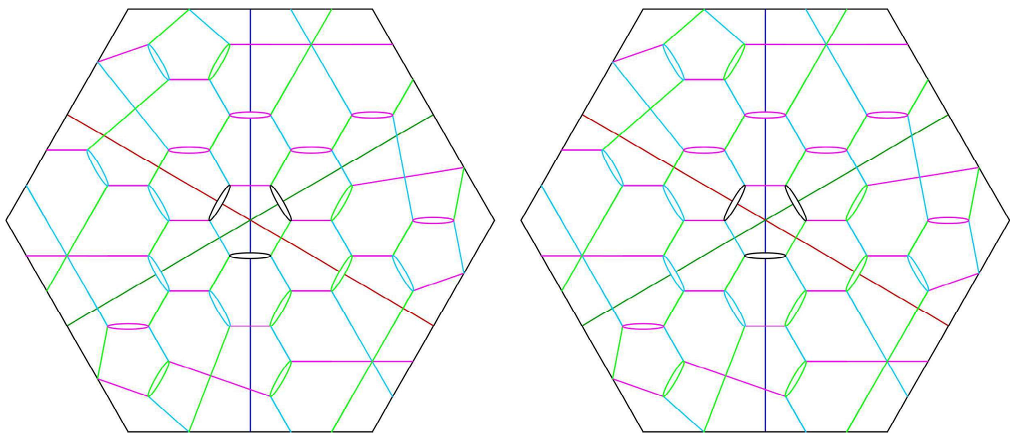






K3

The complex surface K3 is the 2-fold branched cover of $\mathbb{CP}^2$ over a degree 6 curve



Section 0.0 Slide 3

Exotic 4-manifolds

For $d \geq 5$, the degree d hypersurface S_d in $\mathbb{CP}^3$ is an exotic 4-manifold.

S_d is the d-fold branched cover of a degree d curve in $\mathbb{CP}^2$.

There is a homeomorphism $\zeta : \Sigma_{53} \to \Sigma_{53}$ in the Torelli group $\text{Tor}(\Sigma_{53})$ that does not extend across the genus 53 handlebody H_{53} but

$9\mathbb{CP}^2 \# 44\overline{\mathbb{CP}}^2$	S_5
$S^3 \cong H_\alpha \cup_{\phi_1} H_\gamma$	$S^3 \cong H_\alpha \cup_{\zeta \circ \phi_1} H_\gamma$
$\cong H_\beta \cup_{\phi_2} H_\gamma$	$\cong H_\beta \cup_{\zeta \circ \phi_2} H_\gamma$

Lightning Talks III
Tech Topology Conference

December 10, 2017