Three theorems about generating mapping class groups

Justin Lanier
Georgia Tech
(joint with Dan Margalit)

Three theorems about generating mapping class groups: the shock, the hope, and the hunt

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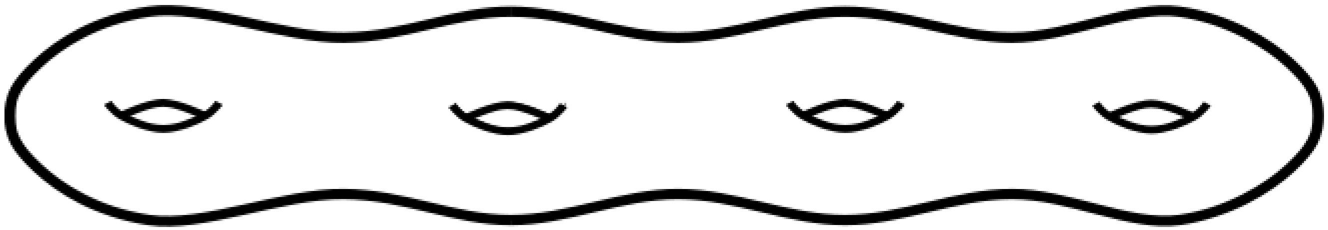
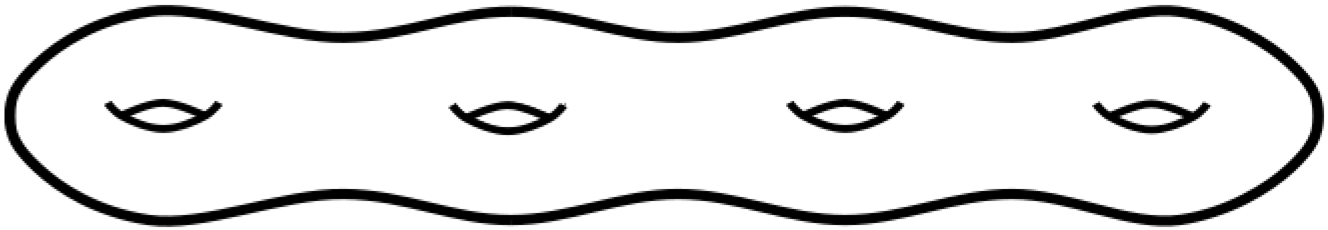
Act 1: the shock



S_g

$$\mathrm{Mod}(S_g) = \mathrm{Homeo}^+(S_g)/\mathit{homotopy}$$



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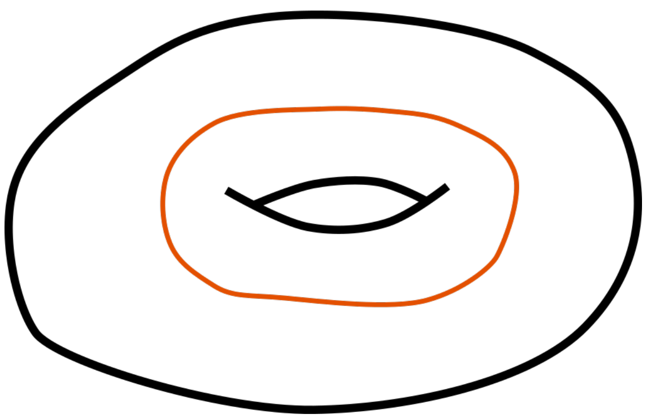
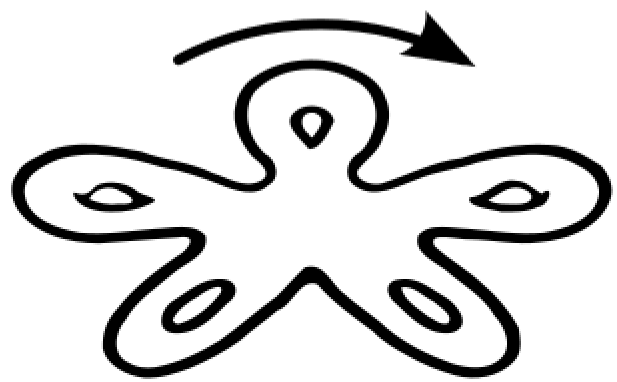
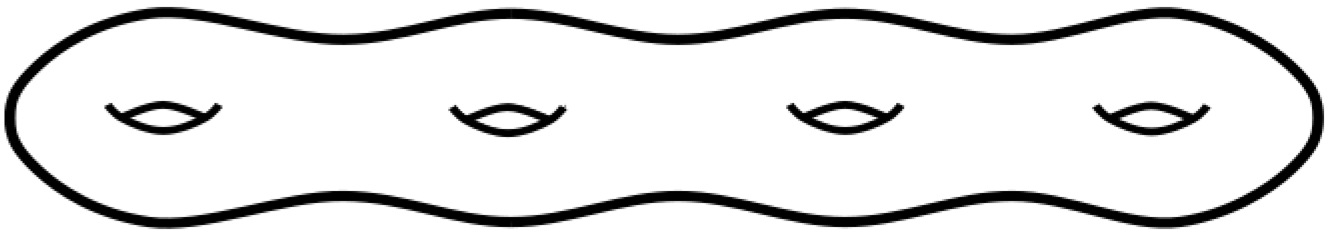


Dehn twist





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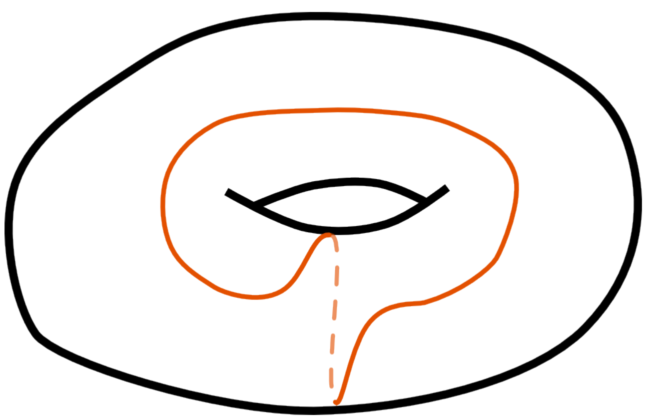

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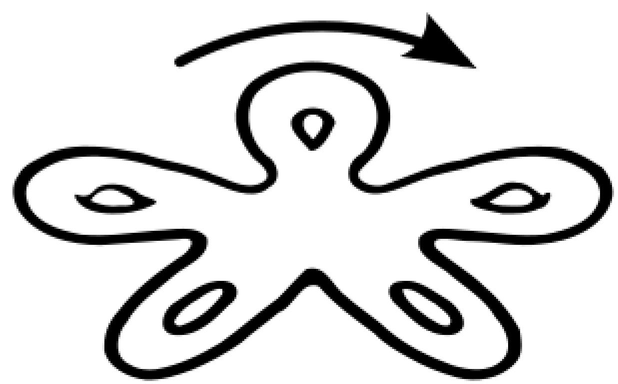

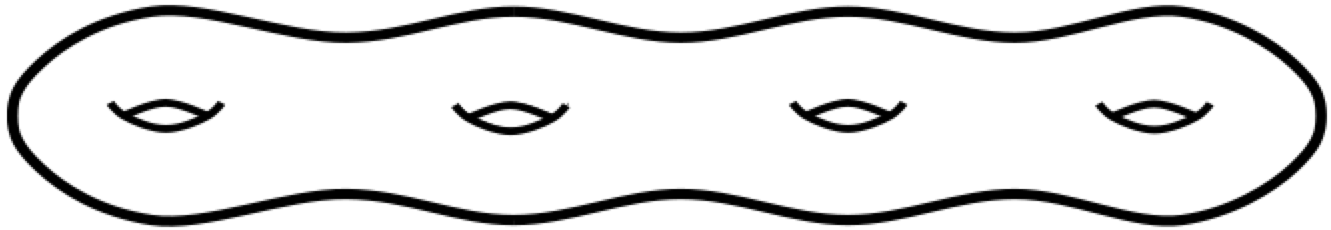

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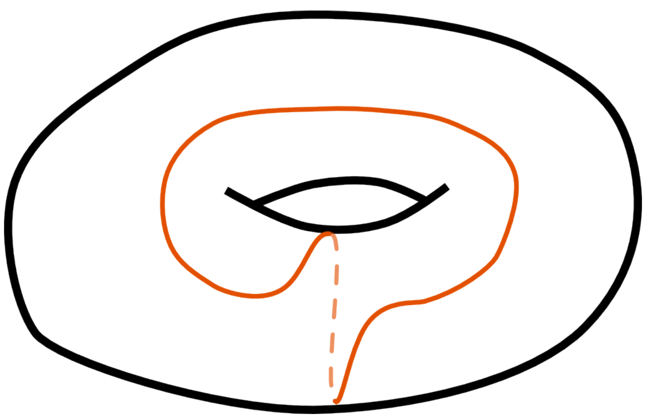
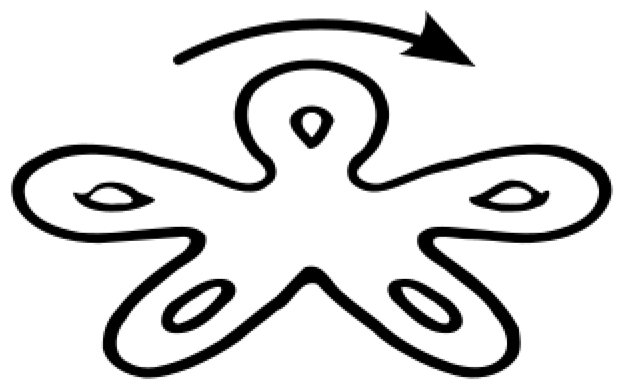


Dehn twist

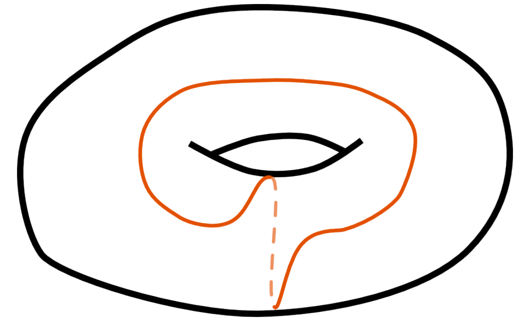
reducible, nonperiodic



reducible, periodic

Theorem (Dehn, 1938)

Dehn twists generate $\mathrm{Mod}(S_g)$.



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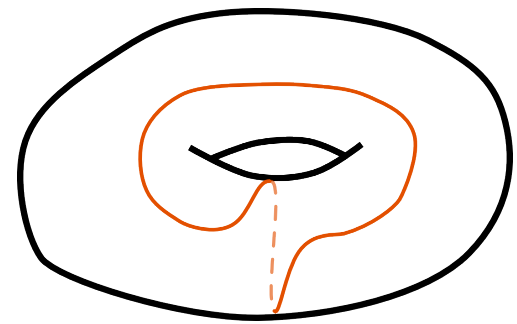
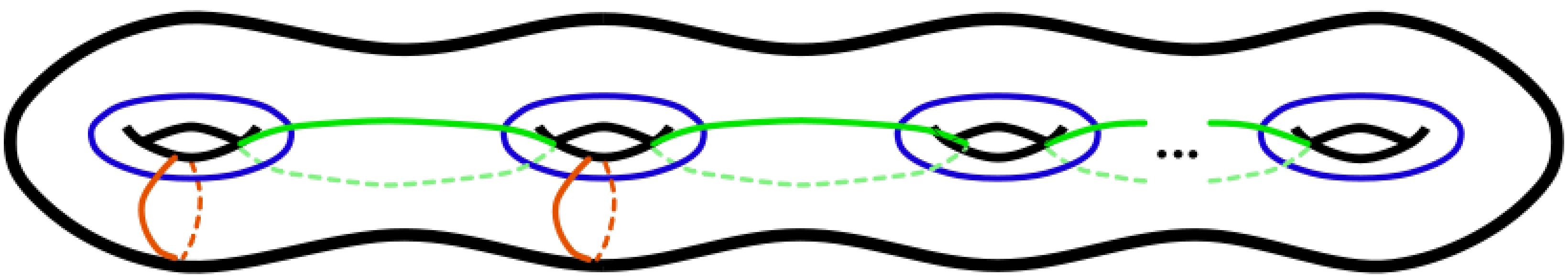
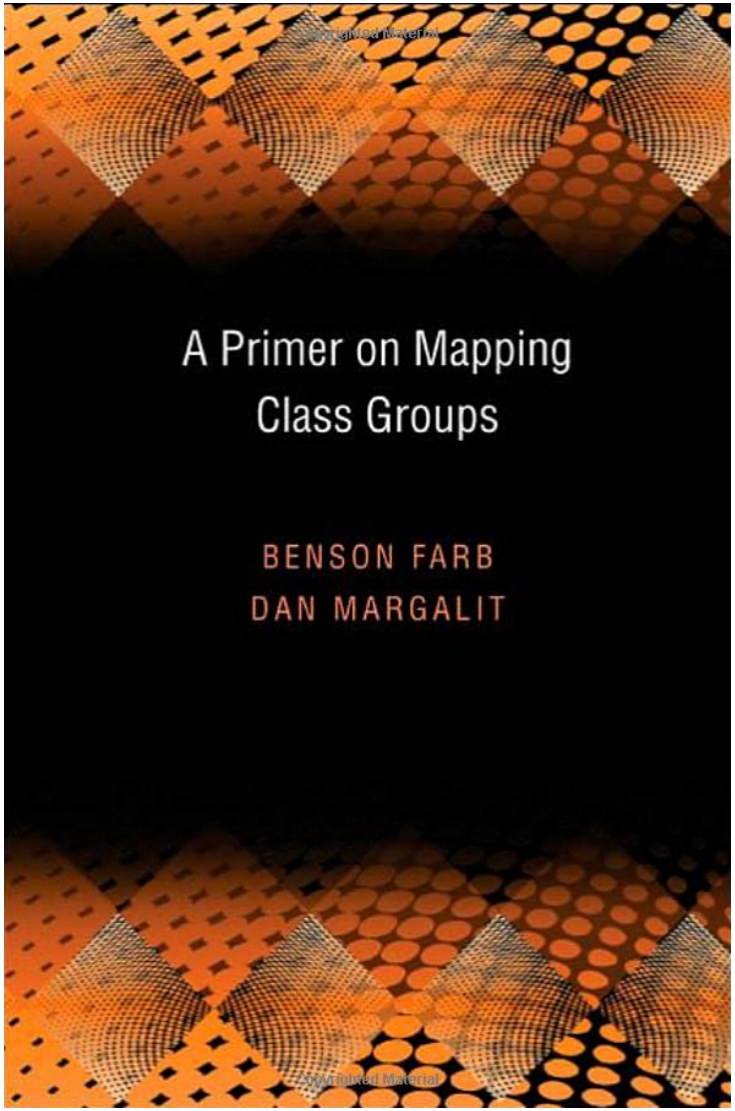




Theorem (Humphries, 1979)

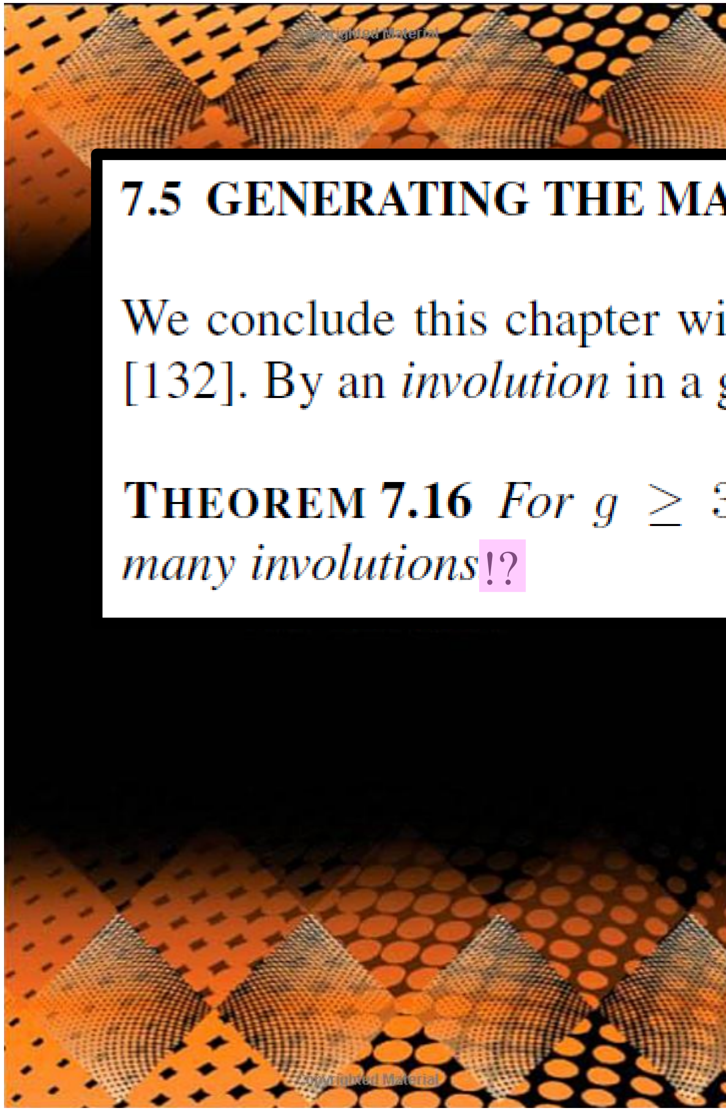
For $g \geq 2$, $2g+1$ Dehn twists generate $\mathrm{Mod}(S_g)$, and this is sharp.


A Primer on Mapping
Class Groups
BENSON FARB
DAN MARGALIT

7.5 GENERATING THE MAPPING CLASS GROUP WITH TORSION

We conclude this chapter with the following curious theorem of Feng Luo [132]. By an *involution* in a group we simply mean any element of order 2.

THEOREM 7.16 *For $g \geq 3$, the group $\mathrm{Mod}(S_g)$ is generated by finitely many involutions.*



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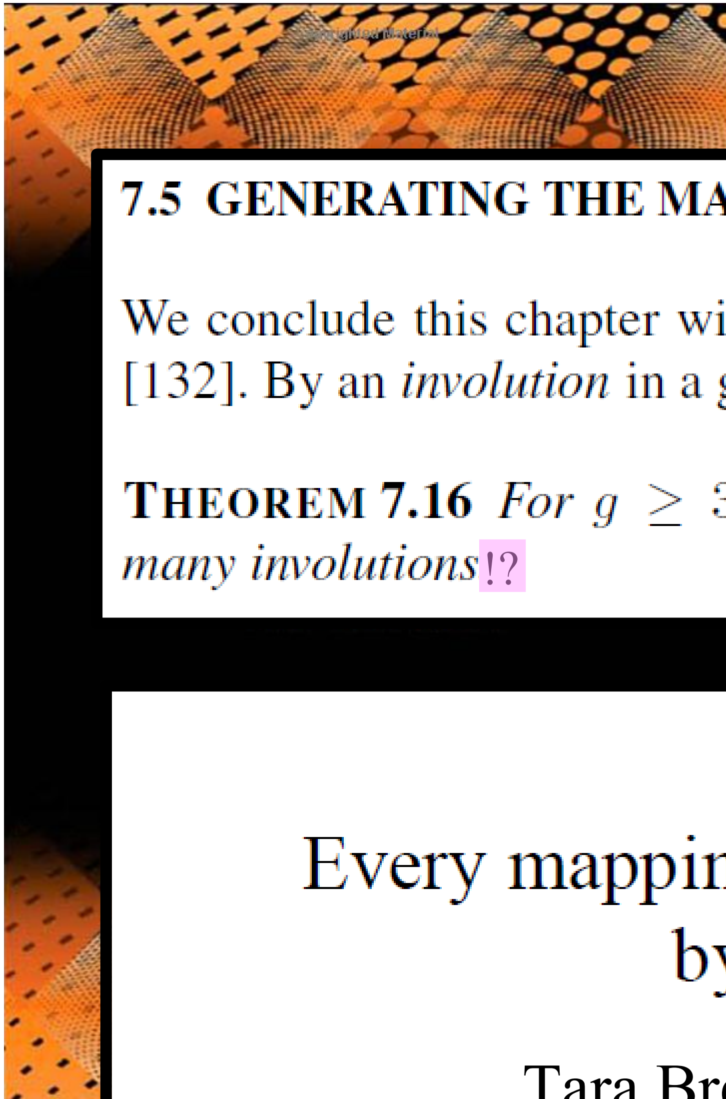

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Every mapping class group is generated by 6 involutions

Tara Brendle and Benson Farb

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There are numbers bigger than 2.

Problem: For $k > 2$, can $\mathrm{Mod}(S_g)$ be generated by elements of order k? How few?

	Order of elements	Number of elements	Genus
Brendle-Farb	2	6	$g \geq 3$
Kassabov	2	4	$g \geq 7$

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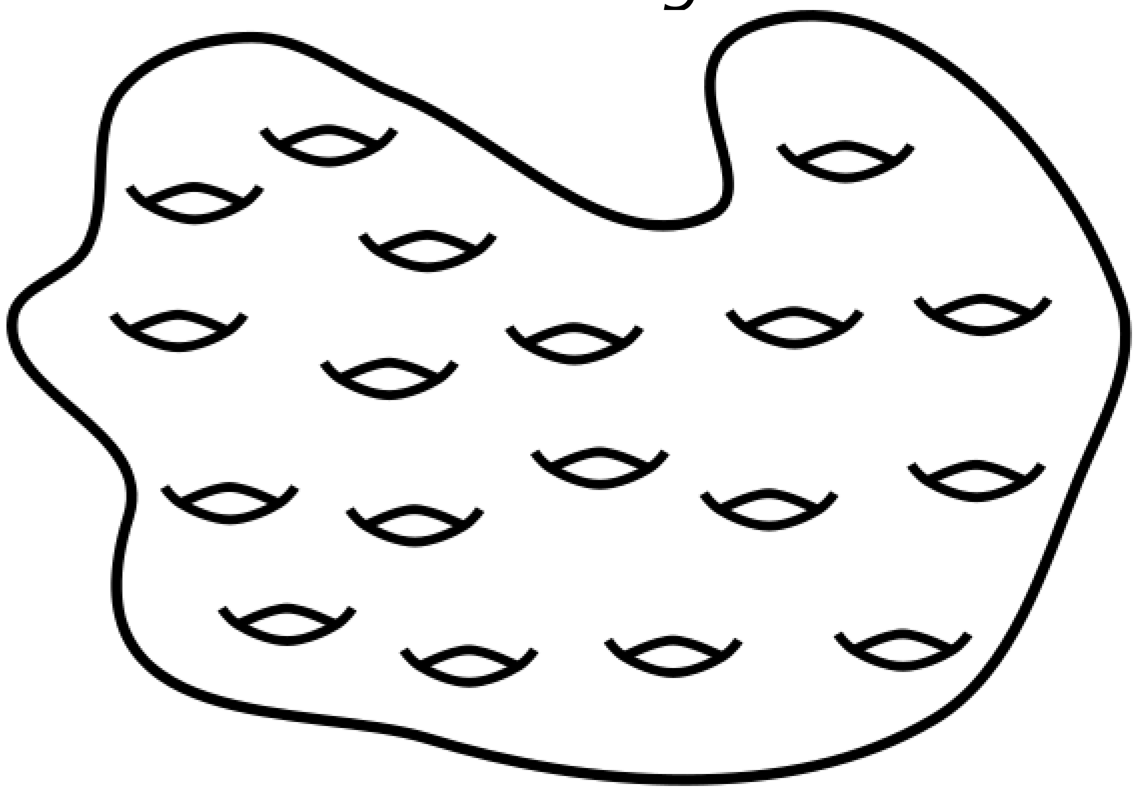
Kassabov	2	4	$g \geq 7$
Monden	3	3	$g \geq 3$
	4	4	$g \geq 3$
Yoshihara	6	3	$g \geq 10$
	6	4	$g \geq 5$

Obstacle:
When do higher-order elements even exist in Mod(S_g)?



Orders of torsion elements in $Mod(S_3)$:

1, 2, 3, 4, 6, 7, 8, 9, 12, 14

Number theoretic conditions for the existence of torsion elements in $\mathrm{Mod}(S_g)$

(i) (the Hurwitz formula) $2(g-1)/n = 2(g'-1) + \sum_{i=1}^{l}(1 - 1/\lambda_i)$.

(ii) (Nielsen [Ni1, (4.6)]) $\sum_{i=1}^{l} \sigma_i/\lambda_i$ is an integer.

(iii) (Wiman [W]) $n \leq 4g + 2$.

(iv) (Harvey [H]) Assume $g \geq 2$. Set $M = \mathrm{lcm}(\lambda_1, \ldots, \lambda_l)$. Then we have:

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(2) M divides n, and if $g' = 0$, then $M = n$.

(3) $l \neq 1$, and, if $g' = 0$, then $l \geq 3$.

(4) If $2|M$, the number of $\lambda_1, \ldots, \lambda_l$ which are divisible by the maximal power of 2 dividing M is even.

(Ashikaga and Ishizaka)

Theorem (L., 2017)

Let $k \geq 6$ and $g \geq (k-1)^2 + 1$. Then $\text{Mod}(S_g)$ is generated by three elements of order k.

Also, $\text{Mod}(S_g)$ is generated by four elements of order 5 when $g \geq 8$.

Strategy:

(Luo, Brendle-Farb)

1) Find some order 2 elements.
2) Write a Dehn twist as a product in these.
3) Show that the order 2 elements generate a subgroup that puts all the Humphries curves in the same orbit.

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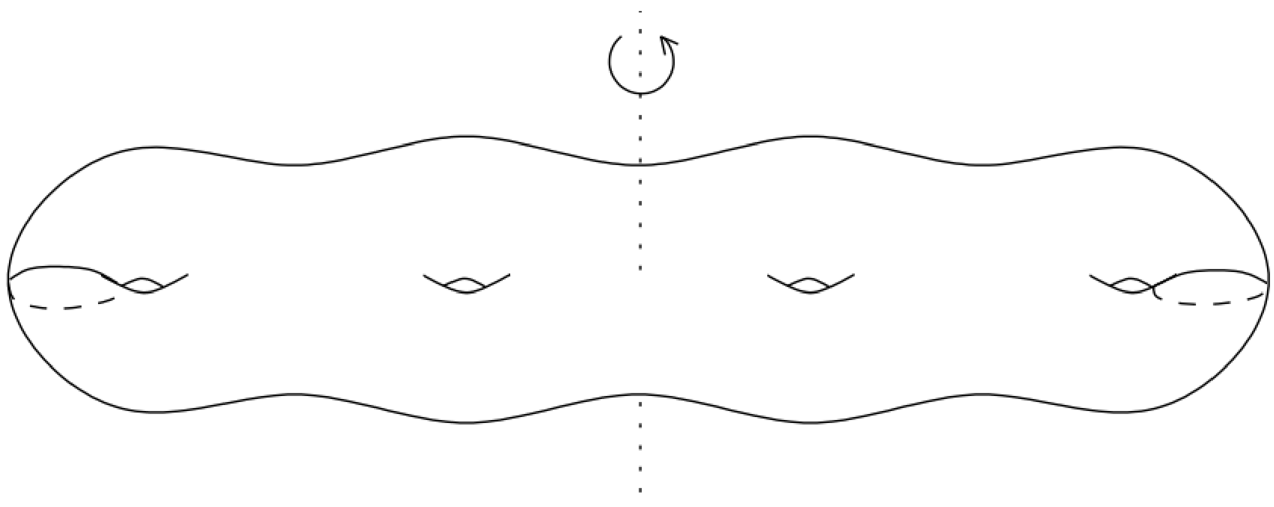
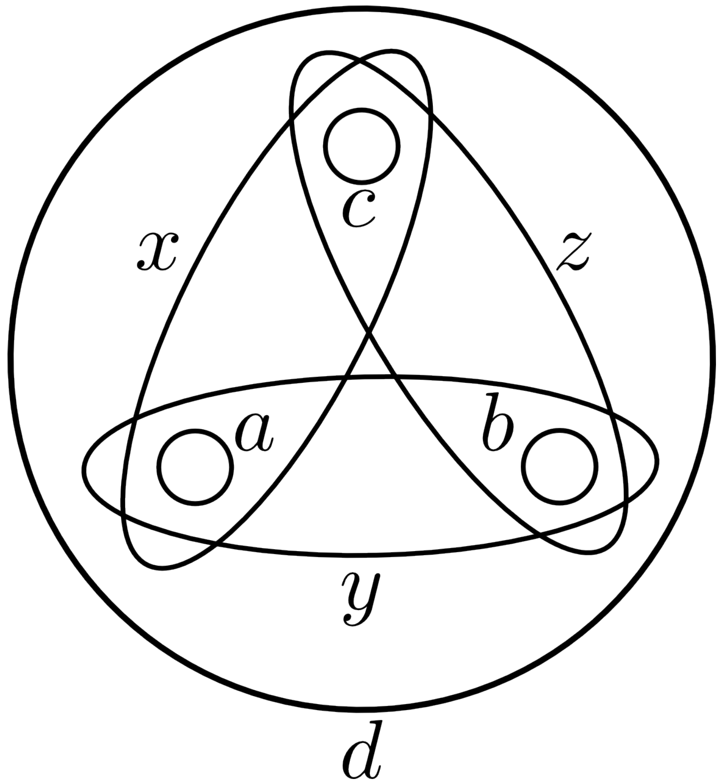


(Luo)

$$T_d = (T_x T_a^{-1})(T_y T_b^{-1})(T_z T_c^{-1})$$



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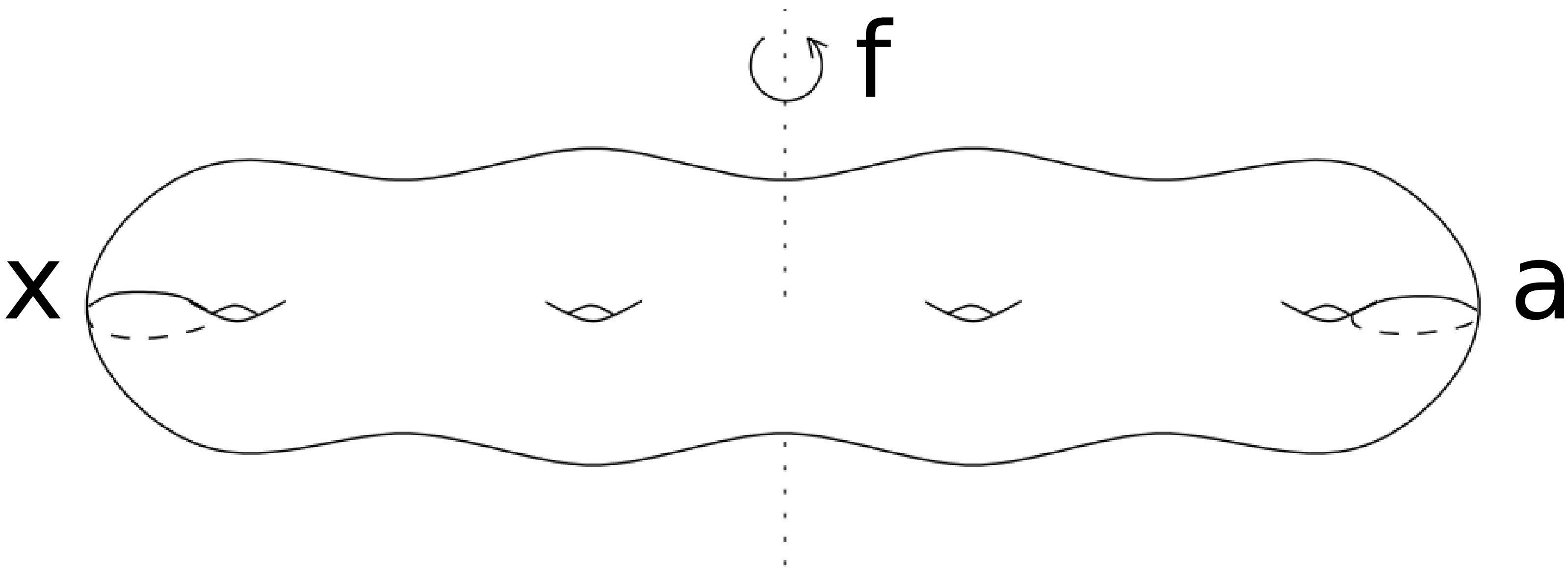


$$T_x T_a^{-1} = T_x (f\, T_x^{-1} f^{-1}) = (T_x f\, T_x^{-1}) f^{-1}$$

(Brendle-Farb)

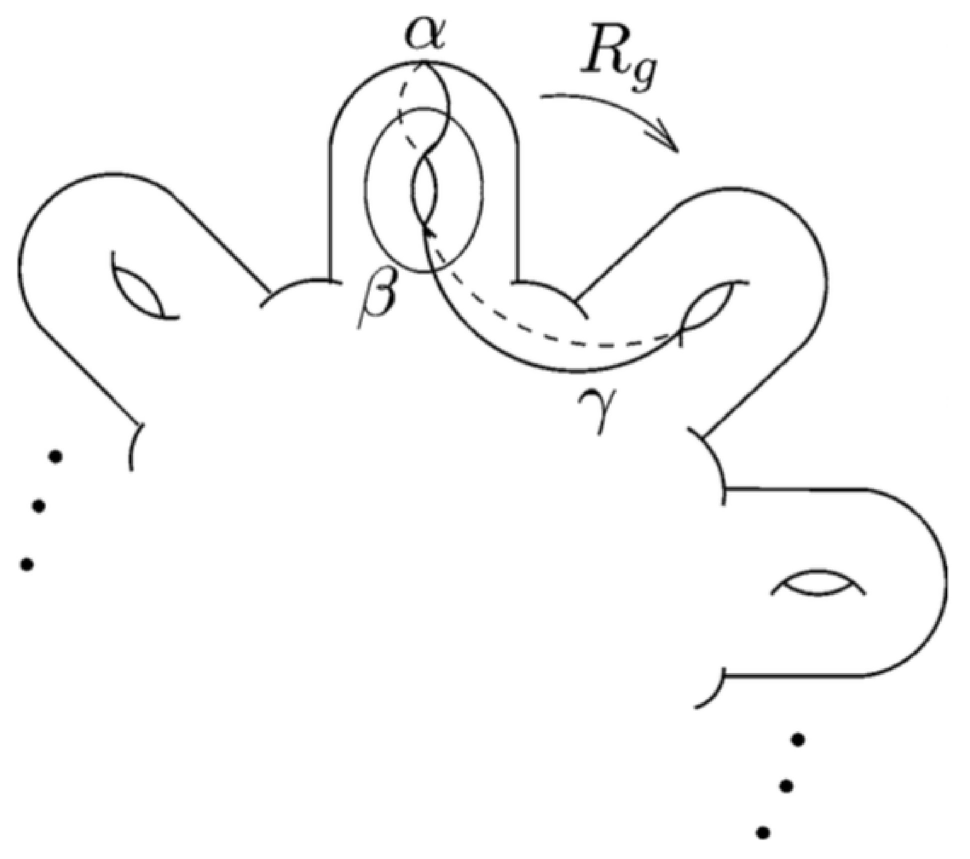


Fig. 4. A generating set for $\mathrm{Mod}_{g,b}$.

(Brendle-Farb)

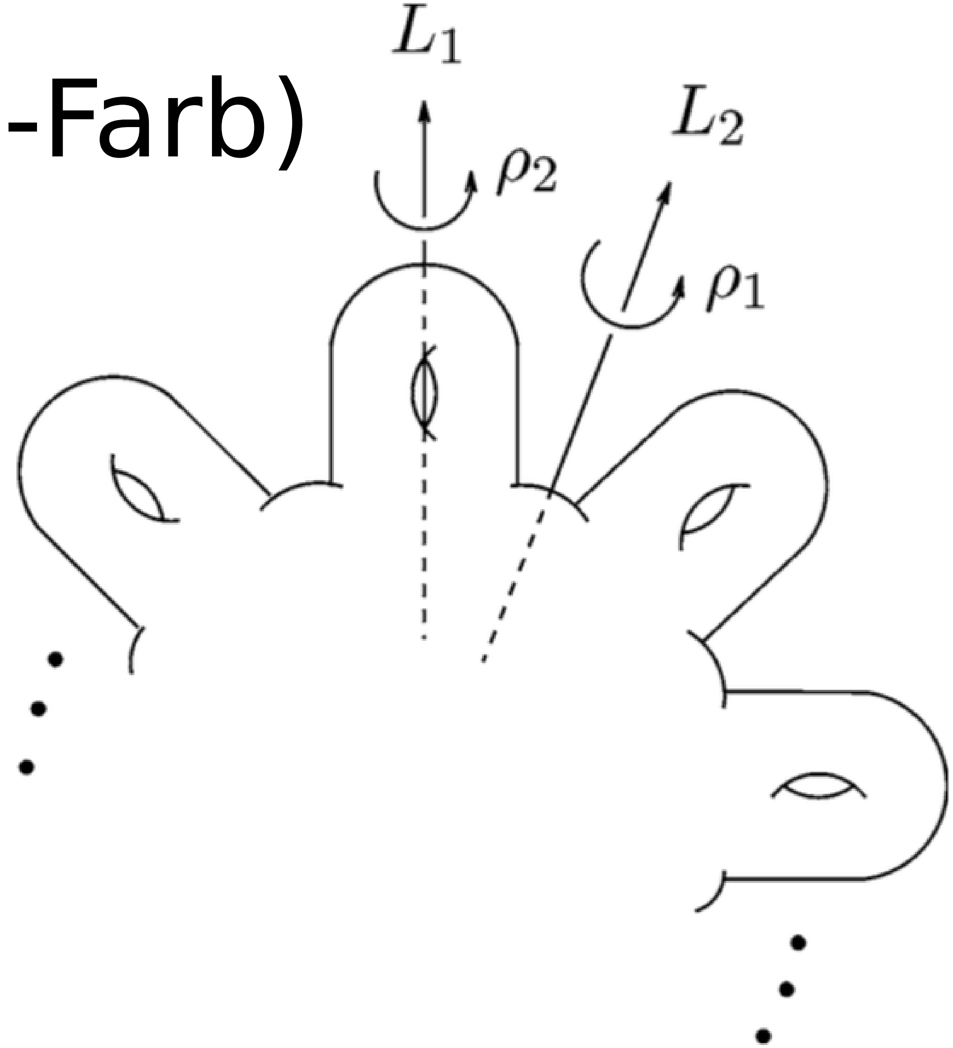
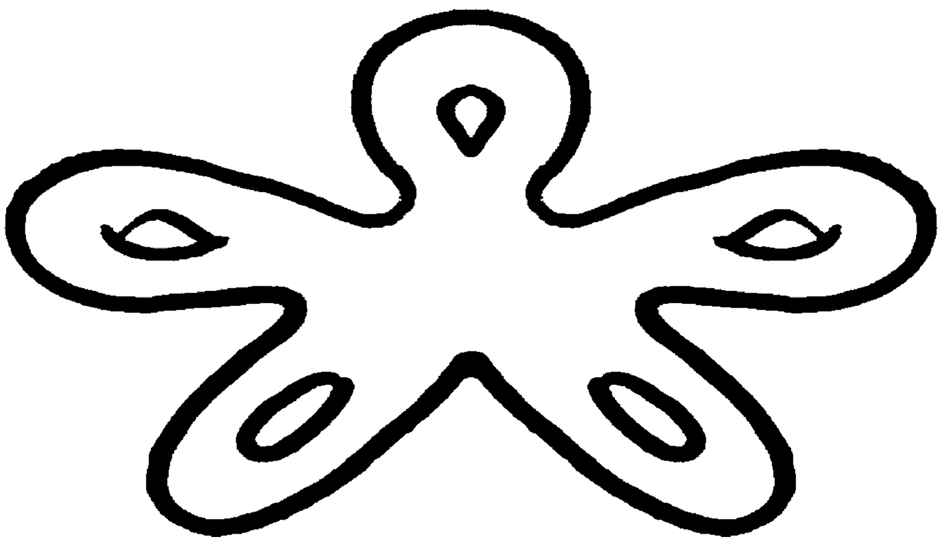


Fig. 5. Two involutions generating R_g .

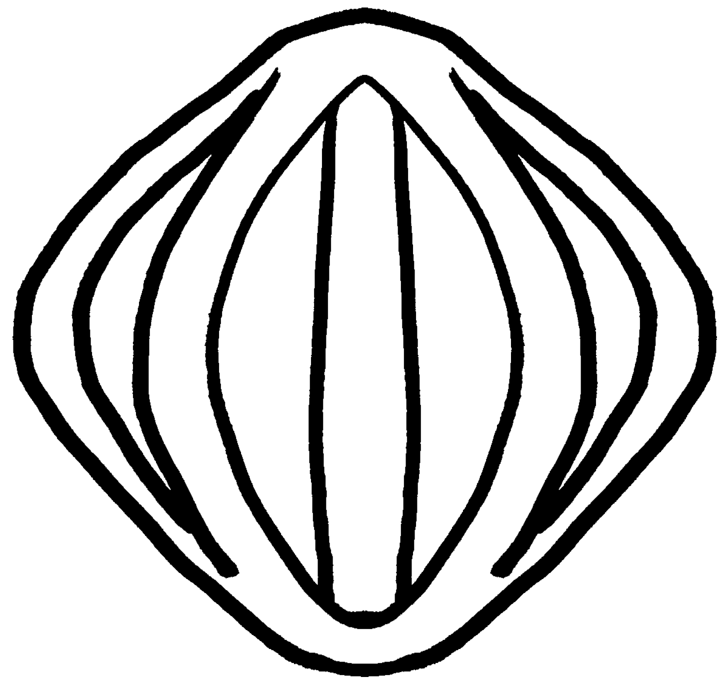
Strategy:

1) Find some order k elements.
2) Write a Dehn twist as a product in these.
3) Show that the order k elements generate a subgroup that puts all the Humphries curves in the same orbit.

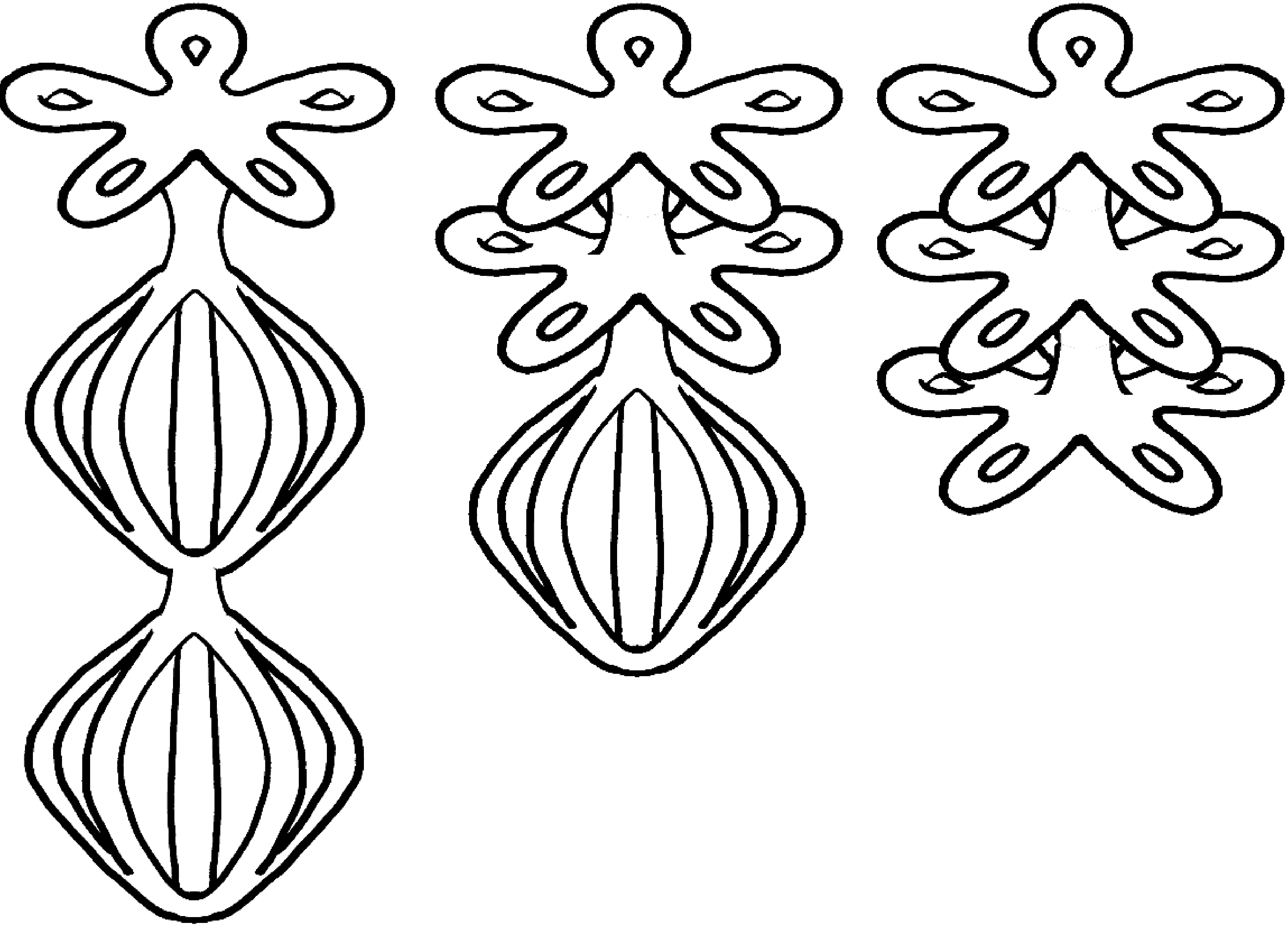
$k = 5$



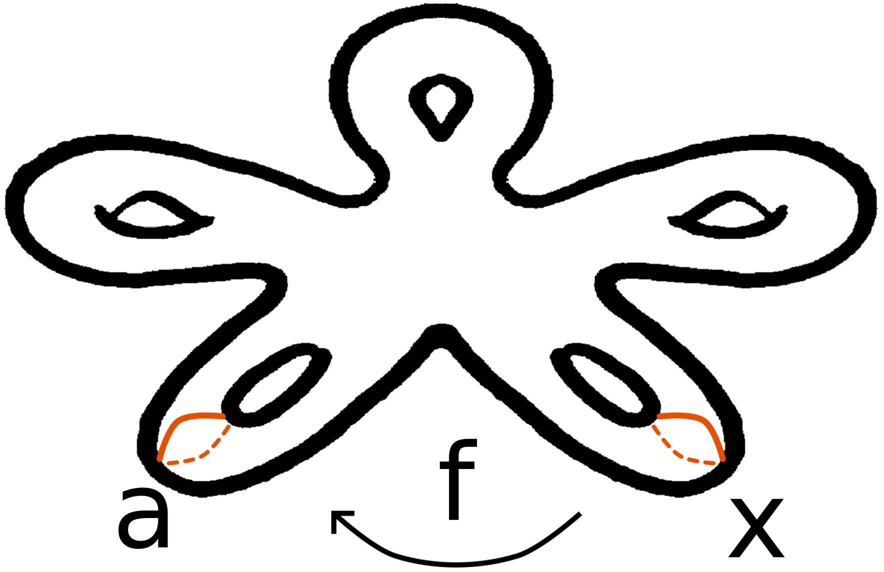
genus 5



genus 4



$$T_d = (T_x T_a^{-1})(T_y T_b^{-1})(T_z T_c^{-1})$$



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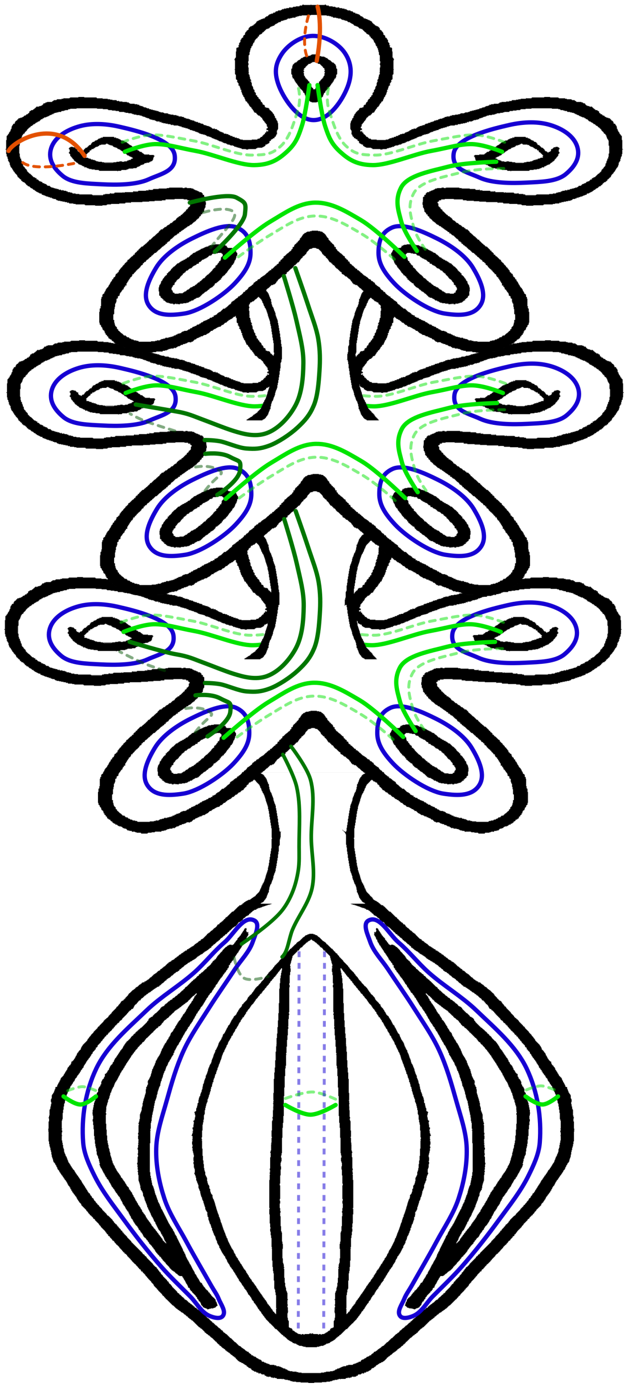
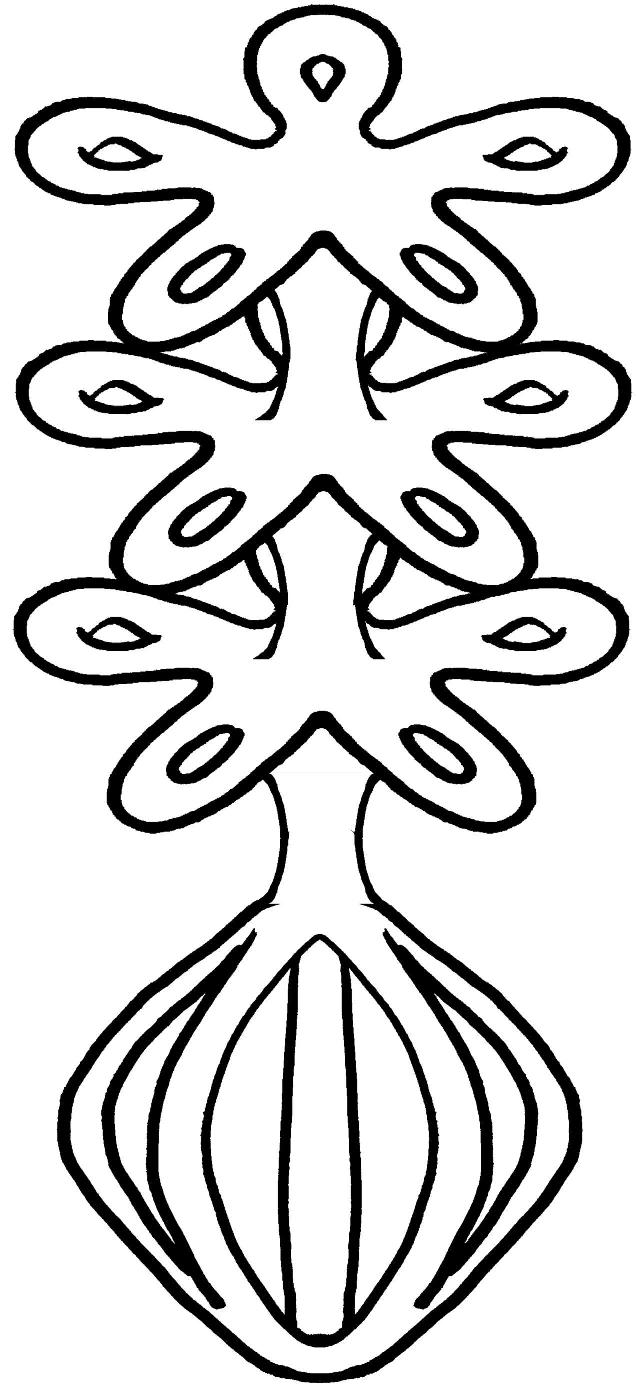
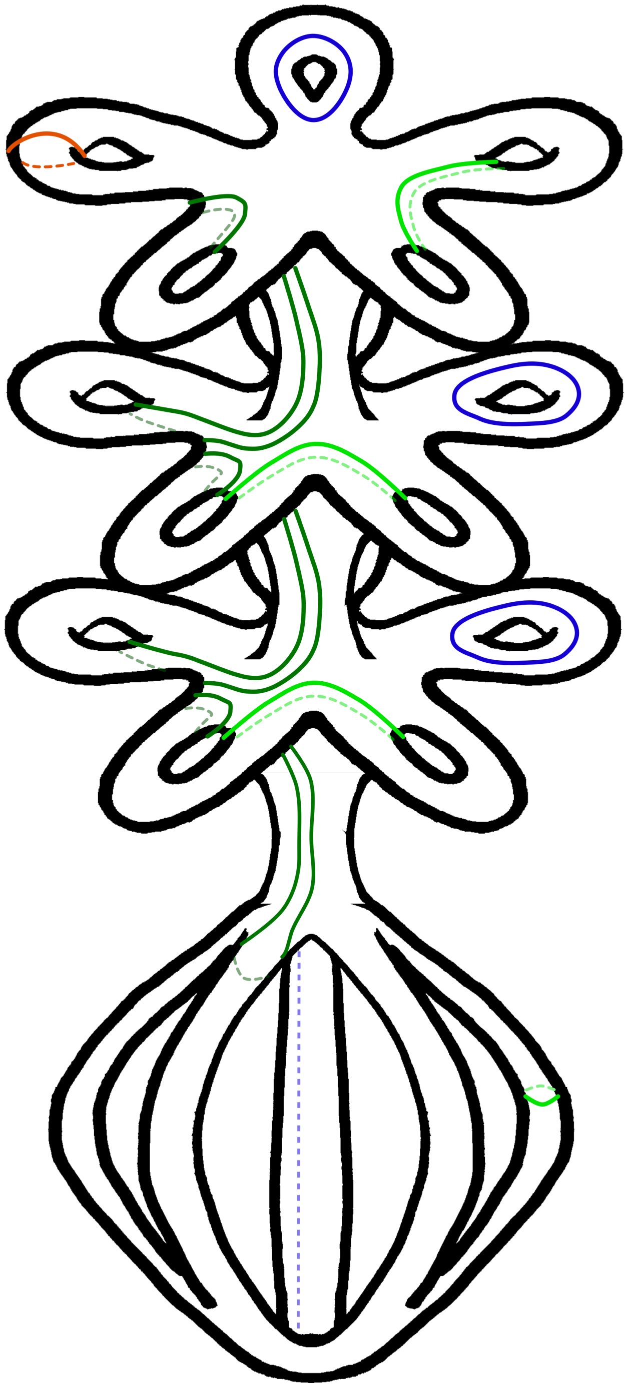
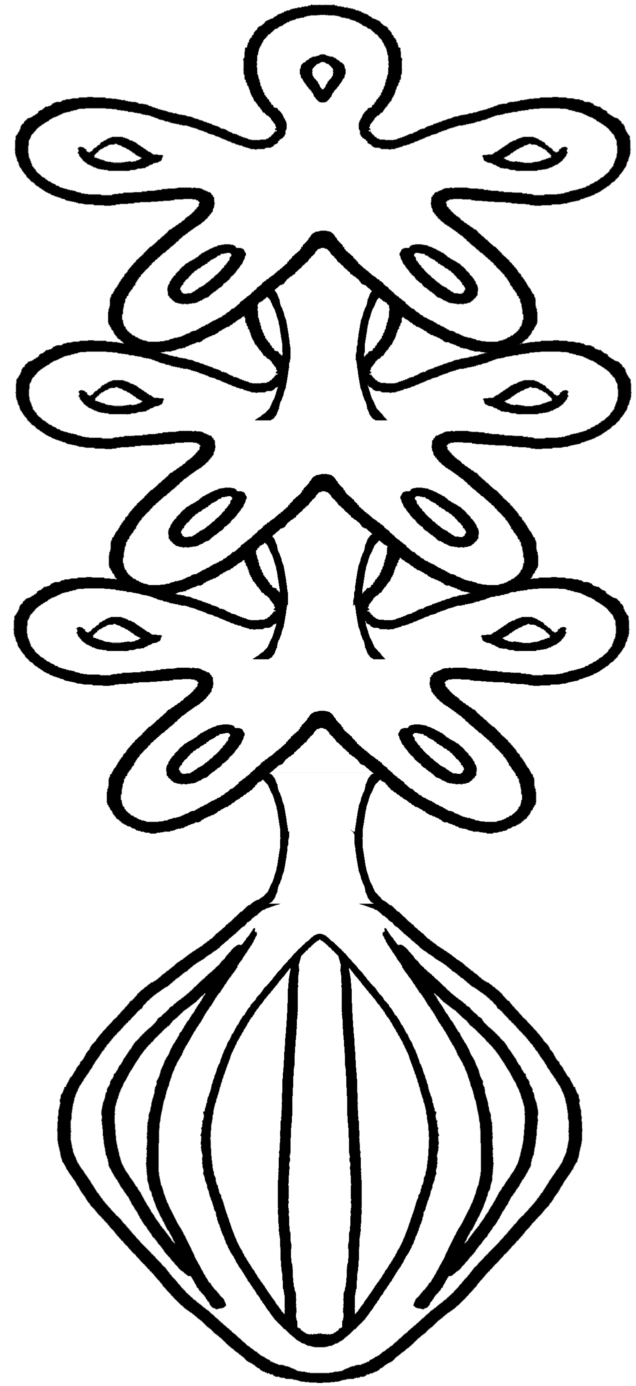
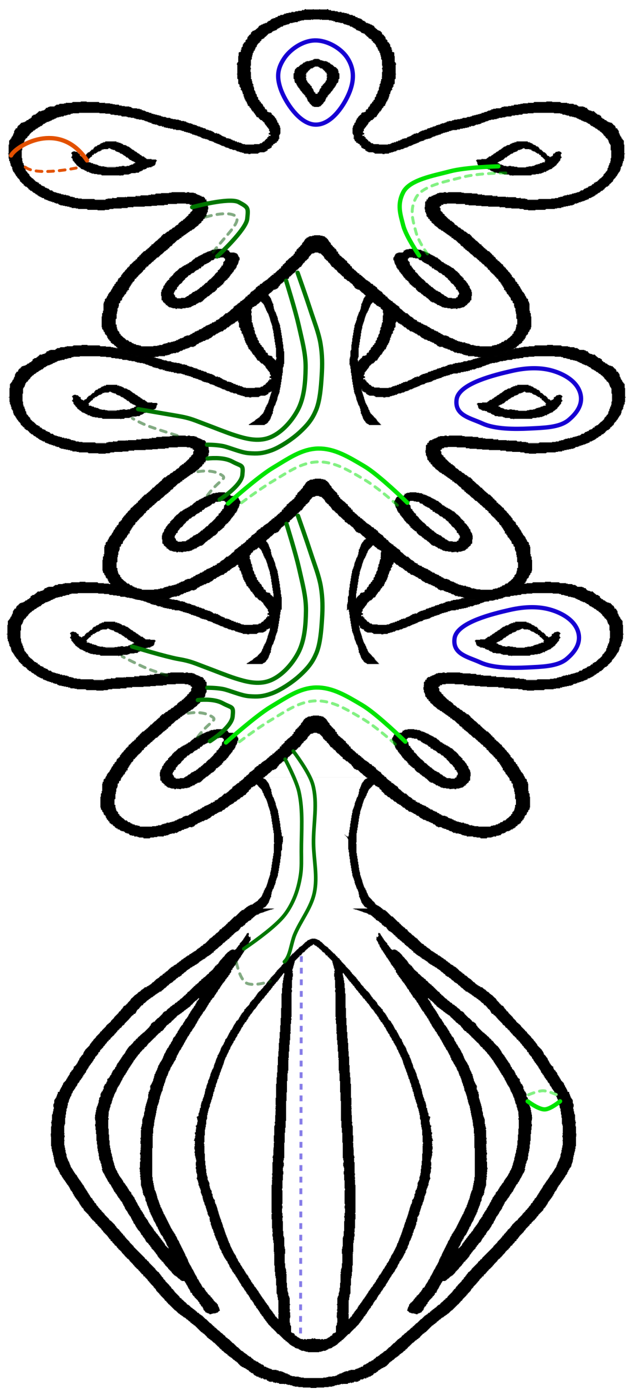
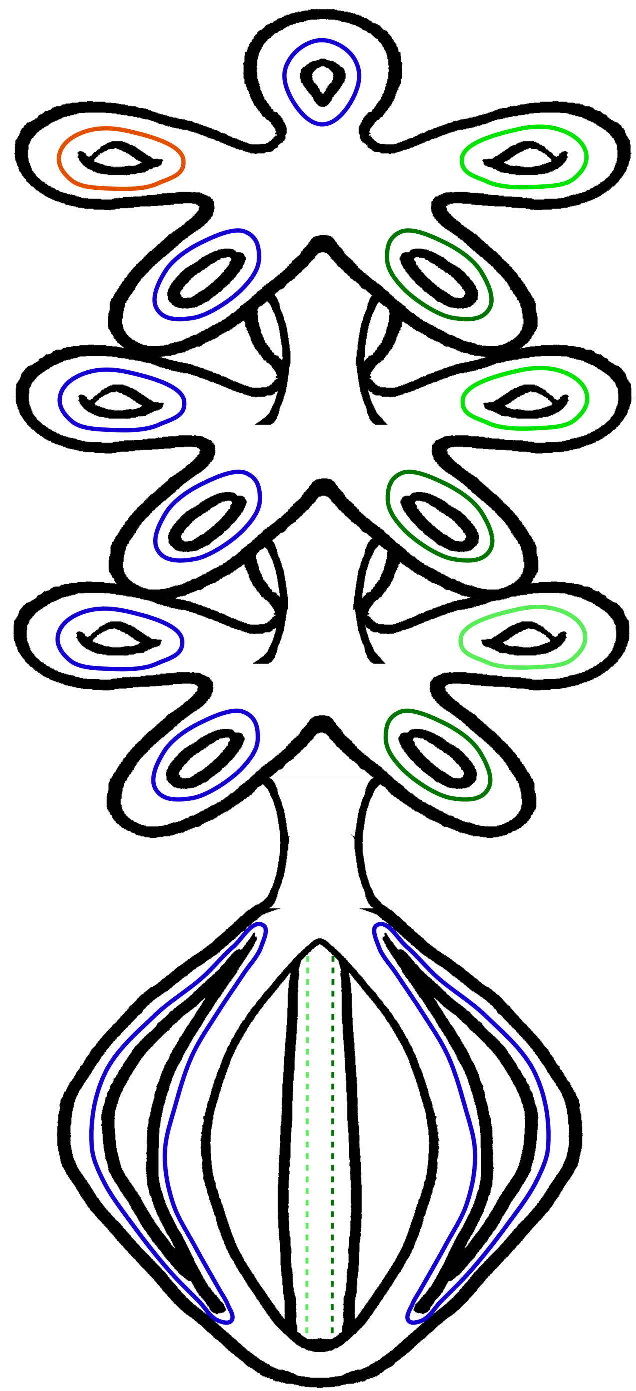




Other groups?

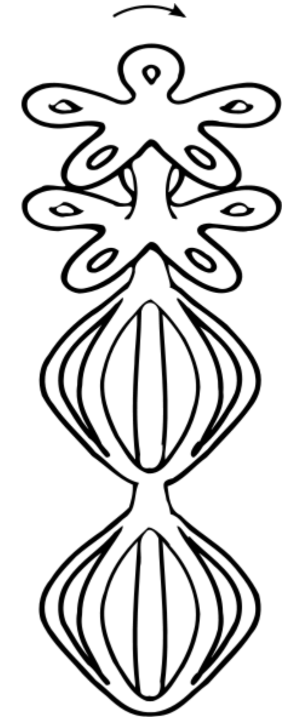
Sharpening these results?

Other periodic elements?

Act 2: the hope

For $g \geq 3$ and $k \geq 3$, if a mapping class can be realized as a rotation of S_g embedded in $\mathbf{R}^3$,
it and three of its conjugates generate $\mathrm{Mod}(S_g)$.

For $g \geq 3$ and $k \geq 3$, if a mapping class can be realized as a rotation of S_g embedded in $\mathbf{R}^3$,
it *normally generates* $\mathrm{Mod}(S_g)$.



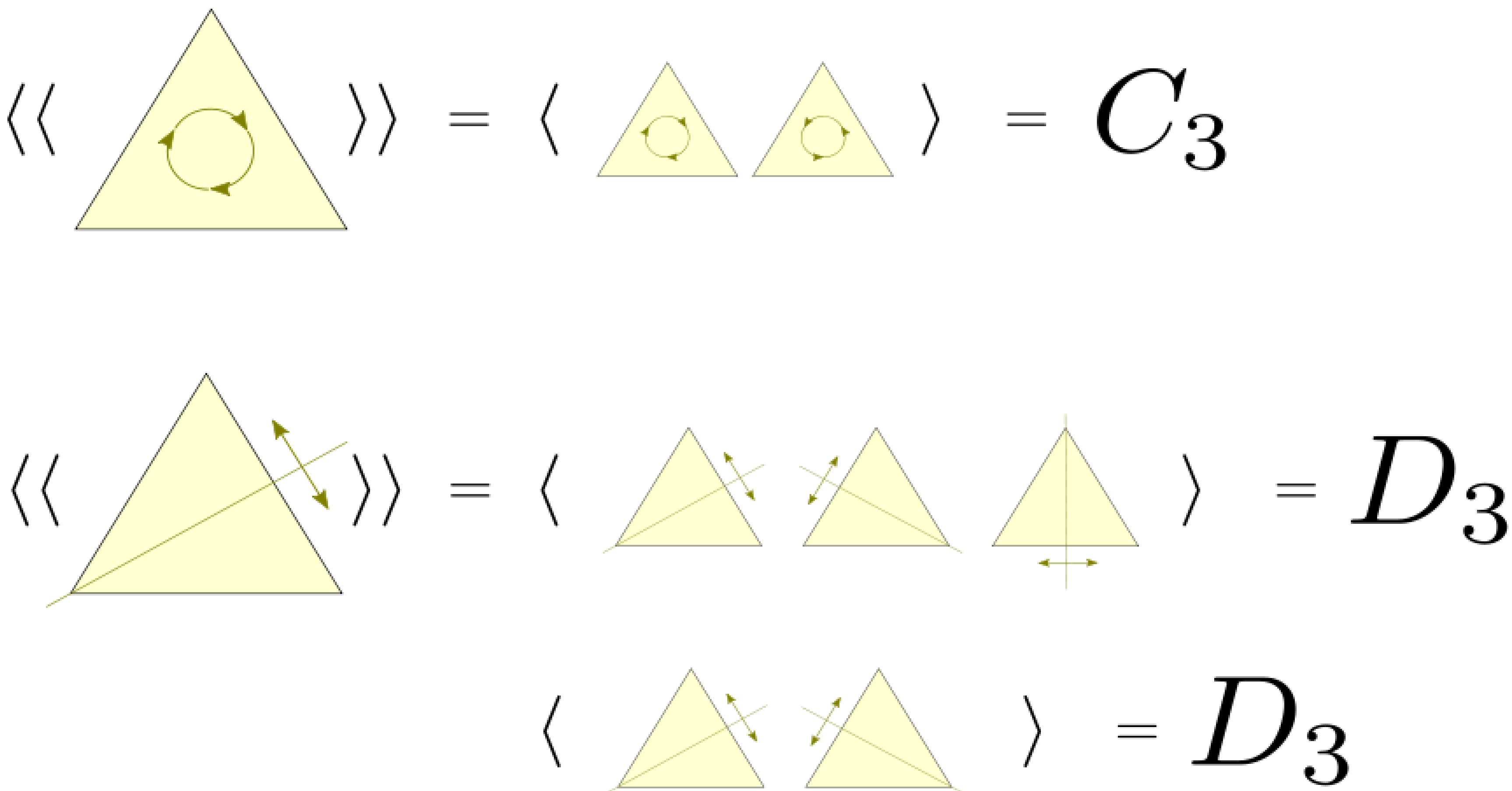
For $g \geq 3$ and $k \geq 3$, if a mapping class can be realized as a rotation of S_g embedded in $\mathbf{R}^3$, it *normally generates* $\mathrm{Mod}(S_g)$.

normal closure

$$\langle\langle g \rangle\rangle := \langle \text{ conjugates of } g \text{ } \rangle$$

normal generator

$$\langle\langle g \rangle\rangle = G$$

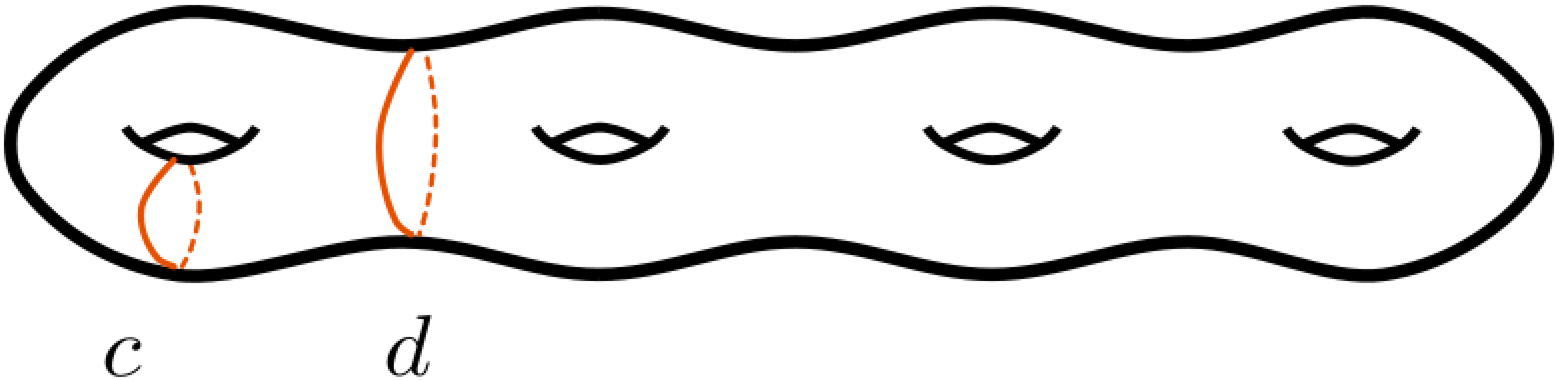

$= C_3$
$= D_3$
$= D_3$

Some normal generators

symmetric groups: transpositions
braid groups: Artin generators
orthogonal groups: reflections

Problem: Characterize the mapping classes that normally generate $\mathrm{Mod}(S_g)$.

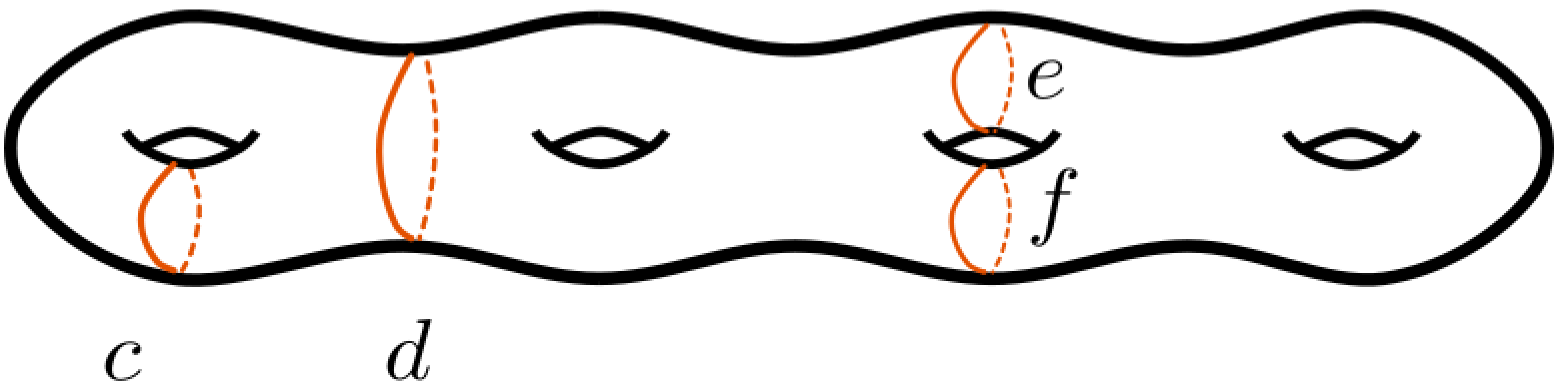
reducible elements



$$\langle\langle T_c \rangle\rangle = \mathrm{Mod}(S_g)$$

$$\langle\langle T_d \rangle\rangle \neq \mathrm{Mod}(S_g)$$

reducible elements

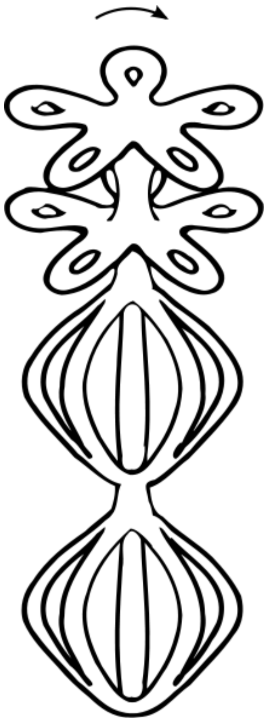


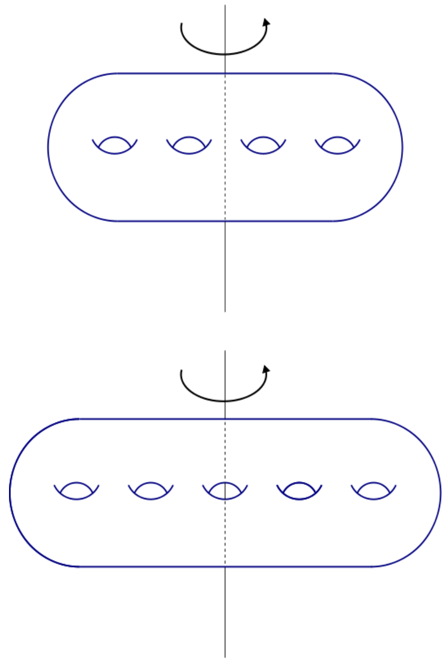
$$\langle\langle T_c \rangle\rangle = \mathrm{Mod}(S_g)$$

$$\langle\langle T_d \rangle\rangle \neq \mathrm{Mod}(S_g)$$

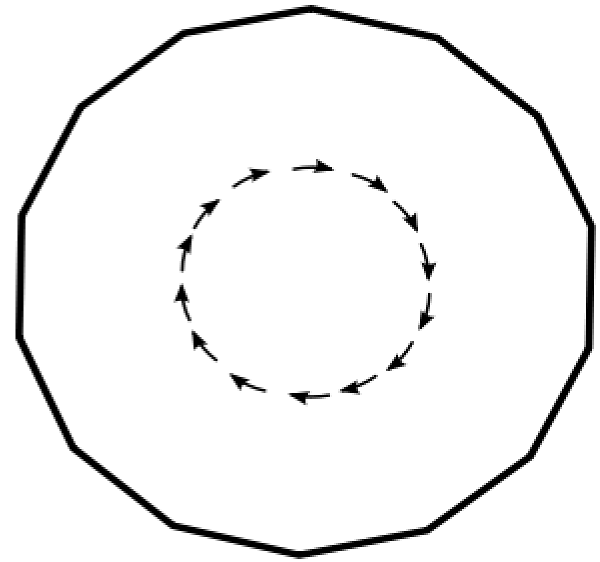
$$\langle\langle T_e T_f^{-1} \rangle\rangle \neq \mathrm{Mod}(S_g)$$

periodic elements



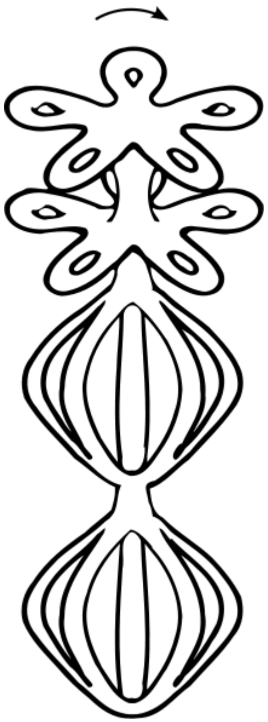


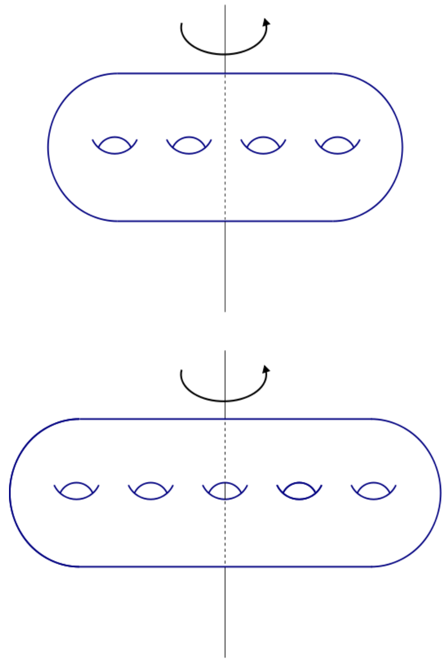
(McCarthy-Papadopoulos, 1987)



(Korkmaz, 2005)

periodic elements





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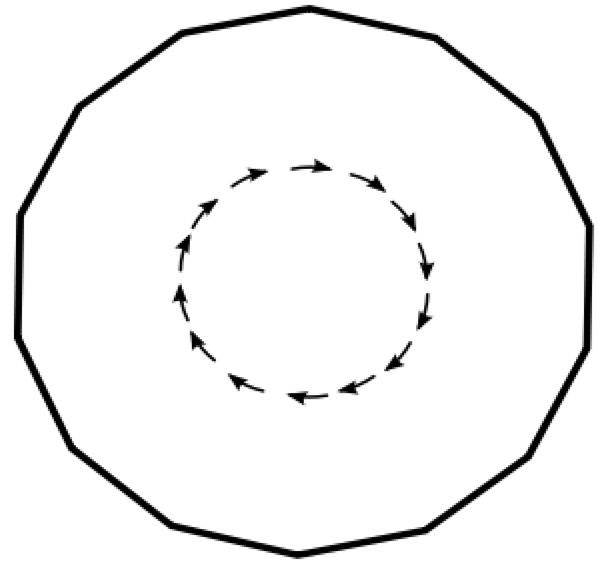
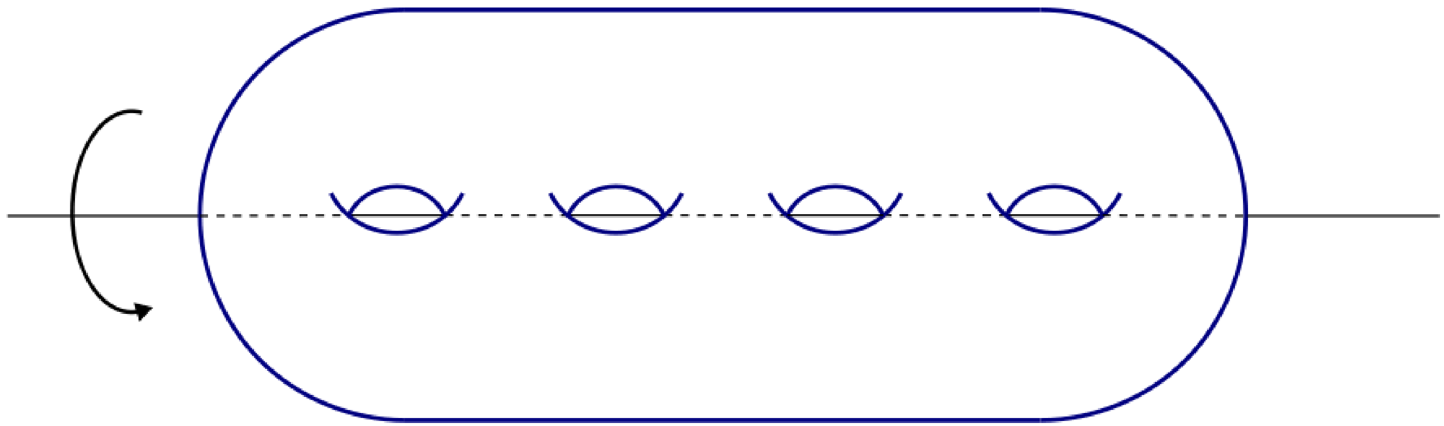


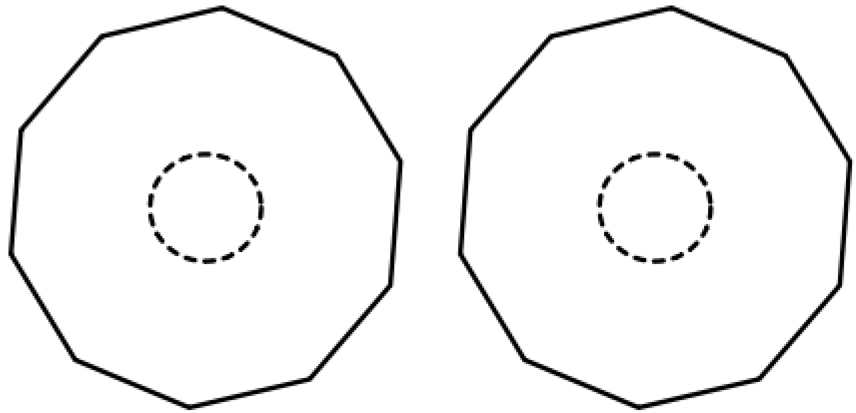
(Korkmaz, 2005)

"Let's be more bold: if $g > 2$ and f is any element of finite order in $\mathrm{Mod}(S_g)$, then the normal closure $\langle\langle f \rangle\rangle$ is $\mathrm{Mod}(S_g)$. In particular, $\mathrm{Mod}(S_g)$ is generated by elements of order $|f|$."



hyperelliptic involution

Obstacle:
How can you even get a handle on all the periodic elements?



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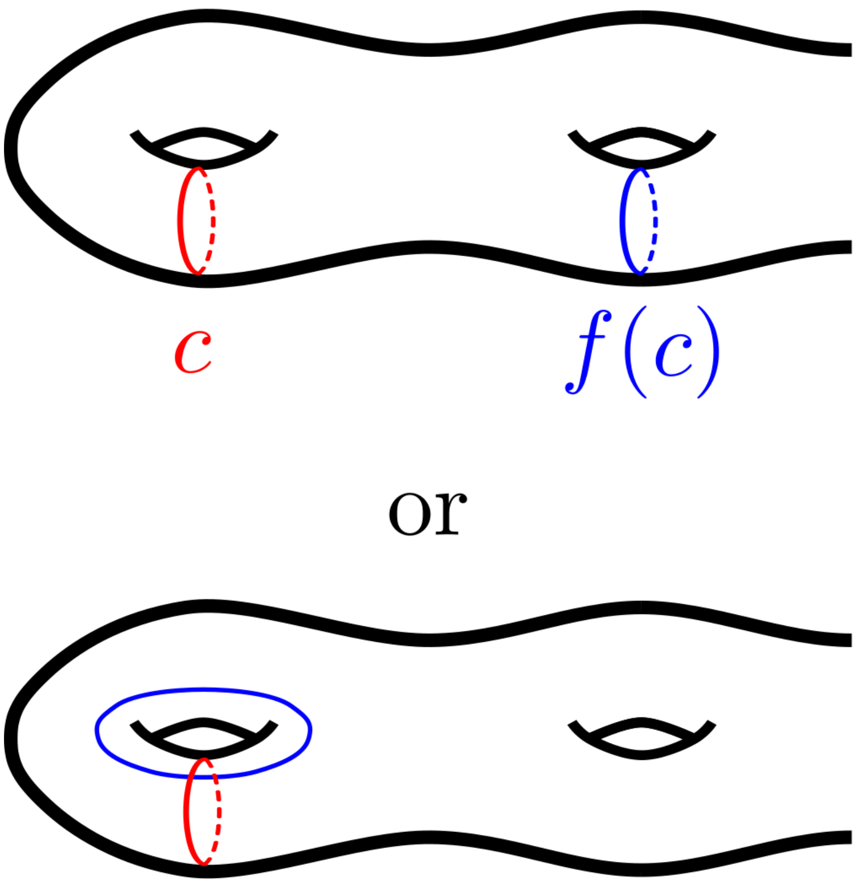
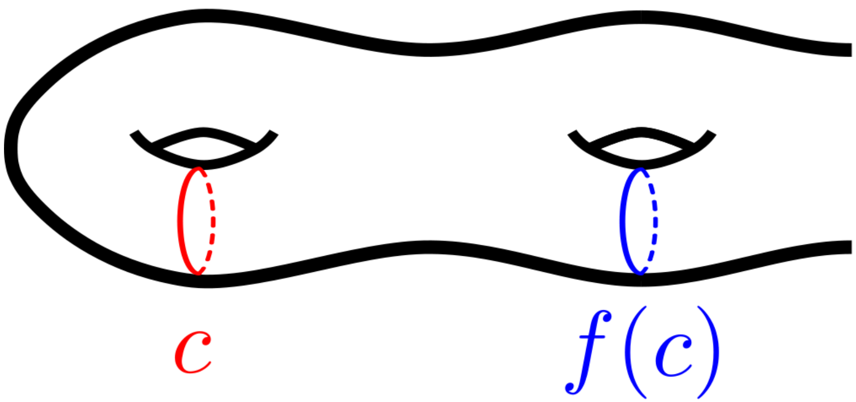
Well-suited curve criteria

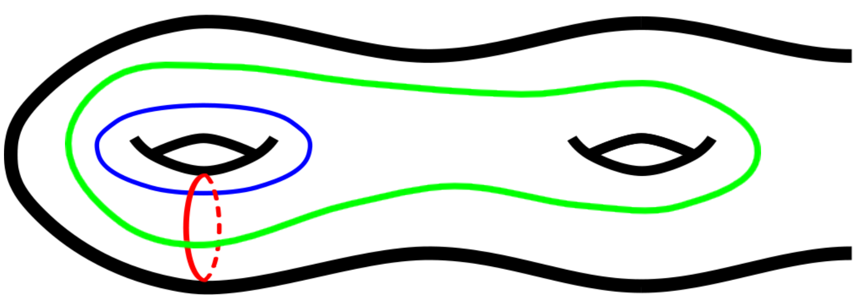


$\Longrightarrow$ normal generator

Well-suited curve criteria



or



(by Luo) $\Longrightarrow$ normal generator

$$(T_b T_r^{-1})(T_r T_g^{-1}) = T_b T_g^{-1}$$

Theorem (L.-Margalit, 2017)

For $g \geq 3$, every periodic mapping class that is not a hyperelliptic involution normally generates $\mathrm{Mod}(S_g)$.

Proof sketch

periodic elements

- free action on S_g
- has a non-identity power that fixes a point in S_g

Proof sketch

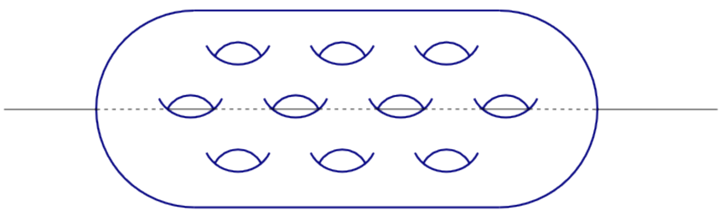
periodic elements

- order 2
- free action on S_g
- has a non-identity power that fixes a point in S_g

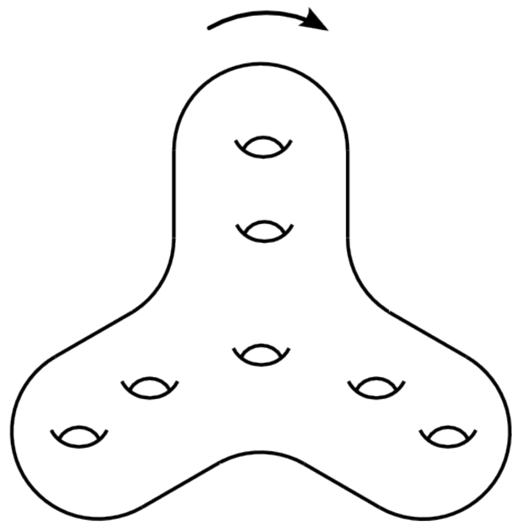
Proof sketch

periodic elements

order 2



free action on S_g

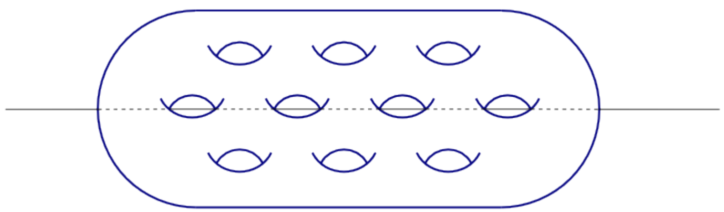


has a non-identity power that fixes a point in S_g

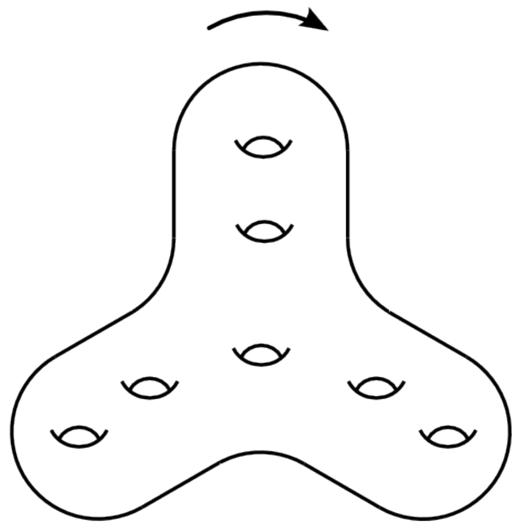
Proof sketch

periodic elements

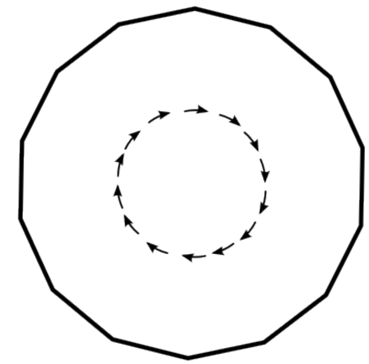
order 2



free action on S_g



has a non-identity power that fixes a point in S_g



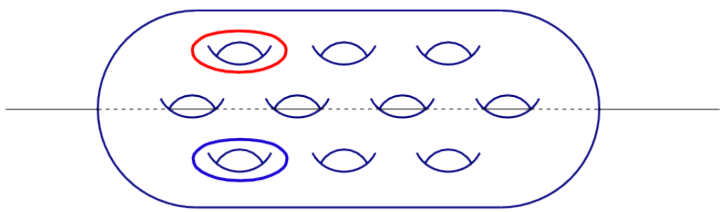
(Kulkarni, 1997)

periodic element with a fixed point $\Longrightarrow$ polygon rotation representative

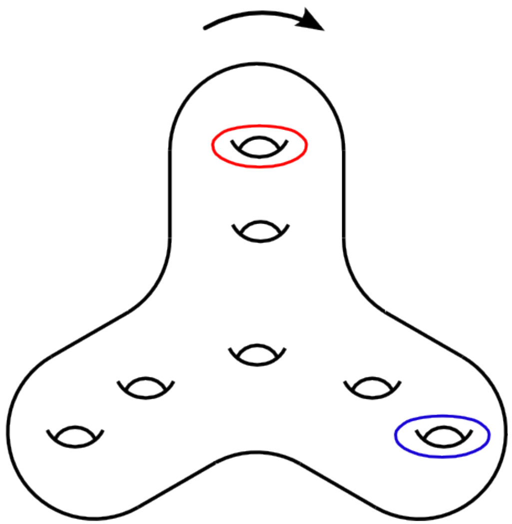
Proof sketch

periodic elements

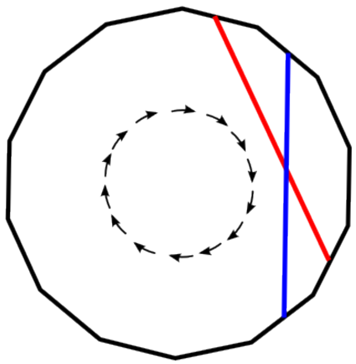
order 2



free action on S_g



has a non-identity power that fixes a point in S_g



(Kulkarni, 1997)

periodic element with a fixed point $\implies$ polygon rotation representative

Consequences:

1) A new proof that the Torelli group is torsion-free!

In fact:

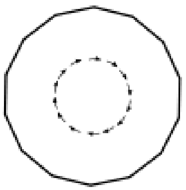
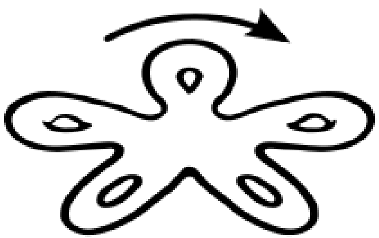
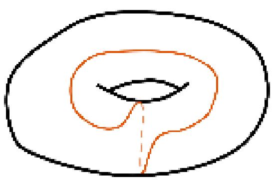
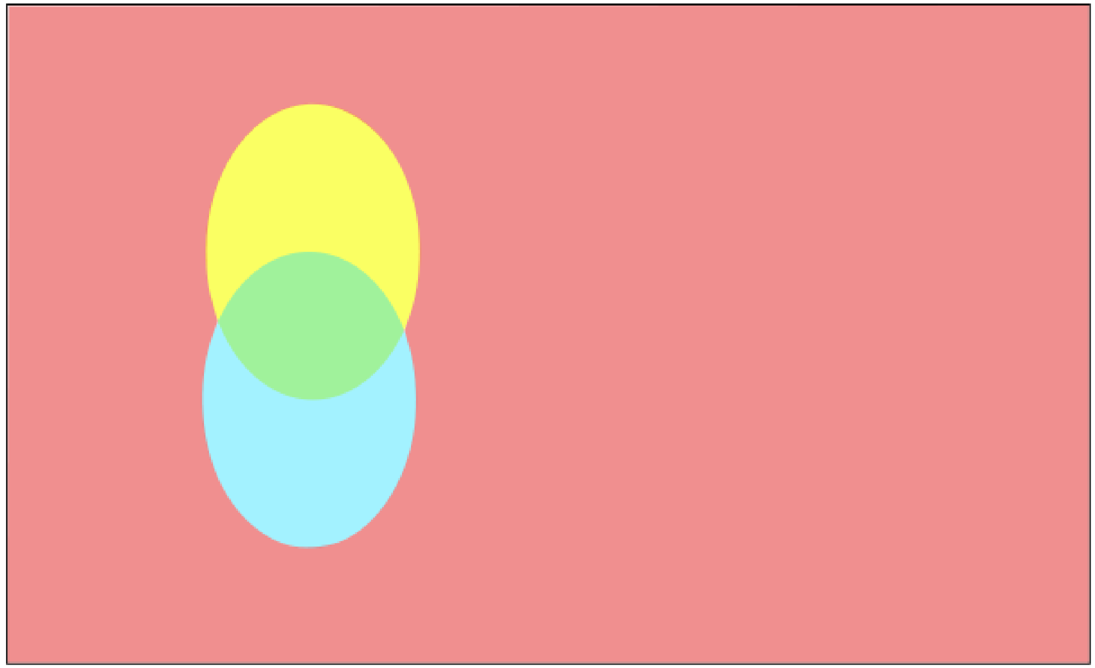
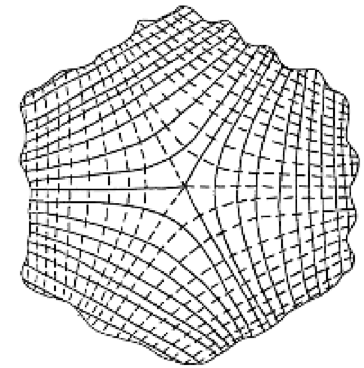
2) Every normal subgroup that does not contain the Torelli subgroup is torsion-free.

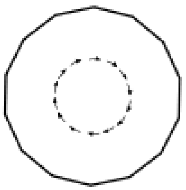
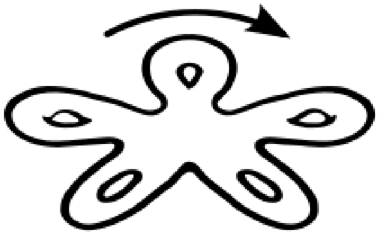
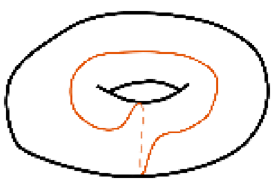
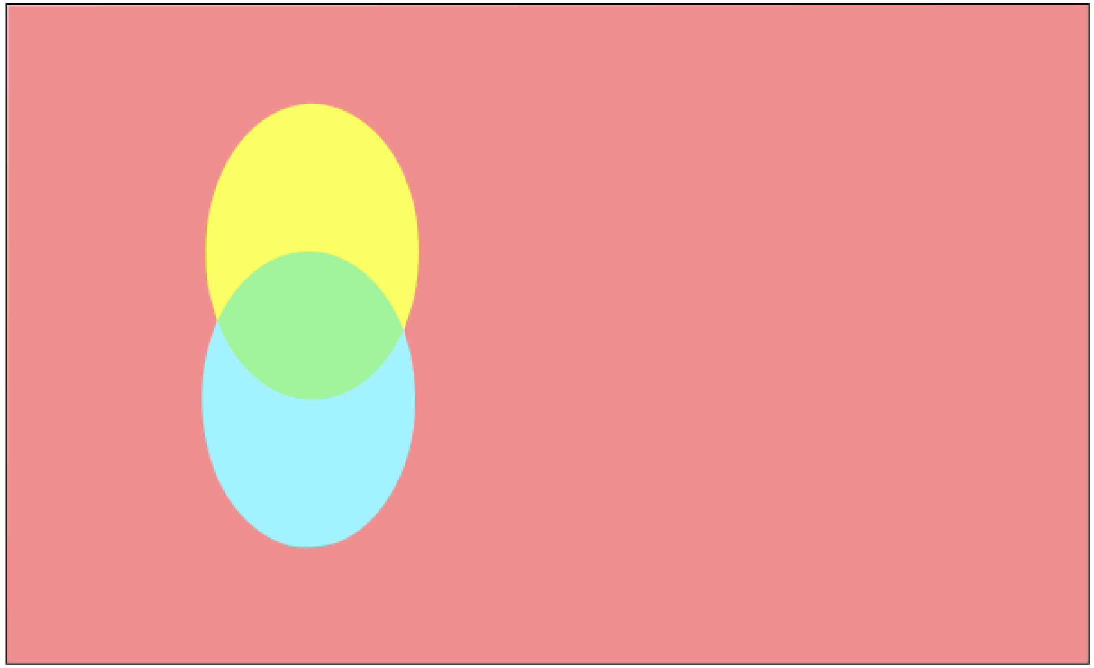
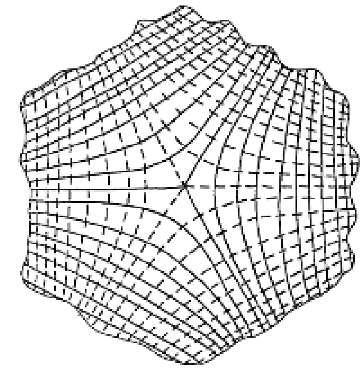
3) A new proof that a homomorphism between different mapping class groups must be trivial!

Theorem (Harvey-Korkmaz, 2005)
Suppose $g \geq 3$ and let $0 \leq h < g$.
Any homomorphism $\mathrm{Mod}(S_g) \to \mathrm{Mod}(S_h)$ has trivial image.

Proof: Where can the order $4g$ element go?

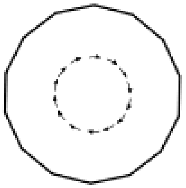
Act 3: the hunt

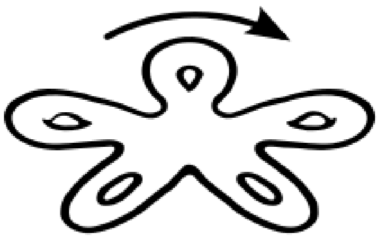






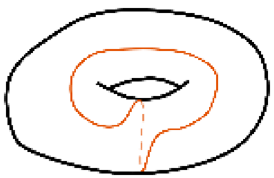






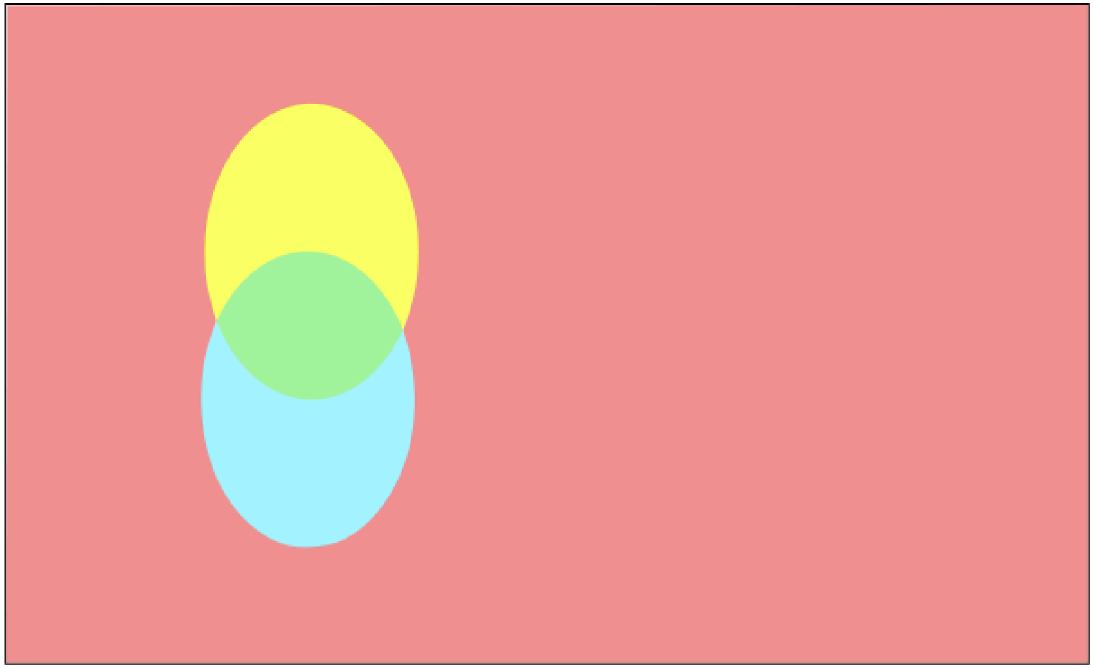
Question. Can the normal closure of a (pseudo-)Anosov map ever be all of M_g?

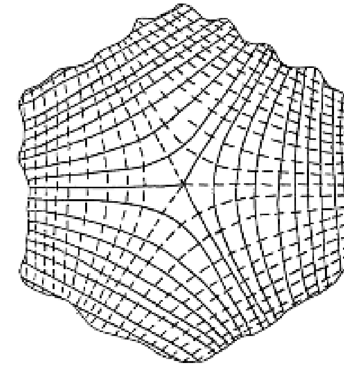
(Long, 1986)







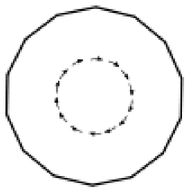


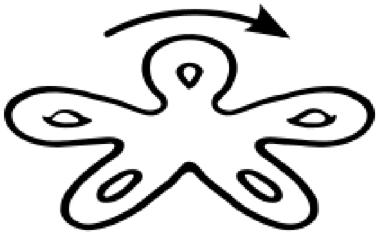


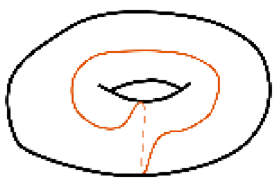
Question. Can the normal closure of a (pseudo-)Anosov map ever be all of M_g?

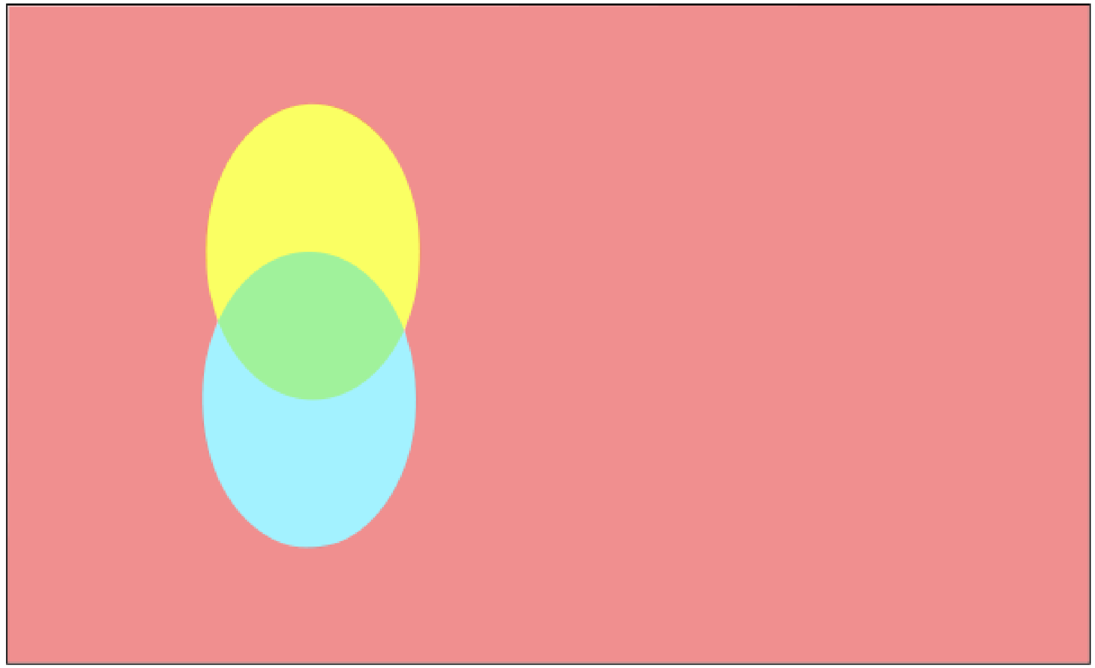
(Long, 1986)

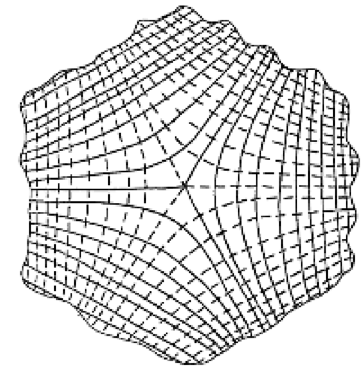
Answer:







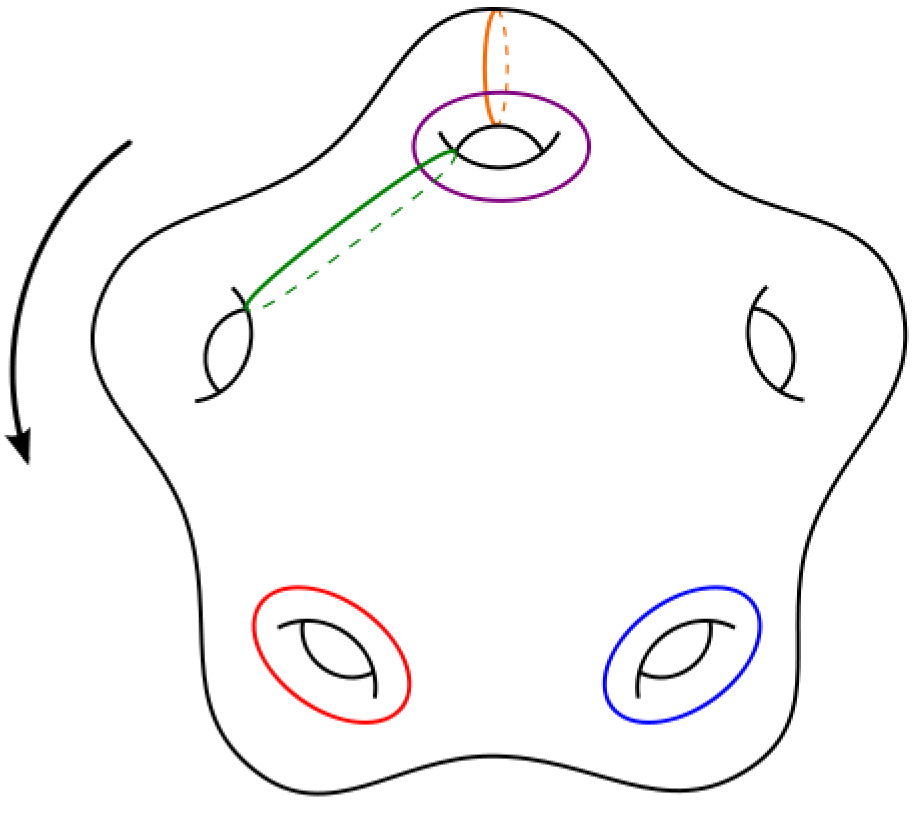




Question. Can the normal closure of a (pseudo-)Anosov map ever be all of M_g?

(Long, 1986)

Answer: Yes!



(Penner, 1988)

Theorem (L.-Margalit, 2017)

For $g \geq 3$, every pseudo-Anosov element with stretch factor less than 1.1 normally generates $\mathrm{Mod}(S_g)$.

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For $g \geq 3$, every pseudo-Anosov element with stretch factor less than 1.1 normally generates $\mathrm{Mod}(S_g)$.

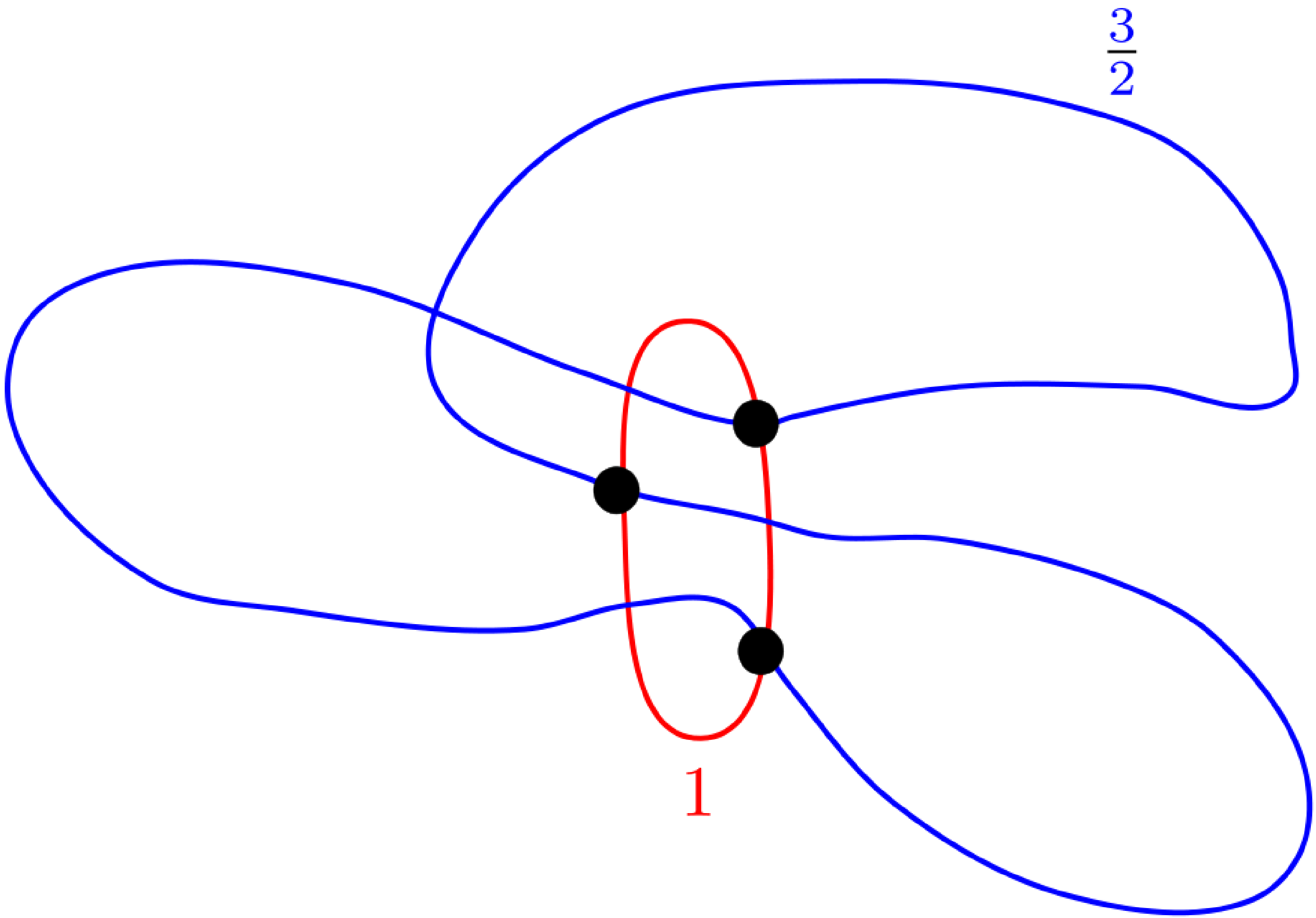
Theorem (Farb-Leininger-Margalit, 2011)

If a pseudo-Anosov lies in the Torelli group, then its stretch fact is at least 1.2.

f with stretch factor less than $3/2$ $\Longrightarrow$ short curve c with $i(c, f(c)) \leq 2$

(Farb-Leininger-Margalit, 2011)

f with stretch factor less than $3/2$ $\Longrightarrow$ short curve c with $i(c, f(c)) \leq 2$



(Farb-Leininger-Margalit, 2011)

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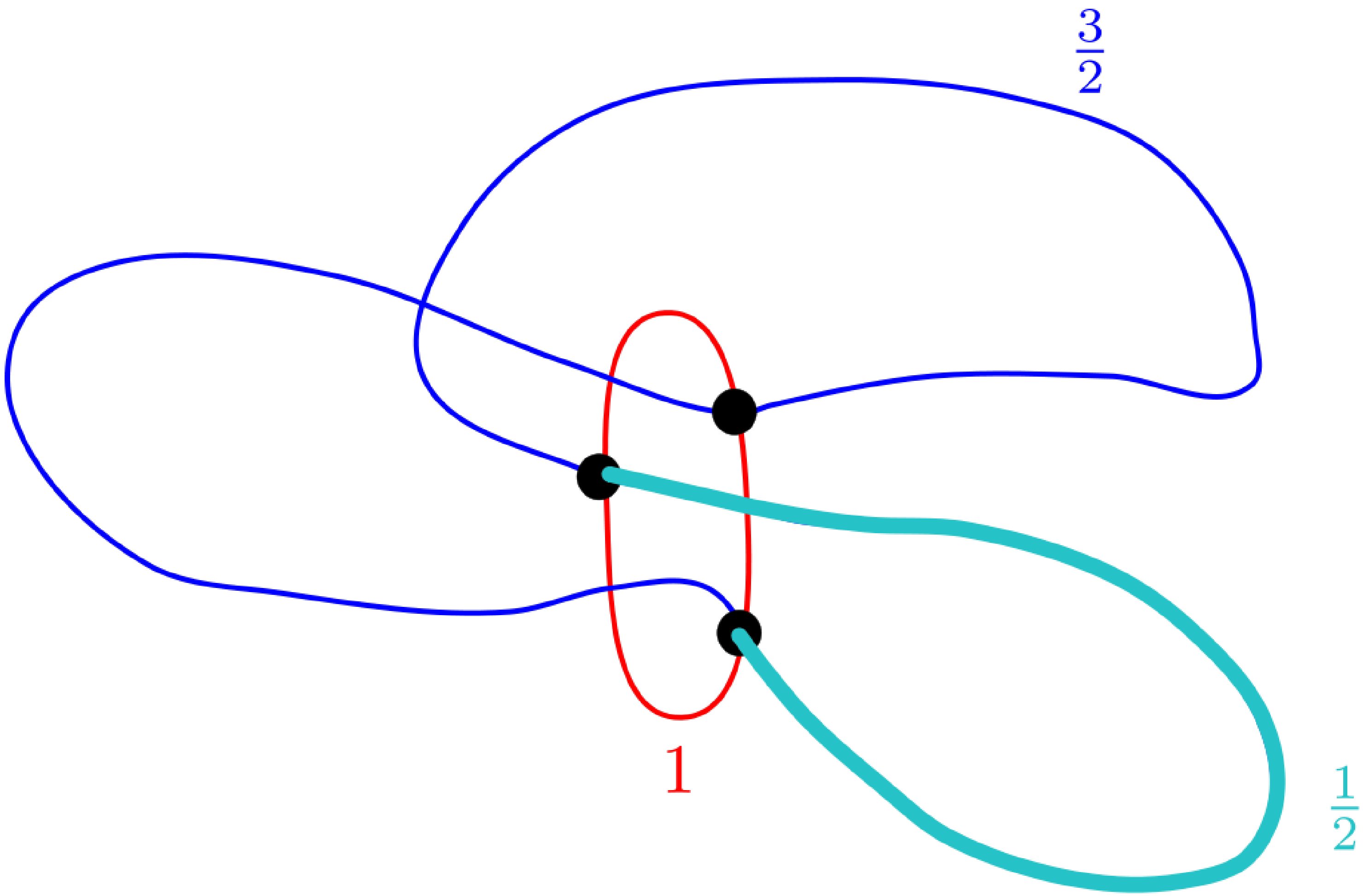

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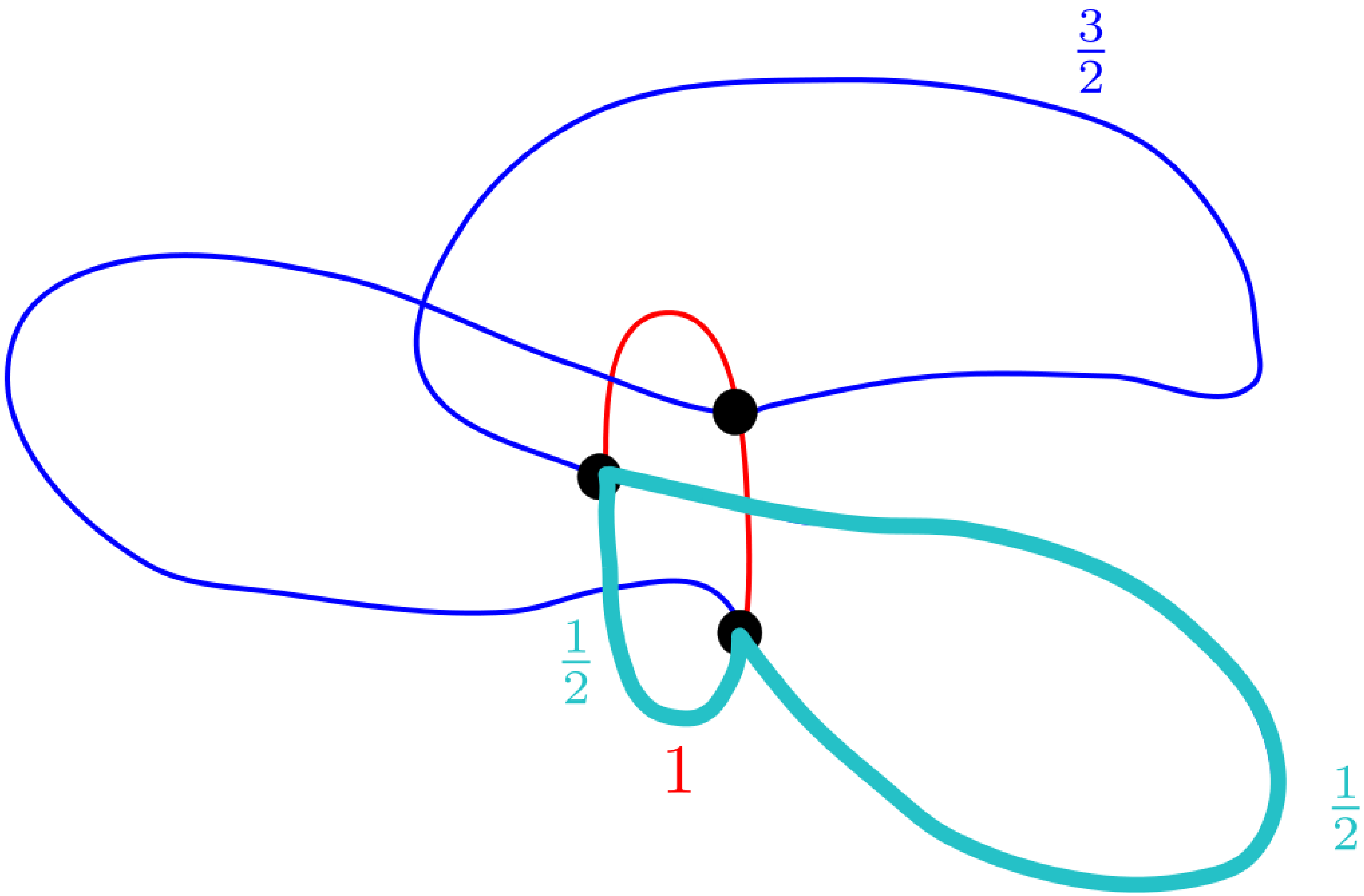

(Farb-Leininger-Margalit, 2011)

$$i(c, f(c)) \leq 2$$

$$i(c, f(c)) \leq 2$$

c nonseparating:

$i(c, f(c)) = 0$, union nonseparating

$i(c, f(c)) = 0$, union separating

$i(c, f(c)) = 1$

$i(c, f(c)) = 2$

c separating:

$i(c, f(c)) = 0$

$i(c, f(c)) = 2$

$$i(c, f(c)) \leq 2$$

c nonseparating:

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c nonseparating:

$i(c, f(c)) = 0$, union nonsepar

$i(c, f(c)) = 0$, union sep g

$i(c, f(c)) = 1$

$i(c, f(c)) =$

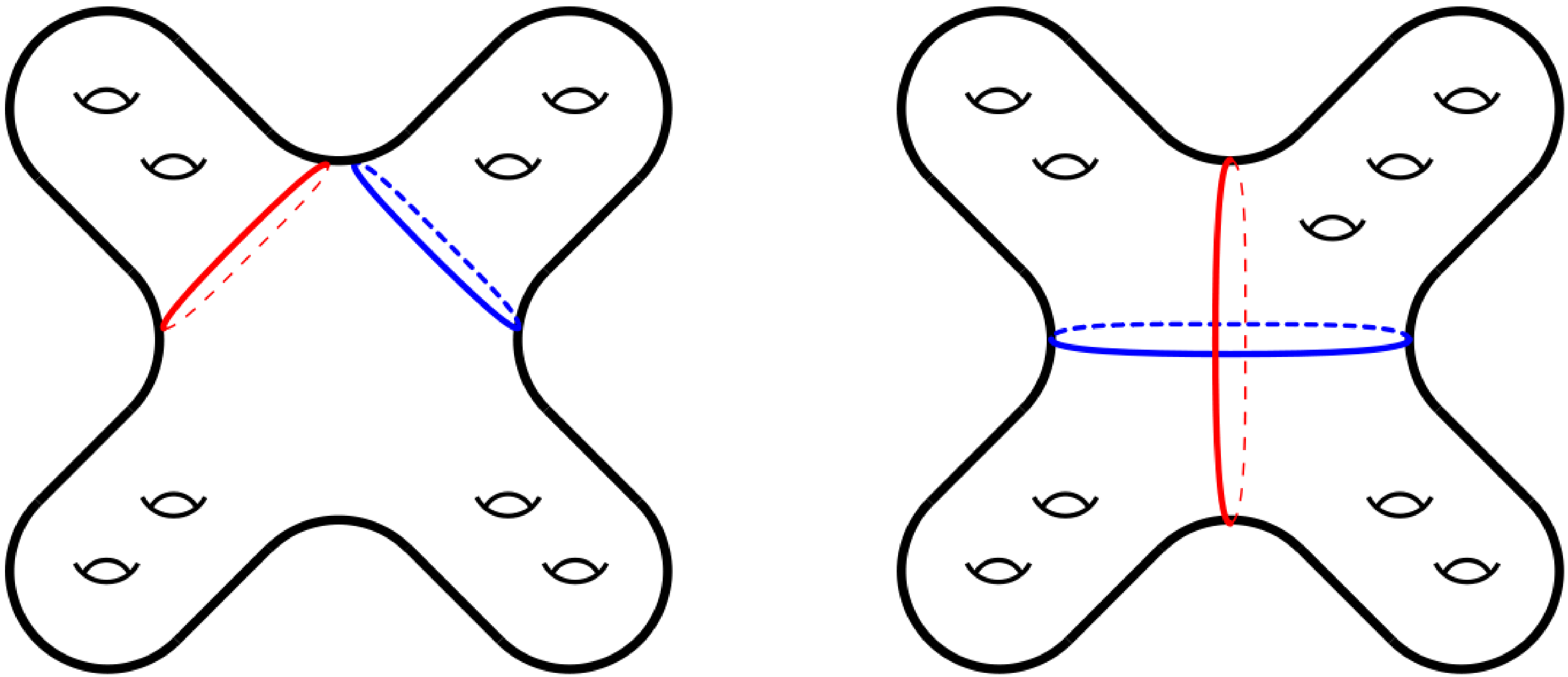
Well-suited curve criteria 2.0!

ating:

$(c, f(c)) = 0$

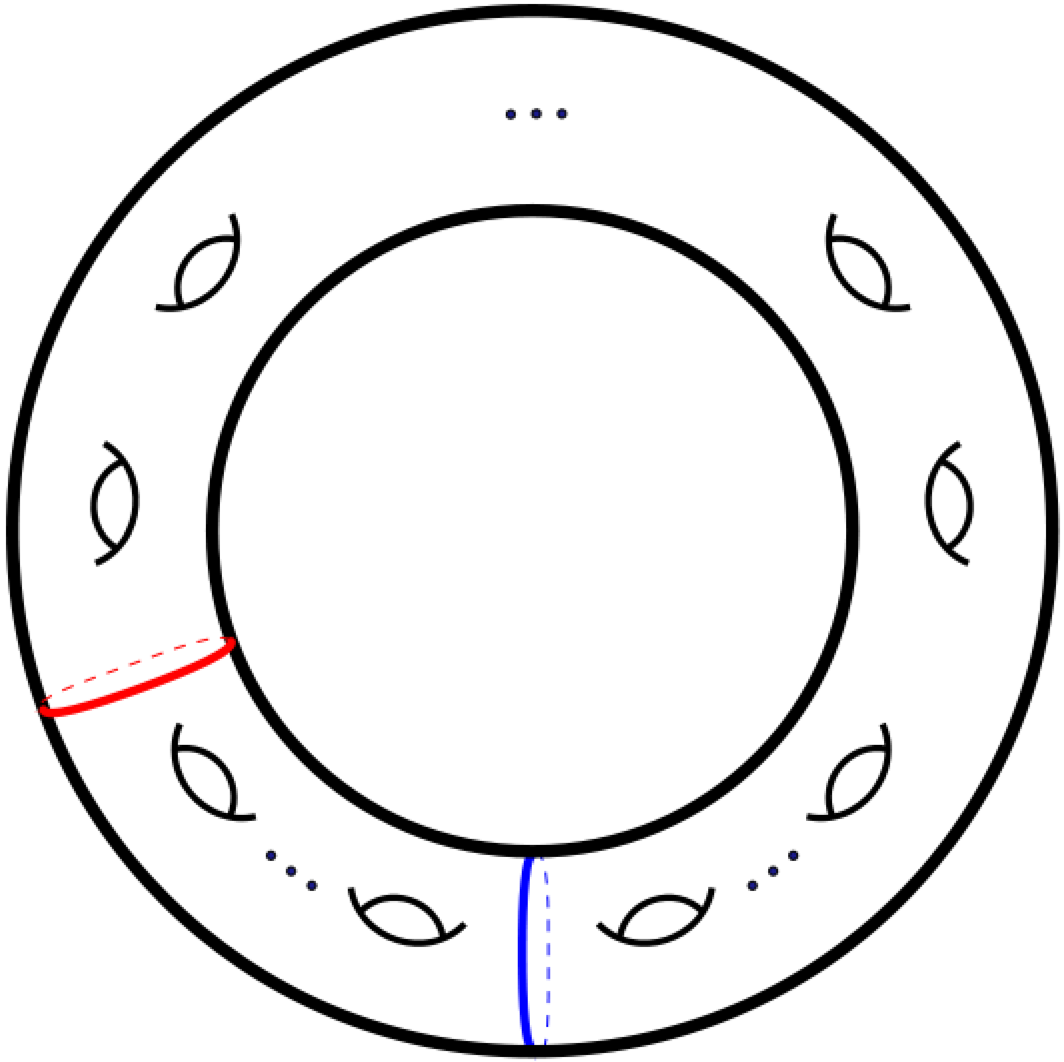
$i(c, f(c)) = 2$

c separating:



$i(c, f(c)) = 0$ $\qquad$ $i(c, f(c)) = 2$

c nonseparating:
$i(c, f(c)) = 0$, union separating



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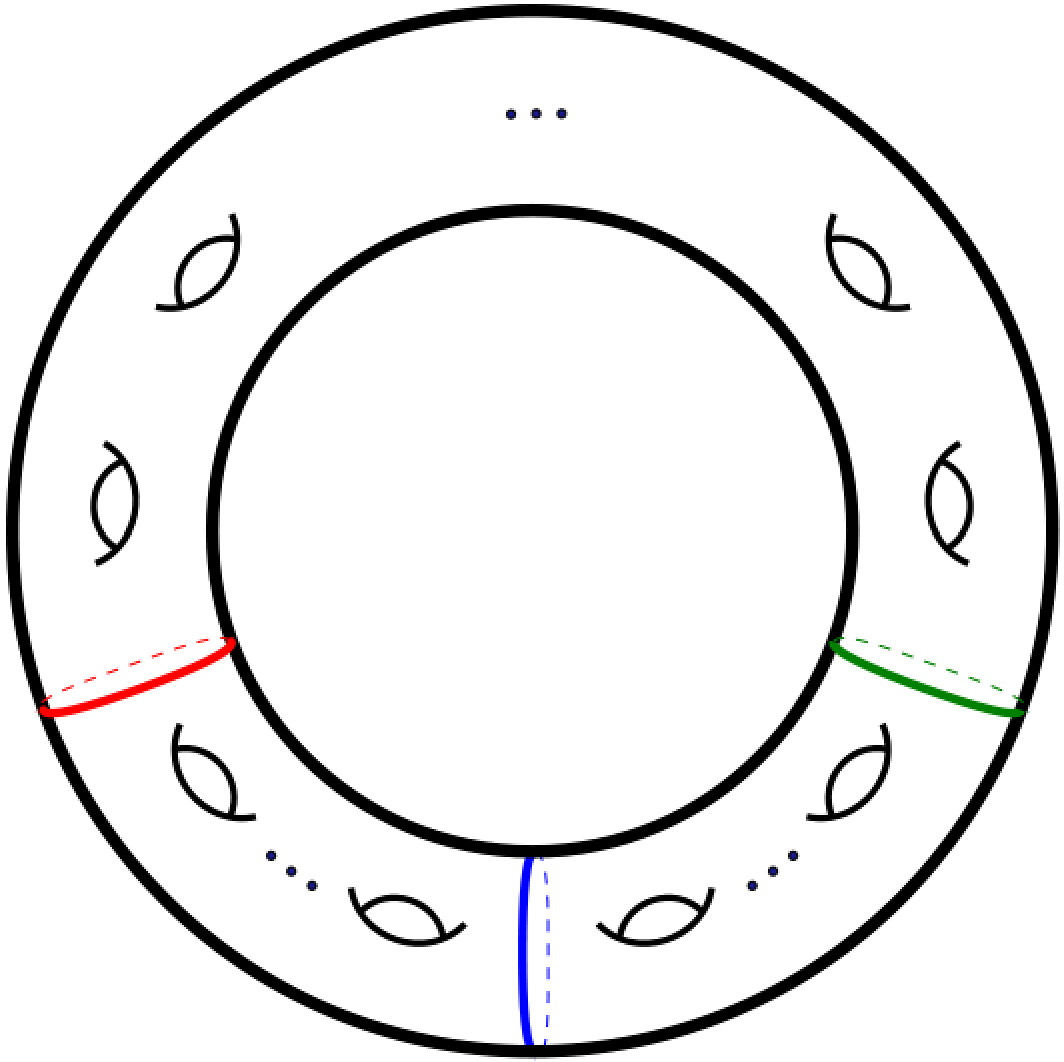

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c nonseparating:

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$i(c, f(c)) = 2$

c separating:

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✓ $i(c, f(c)) = 2$

$$i(c, f(c)) \leq 2$$

c nonseparating:

✓ $i(c, f(c)) = 0$, union nonseparating

✓ $i(c, f(c)) = 0$, union separating

✓ $i(c, f(c)) = 1$

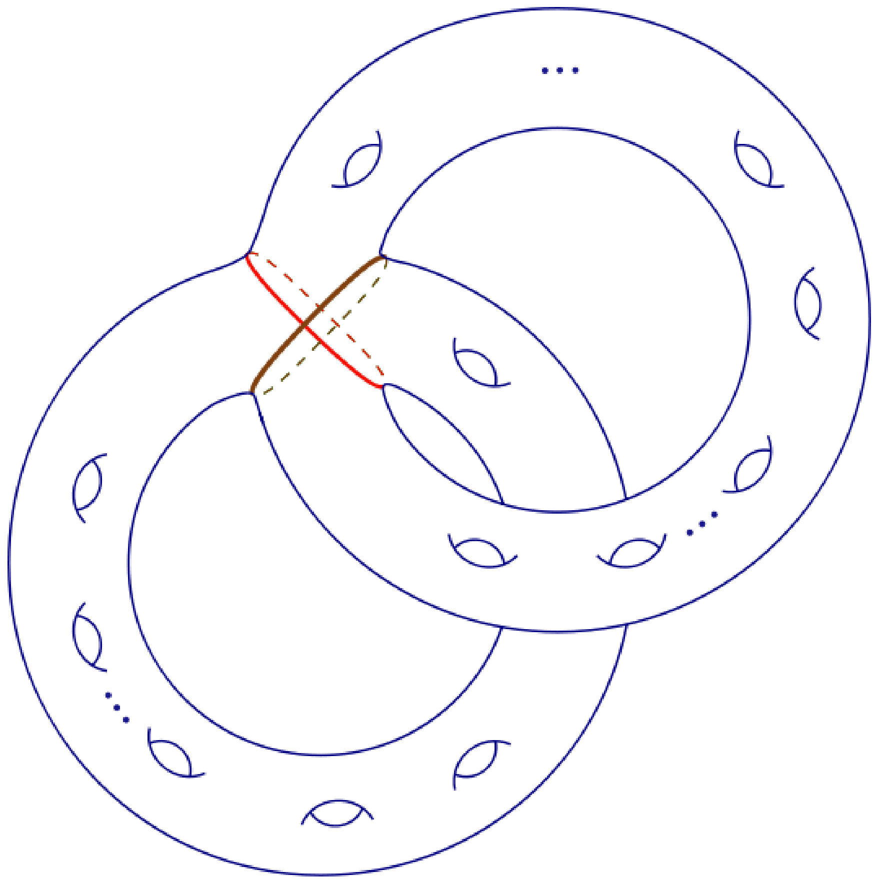
$i(c, f(c)) = 2$

Obstacle

c separating:

✓ $i(c, f(c)) = 0$

✓ $i(c, f(c)) = 2$



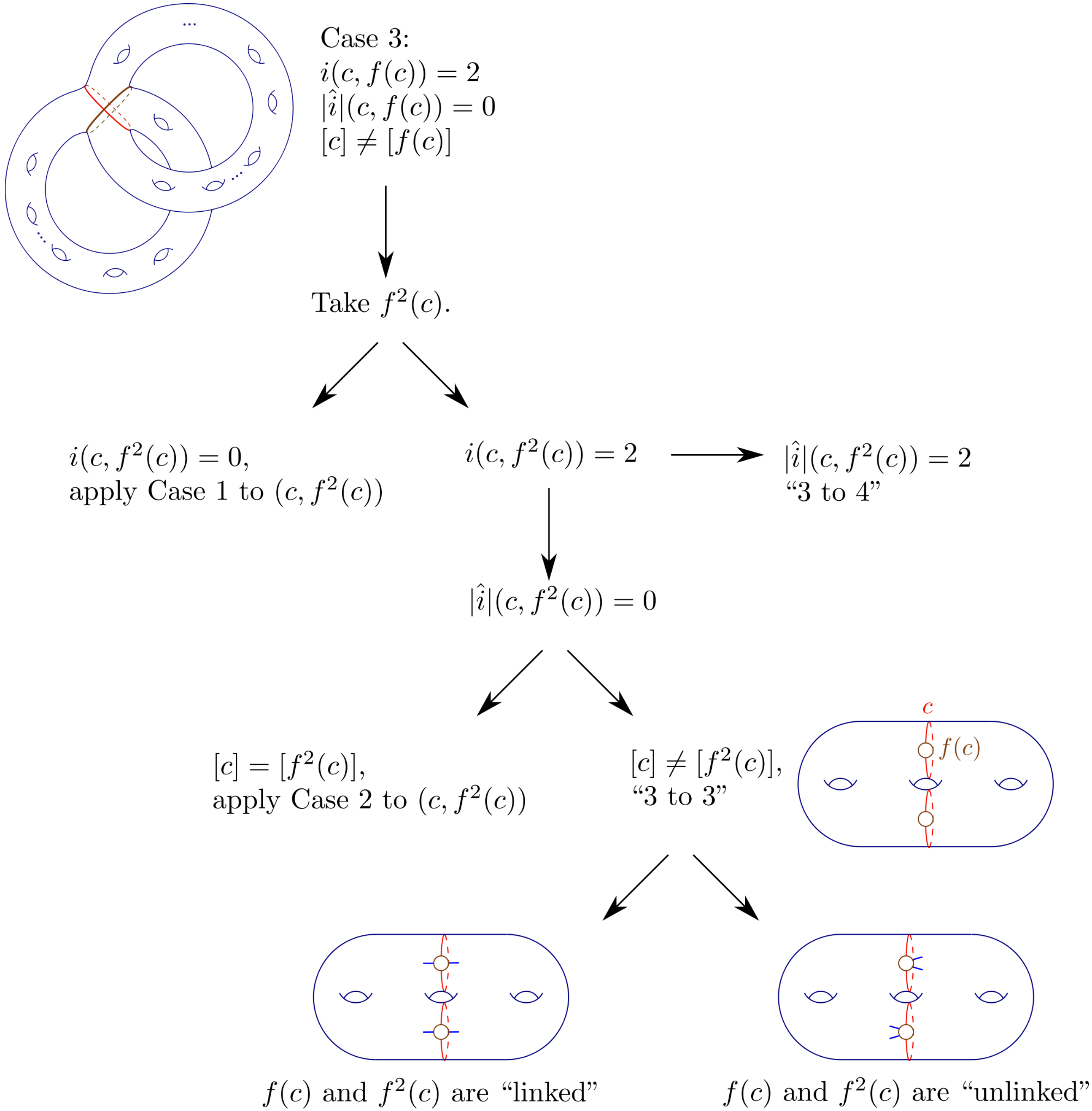
Case 3:

$i(c, f(c)) = 2$

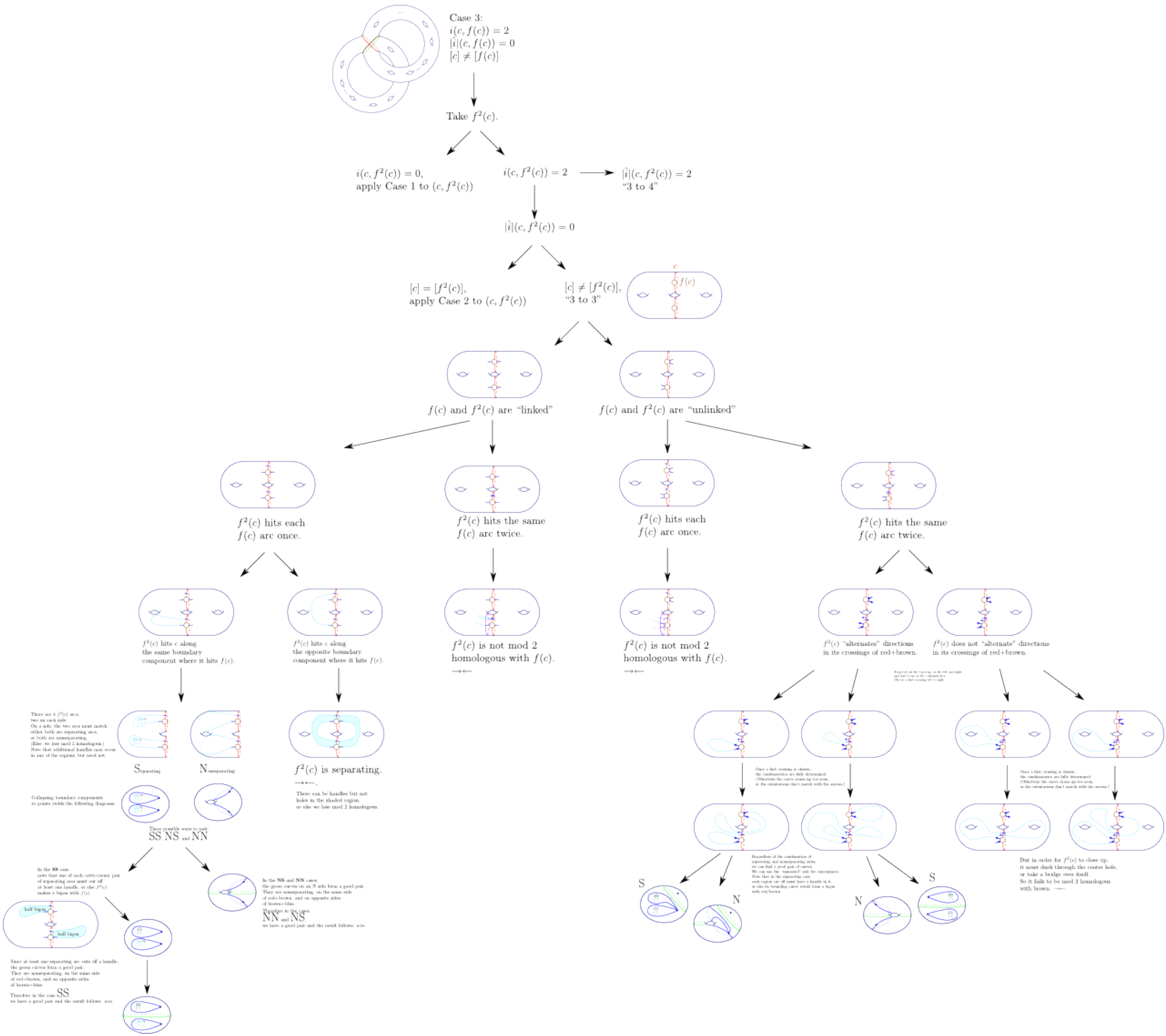
$|\hat{i}|(c, f(c)) = 0$

$[c] \neq [f(c)]$

Take $f^2(c)$.



Case 3:
$i(c, f(c)) = 2$
$|\hat{i}|(c, f(c)) = 0$
$[c] \neq [f(c)]$
Take $f^2(c)$.
$i(c, f^2(c)) = 0$, apply Case 1 to $(c, f^2(c))$
$i(c, f^2(c)) = 2$
$|\hat{i}|(c, f^2(c)) = 2$ "3 to 4"
$|\hat{i}|(c, f^2(c)) = 0$
$[c] = [f^2(c)]$, apply Case 2 to $(c, f^2(c))$
$[c] \neq [f^2(c)]$, "3 to 3"
c
$f(c)$
$f(c)$ and $f^2(c)$ are "linked"
$f(c)$ and $f^2(c)$ are "unlinked"



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$i(c, f^2(c)) = 2$
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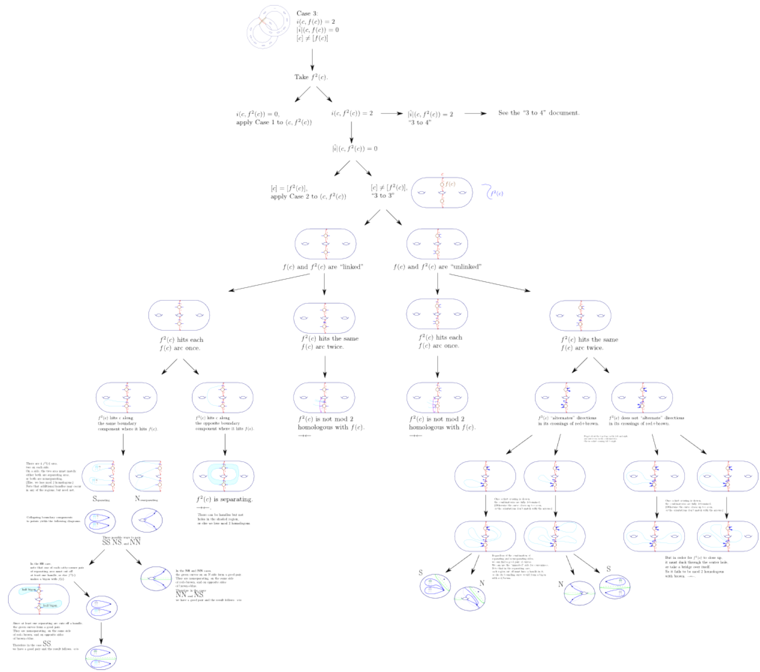
$|\hat{i}|(c, f^2(c)) = 0$
$[c] = [f^2(c)]$, apply Case 2 to $(c, f^2(c))$
$[c] \neq [f^2(c)]$, "3 to 3"
c
$f(c)$
$f(c)$ and $f^2(c)$ are "linked"
$f(c)$ and $f^2(c)$ are "unlinked"
$f^2(c)$ hits each $f(c)$ arc once.
$f^2(c)$ hits the same $f(c)$ arc twice.
$f^2(c)$ hits each $f(c)$ arc once.
$f^2(c)$ hits the same $f(c)$ arc twice.
$f^2(c)$ is not mod 2 homologous with $f(c)$.
$f^2(c)$ is not mod 2 homologous with $f(c)$.
$f^2(c)$ is separating.
S
N
S
N
N
S
SS
NN
NS



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$i(c, f(c)) = 2$
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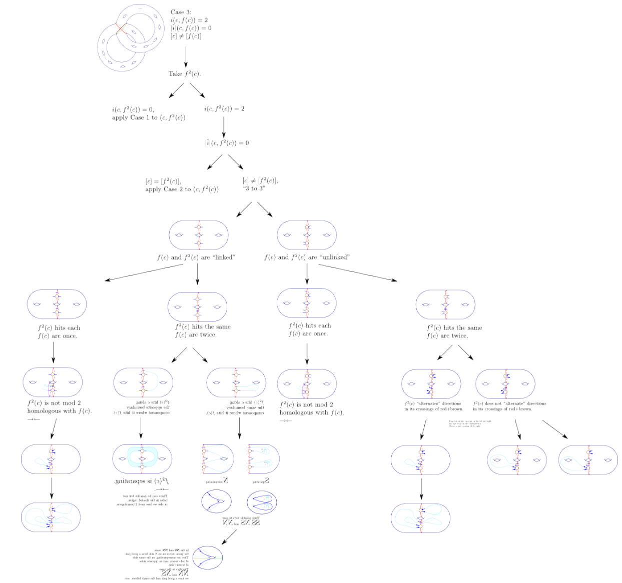
$i(c, f^2(c)) = 2$
$\hat{i}(c, f^2(c)) = 2$
"3 to 4"
See the "3 to 4" document.
$\hat{i}(c, f^2(c)) = 0$
$[c] = [f^2(c)]$,
apply Case 2 to $(c, f^2(c))$
$[c] \neq [f^2(c)]$,
"3 to 3"
$f(c)$ and $f^2(c)$ are "linked"
$f(c)$ and $f^2(c)$ are "unlinked"
$f^2(c)$ hits each
$f(c)$ arc once.
$f^2(c)$ hits the same
$f(c)$ arc twice.
$f^2(c)$ is not mod 2
homologous with $f(c)$.
$f^2(c)$ "alternates" directions
in its crossings of red+brown.
$f^2(c)$ does not "alternate" directions
in its crossings of red+brown.
$f^2(c)$ is separating.



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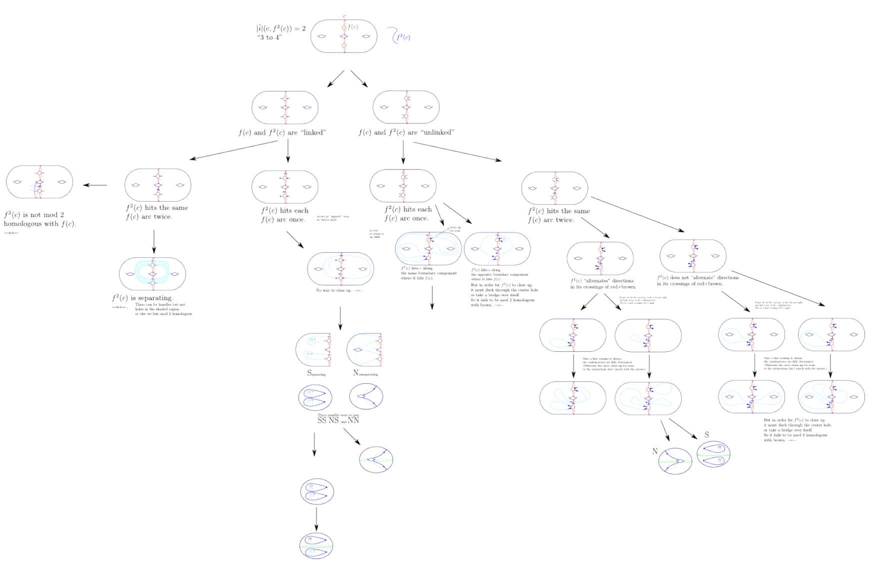

$\hat{i}(c, f^2(c)) = 2$
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$f^2(c)$ is not mod 2
homologous with $f(c)$.
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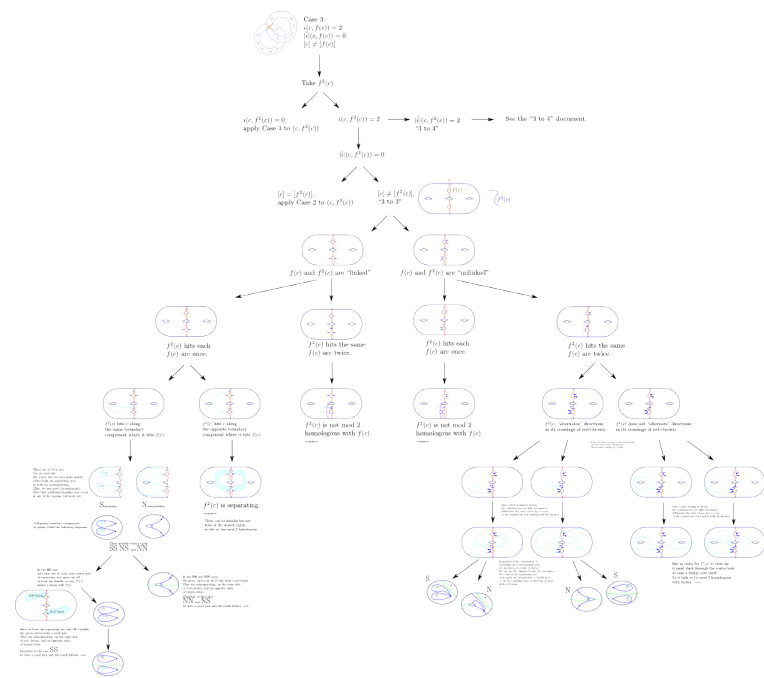
$f(c)$ arc twice.
$f^2(c)$ hits each
$f(c)$ arc once.
$f^2(c)$ is separating.
No way to close up.
$f^2(c)$ "alternates" directions
in its crossings of red+brown.
$f^2(c)$ does not "alternate" directions
in its crossings of red+brown.
SS NS and NN



Case 3:
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$i(c, f^2(c)) = 0$,
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"3 to 4"
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$[c] = [f^2(c)]$,
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Theorem (L.-Margalit, 2017)

For $g \geq 3$, every pseudo-Anosov element with stretch factor less than 1.1 normally generates $\mathrm{Mod}(S_g)$.

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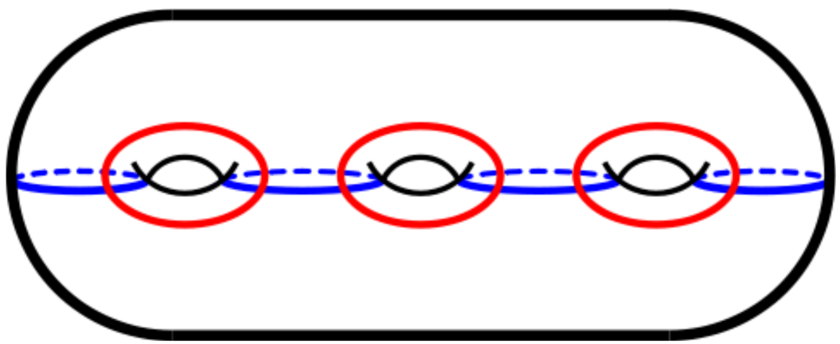
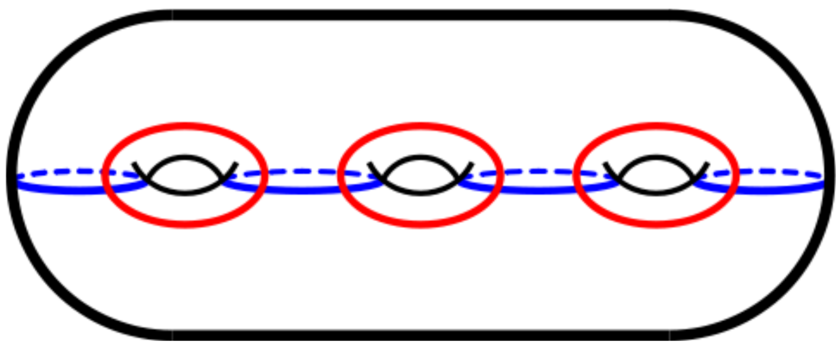


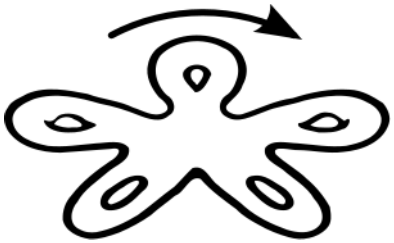
Theorem (Dahmani-Guirardel-Osin, 2017)

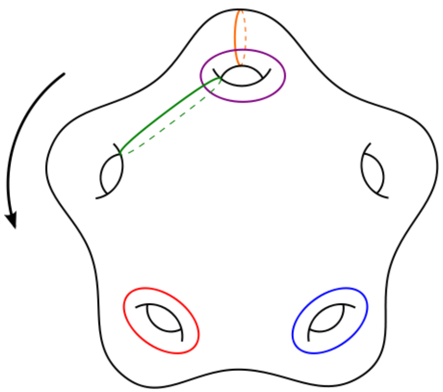
There exist pseudo-Anosovs whose normal closures are infinitely-generated, all pseudo-Anosov free groups.

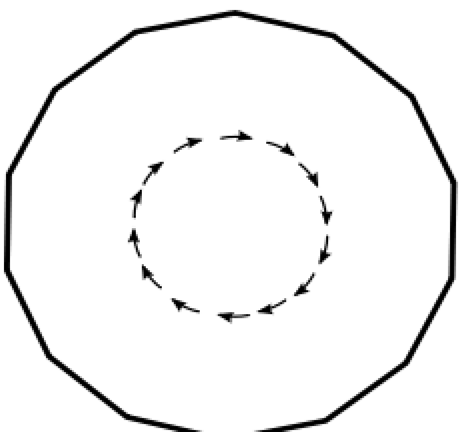


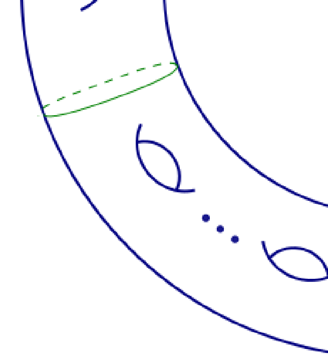
$\Longrightarrow$ large stretch factor pseudo-Anosovs

Normal generators
for mapping class groups
are abundant.









Theorem (L., 2017)

Let $k \geq 6$ and $g \geq (k-1)^2 + 1$.
Then $\mathrm{Mod}(S_g)$ is generated by three elements of order k.

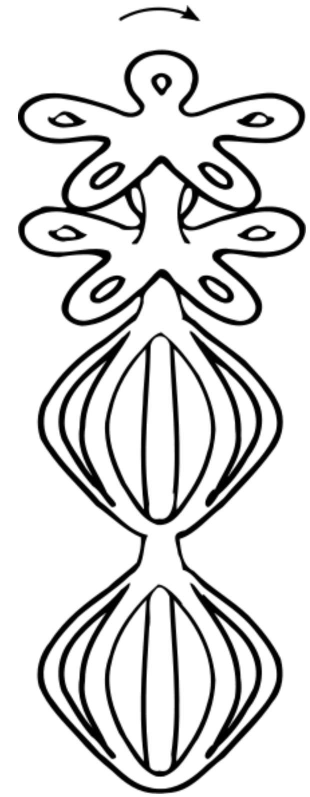
Also, $\mathrm{Mod}(S_g)$ is generated by four elements of order 5 when $g \geq 8$.

Theorem (L.-Margalit, 2017)

For $g \geq 3$, every periodic mapping class that is not a hyperelliptic involution normally generates $\mathrm{Mod}(S_g)$.

Theorem (L.-Margalit, 2017)

For $g \geq 3$, every pseudo-Anosov element with stretch factor less than 1.1 normally generates $\mathrm{Mod}(S_g)$.



Thanks.