Juanita Pinzon-Calcedo   Satellites of infinite rank in concordance g.p.

Review

$$\mathcal{C}_\infty \twoheadrightarrow \mathcal{C}_{top} \twoheadrightarrow \mathcal{C}_{alg} \twoheadrightarrow 1.$$

Def K_1 and K_2 are $\dfrac{\text{smoothly}}{\text{topologically}}$ concordant if $K_1 \# m(K_2)$ bounds a disk $\mathbb{D}^2 \xhookrightarrow[\text{locally flat}]{\text{smooth}} \mathbb{B}^4$.

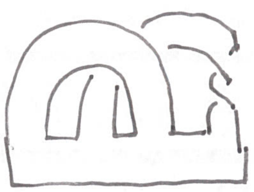
Thm (Fox-Milnor 66) $\mathcal{C}_\infty$ and $\mathcal{C}_{top}$ are abelian groups.

Q. Are $\mathcal{C}_\infty$, $\mathcal{C}_{top}$, $\mathcal{C}_{alg}$ different?

- Non-trivial?
- Finite / infinite order?
- Linearly independent families?

Concordance: Study of knots from the perspective of surfaces.

Thm [Seifert 34] For every knot $K \subseteq S^3$, there exists a surface $F \subseteq S^3$ s.t. $\partial F = K$.



$$g_3(K) = \min\{\, \text{genus}(F) \mid F \subseteq S^3 \ \& \ \partial F = K \,\}.$$

Def For a Seifert surface F of K,

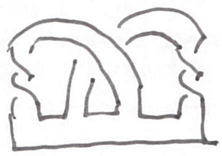
$$V = \begin{bmatrix} lk(a, a^+) & lk(a, b^+) \\ lk(b, a^+) & lk(b, b^+) \end{bmatrix}$$

is called a Seifert matrix.

Note
Alexander poly
$\Delta_K(t) = \det(V - tV^T)$
Levine-Tristram sig
$\sigma_K(\omega) = \operatorname{sig}\left((1-\omega)V + (1-\bar{\omega})V^T\right)$ $\omega \in S^1$

Facts

· $T_{2,3}$



$\sigma_{T_{2,3}}(-1) = -2 \Rightarrow T_{2,3}$ has infinite order.

Thm [Fox-Milnor] K slice $\Rightarrow \Delta_K(t) = f(t)\cdot f(t^{-1})$.

Thm [Levine, Tristram] · K slice $\Rightarrow \sigma_K(\omega) = 0$ for every $\omega \in S^1$.

· $\sigma_{K_1 \# K_2}(\omega) = \sigma_{K_1}(\omega) + \sigma_{K_2}(\omega)$.

Thm [Litherland] $\{T_{p,q}\}$ are linearly independent in $\mathcal{C}_{alg}$.

Def A satellite knot $P(K)$ is the result of tying up $P \subseteq D^2 \times S^1$ into K.

For $P(K)$ a satellite knot

· $\Delta_{P(K)}(t) = \Delta_{P(U)}(t) \cdot \Delta_K(t^w)$.

$w=0 \quad \Delta_{P(K)}(t) = \Delta_{P(U)}(t)$

· $\sigma_{P(K)}(\omega) = \sigma_{P(U)}(\omega) + \sigma_K(\omega^w)$.

$w=0 \quad \sigma_{P(K)}(\omega) = \sigma_{P(U)}(\omega)$

For $P: \mathcal{C}_\infty \longrightarrow \mathcal{C}_\infty$ injective / surjective are hard questions

we ask a different one!

Q. Does there exist a L.I. family $\{K_i\}_{i\in\mathbb{N}}$ s.t. $\{P(K_i)\}_{i\in\mathbb{N}}$ is also L.I.?

Answer: YES! In most cases. Moreover $K_i = T_{p_i,q_i}$

Case I: $w \neq 0$ follow the proof of L.I. of torus knots and use $\sigma(w)$.

Case II: $w = 0$ interesting.

<u>Thm</u> [Freedman] If K has a trivial Alexander polynomial, then K is topologically slice.

$\left(\mathrm{lk}(a,\eta), \mathrm{lk}(b,\eta) \right) = \vec{x}$ $\quad V$ seifert matrix for $P \subseteq S^3 \setminus \eta$.

$$\ell = \vec{x}\,(V+V^T)^{-1}\,\vec{x}^{\,T}$$

<u>Thm</u> [Hedden-PC] If $\ell \neq 0$, then image of $P: \mathcal{C} \to \mathcal{C}$ has infinite rank.

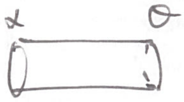
The proof involves SO(3)-gauge theory and Chern-Simons invariants.

<u>Thm</u> [Furuta, Fintushel-Stern, Hedden-Kirk].

For 3-mfld Σ, let $\tau(\Sigma) = \min\{ CS(\alpha,\theta) \mid \alpha$ is a flat connection $\}$.
θ is a trivial connection

ASD connection

$$CS(\alpha,\theta) = \int_{\Sigma\times[0,1]} \mathrm{Tr}\left(F(A_t)\wedge F(A_t)\right)$$



Let $\{\Sigma_i\}_{i=1}^{N}$ be a family of 3-mflds with $H_*(\Sigma;\mathbb{Z}_2) \cong H_*(S^3;\mathbb{Z}_2)$.

and $\Sigma_{N+1} = S^3_{r/s}(T_{p,q})$.

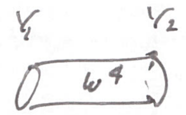
$$\frac{s}{r(spq-r)} \leq \min\left\{ \tau(\Sigma_1), \tau(-\Sigma_1), \dots, \tau(-\Sigma_N), \frac{1}{r}, \frac{1}{s}, \frac{1}{spq-r} \right\}.$$

then no combination of the Σ_i's cobounds a 4-mfld X

with $H_1(X;\mathbb{Z}_2)=0$ and negative definite.

Consequence.

$\Theta^3_{\mathbb{Z}_2}$: Objects : classes of 3-mflds s.t. $Y_1 \sim Y_2$



if $\exists\, W^4$ smooth $H_*(W;\mathbb{Z}_2) \cong H_*(B^4;\mathbb{Z}_2)$

$\partial W = Y_1 \amalg -Y_2$.

$\Sigma_2 : \mathcal{C}_{\text{smooth}} \longrightarrow \Theta^3_{\mathbb{Z}_2}$ is a homomorphism.

Translate $\#_i K_i = \partial D^2$ into $\#_i \Sigma_2(K_i) = \partial W^4$.