Lightning Talks III
Tech Topology Conference

December 8, 2019

Statistics of Random Square-tiled Surfaces

Sunrose Shrestha
Tufts University

Square-tiled Surfaces (STSs)

Finite collection of axis parallel Euclidean unit squares, glued edge-to-edge via translations.

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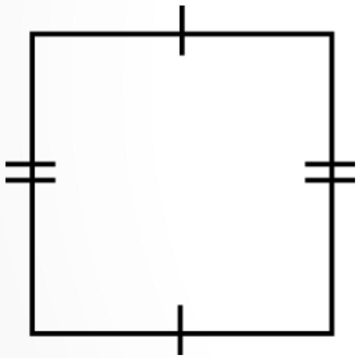

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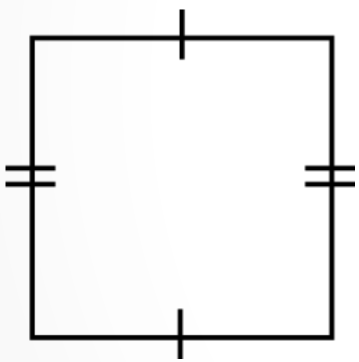

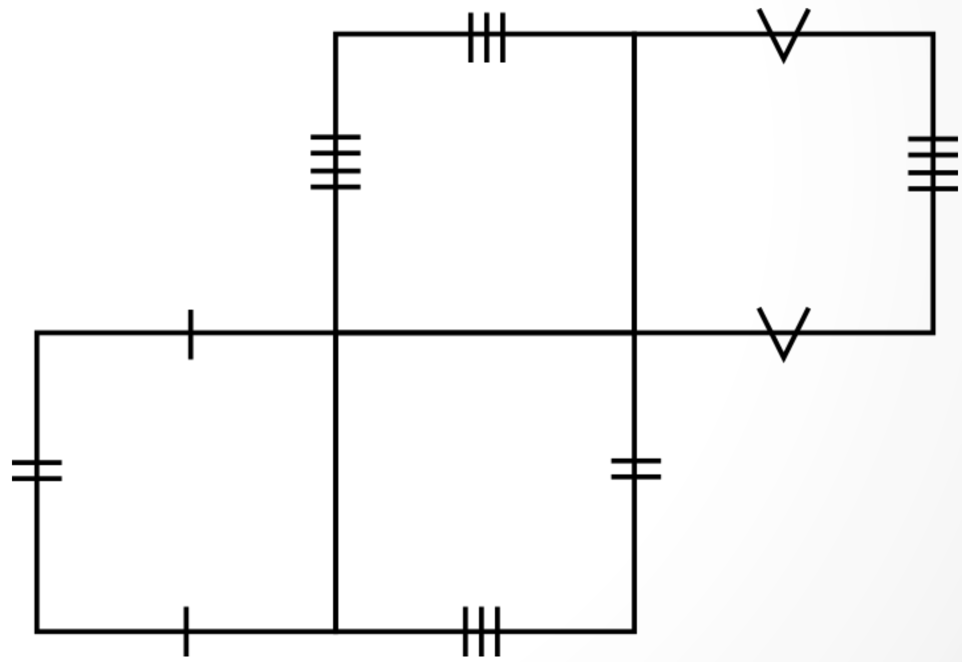

Cone points

STSs are flat except for finitely many points, called *cone points / singularities* with angle $2\pi(k+1)$ for $k \geq 1$

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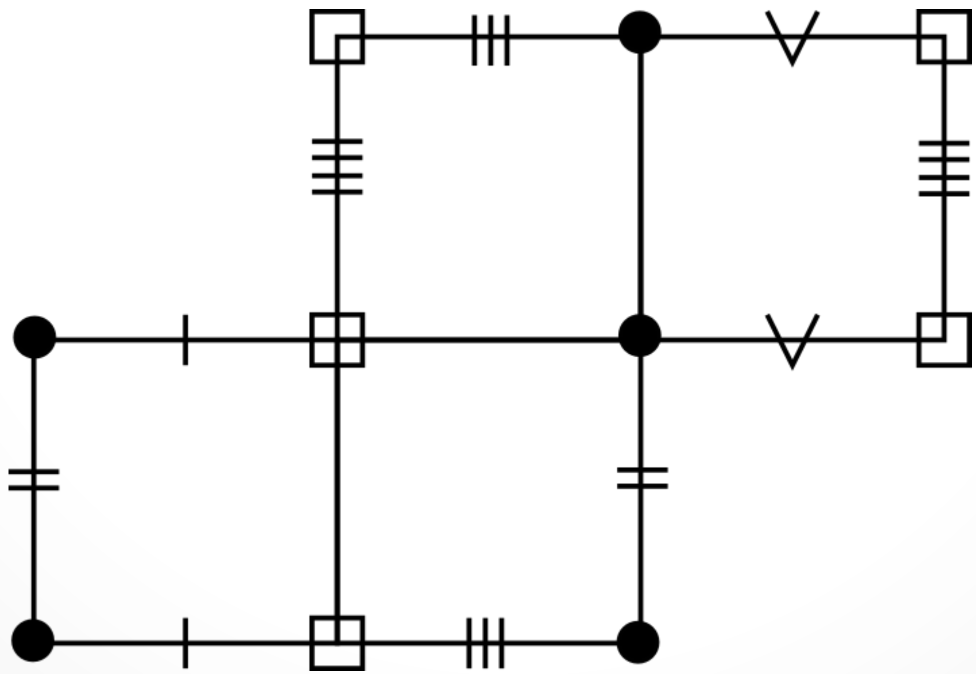

Saddle Connection/Holonomy Vector

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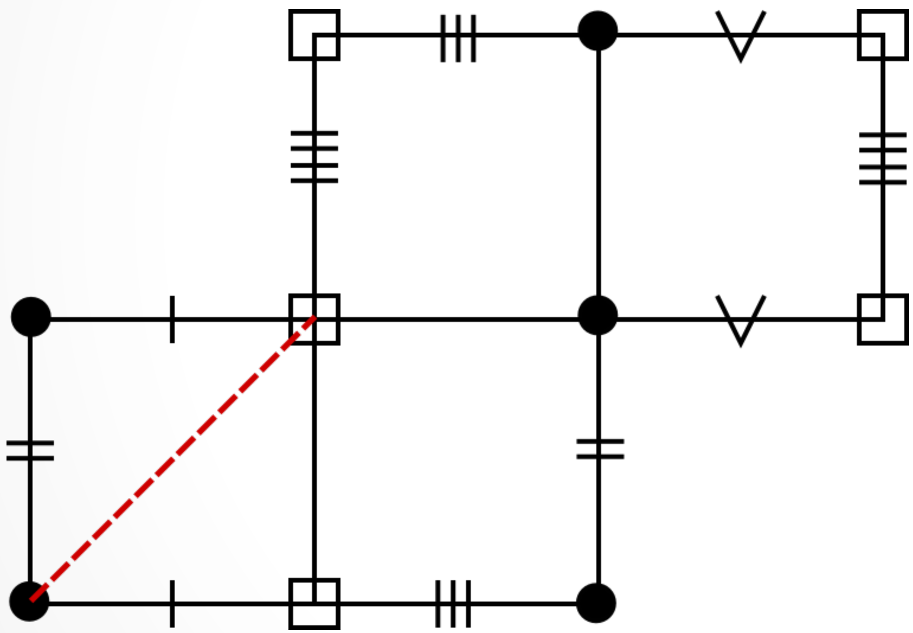

(1,1)

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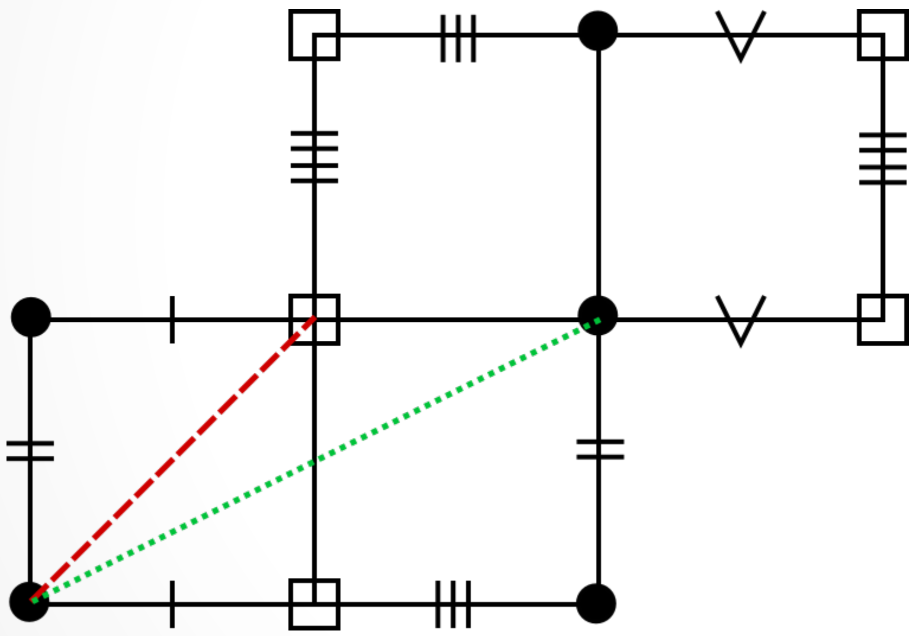

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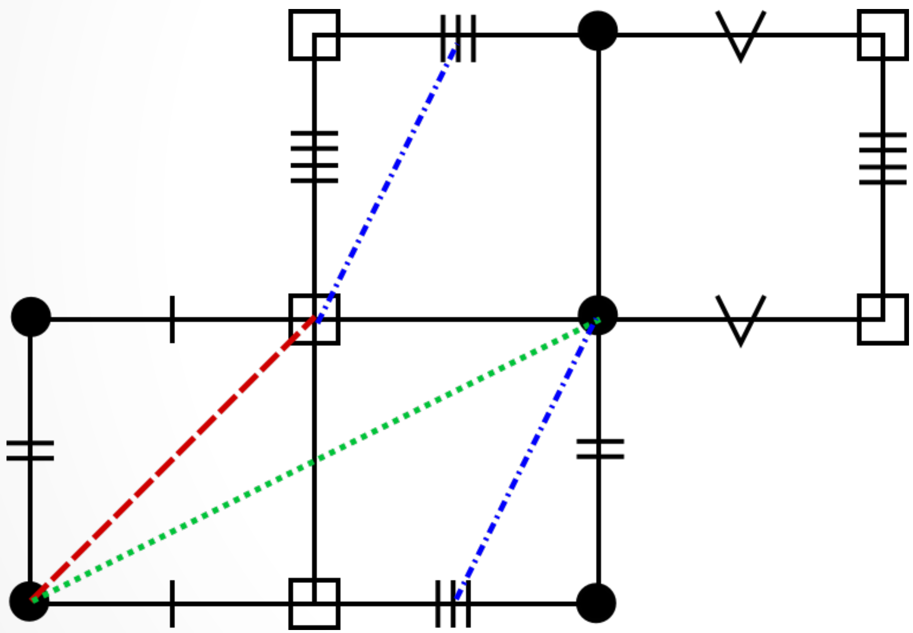

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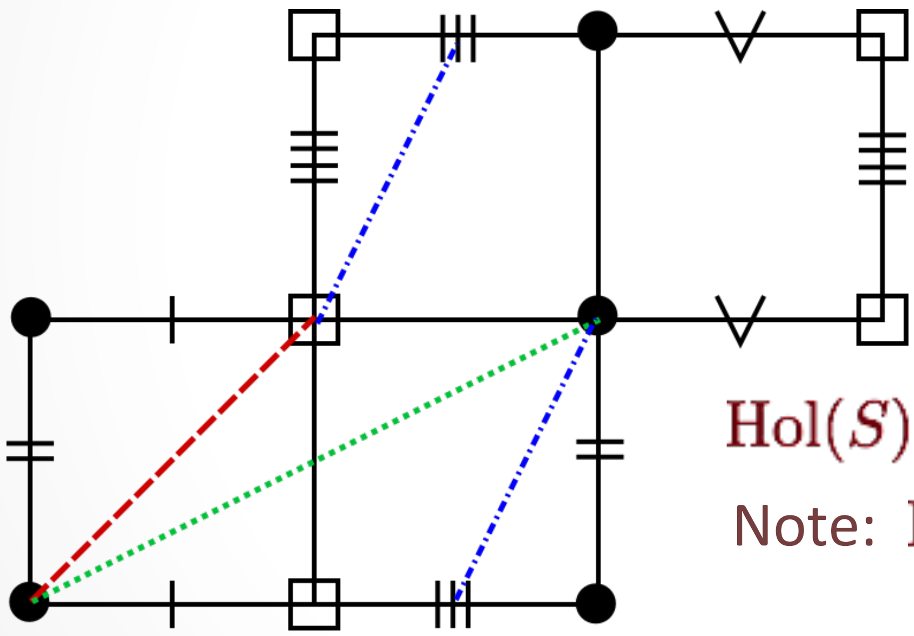

(1,1)

(2,1)

(1,2)

$\mathrm{Hol}(S)$ for holonomy vectors of S.

Note: $\mathrm{Hol}(S) \subset \mathbb{Z}^2$

Holonomy Examples

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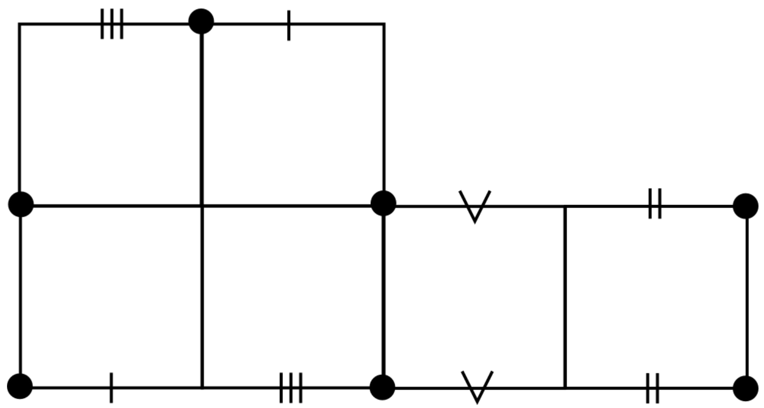

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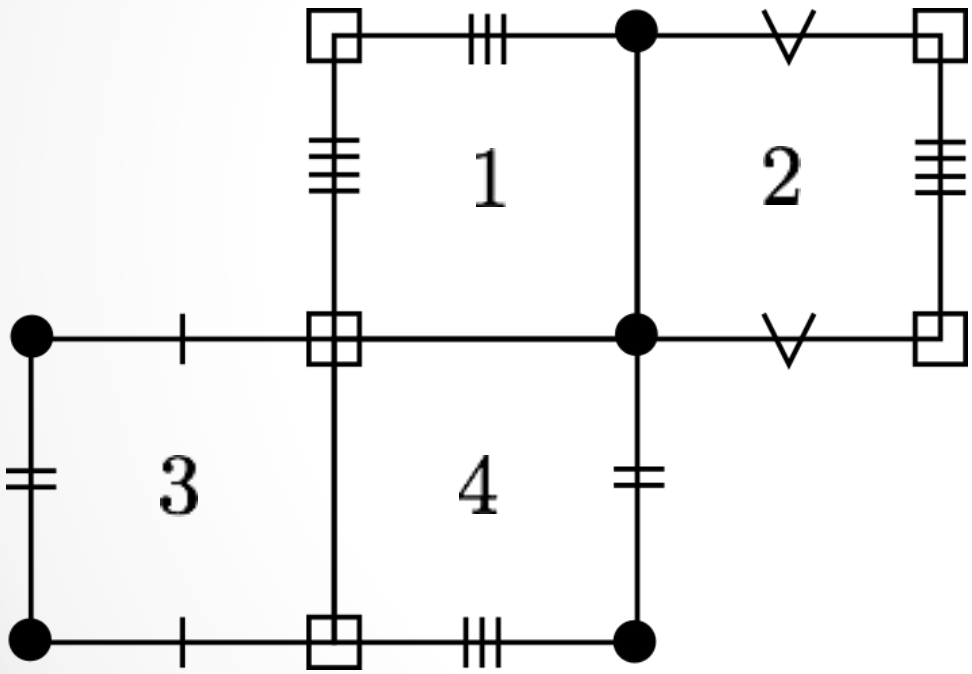

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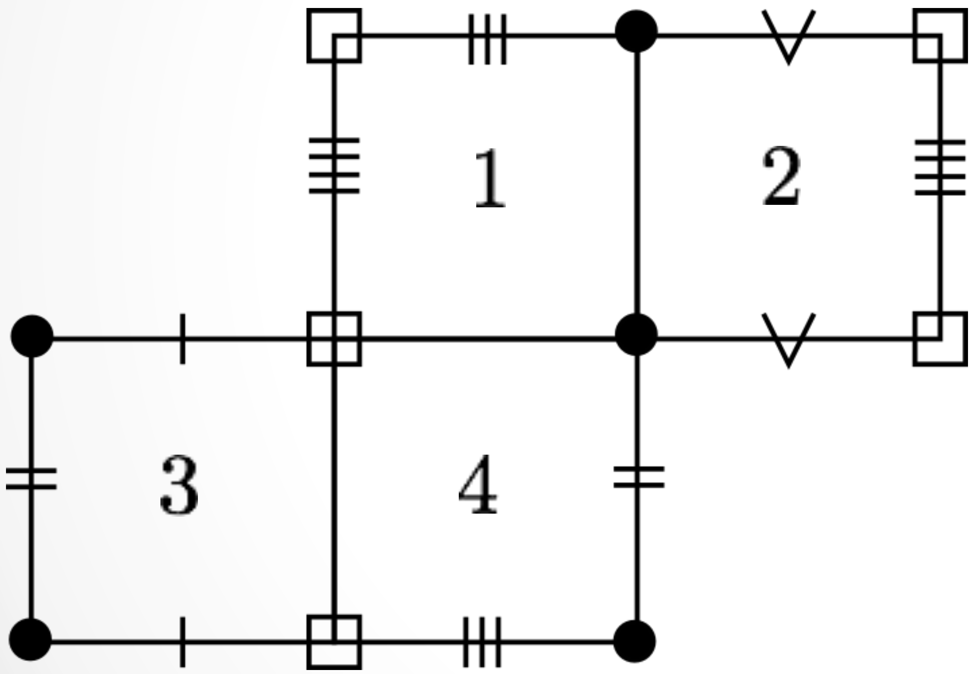

$$\sigma = (12)(34)$$

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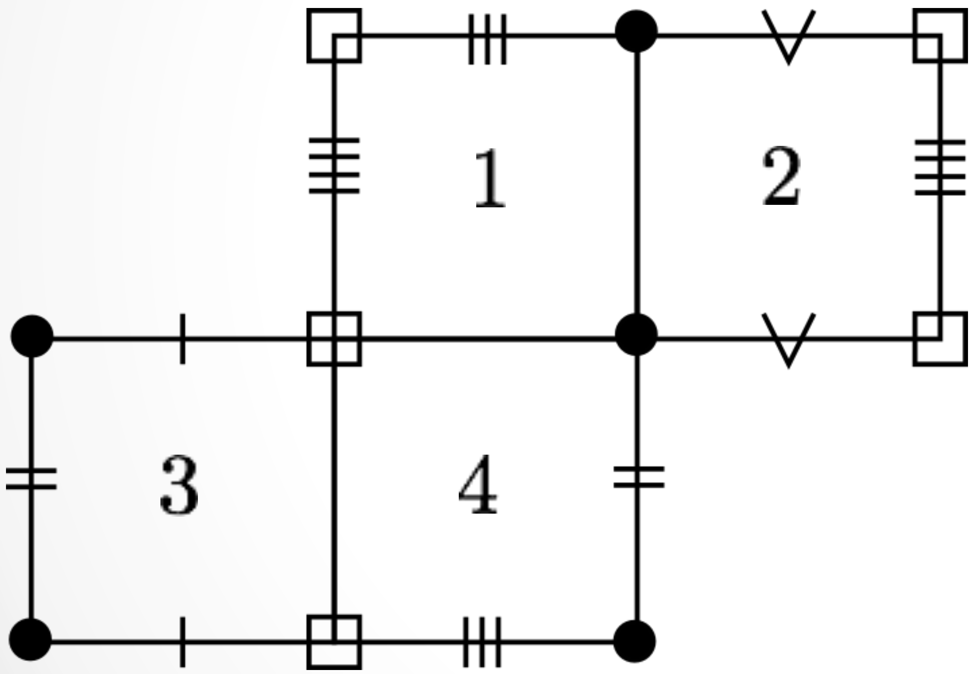

$\sigma = (12)(34)$

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Topology Result

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$$\mathbb{E}(genus) = \frac{n}{2} - \frac{\ln n}{2} - \gamma + o(1)$$

Note:

- In fact, distribution is asymptotically normal.
- My method generalizes to other even-gon-tiled surfaces.

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Theorem 2 (S): For a random n-square-tiled surface, S

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$$\Pr(S \text{ has } \mathrm{Hol}(S) \supset \mathrm{RP}) \to 1 \text{ as } n \to \infty$$

Demo!
(time permitting..)

Thank You!

The Word Problem for ART $\left(\widetilde{A_2}\right)$

Tech Topology Conference, Georgia Institute of Technology

Ashlee Kalauli

December 8, 2019

The Word Problem for Artin Groups

The Word Problem: (Dehn 1910)

Given a group $G = \langle S \mid R \rangle$ with a finite generating set S and relations R, can you decide which words are equivalent to the identity?

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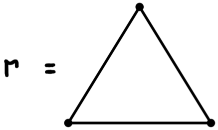

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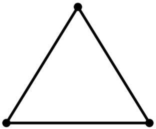
r =



$$\mathrm{ART}\left(\widetilde{A_2}\right) = \langle a, b, c \mid aba = bab, bcb = cbc, aca = cac \rangle$$

A Solution

Theorem (McCammond, Sulway, 2017):

$\textsc{Art}(\widetilde{A_2})$ is a torsion-free, centerless group with a solvable word problem.

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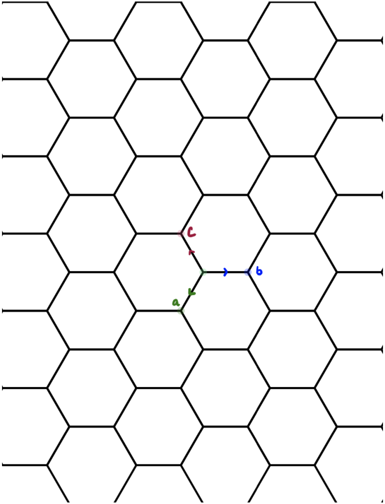
$$\begin{array}{ccccc} \textsc{Art}(\widetilde{A_2}) & \cong & \textsc{Art}^*(\widetilde{A_2}, w) & \hookrightarrow & \textsc{Gar}(\widetilde{A_2}, w) \\ \downarrow\!\!\!\downarrow & & \downarrow\!\!\!\downarrow & & \\ \textsc{Cox}(\widetilde{A_2}) & \cong & \textsc{Cox}(\widetilde{A_2}, w) & \hookrightarrow & \textsc{Cryst}(\widetilde{A_2}, w) \end{array}$$

A Solution



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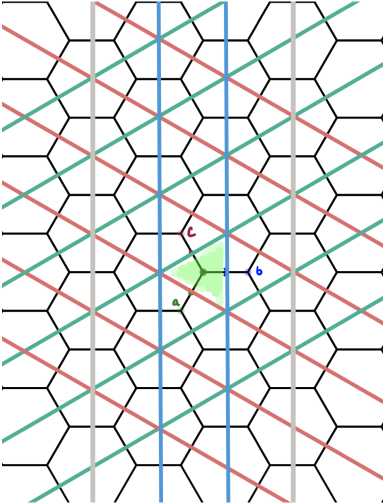

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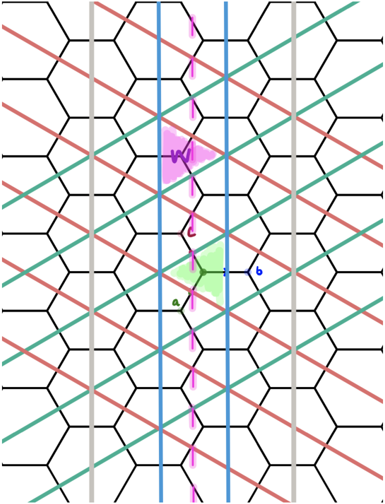

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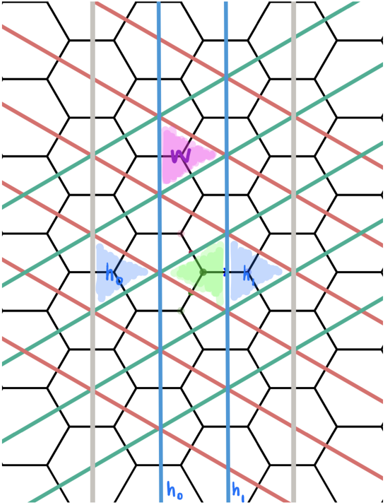

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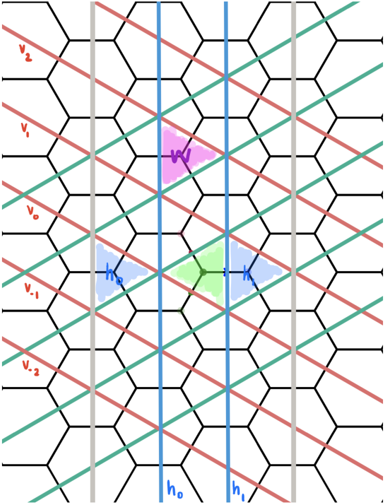

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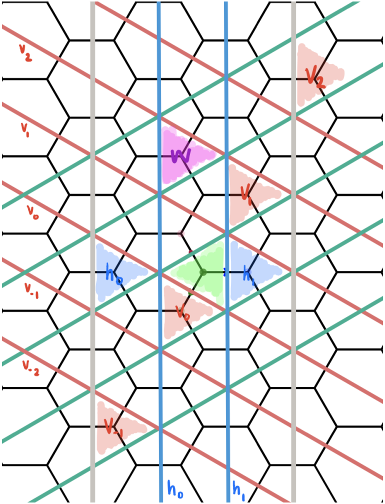

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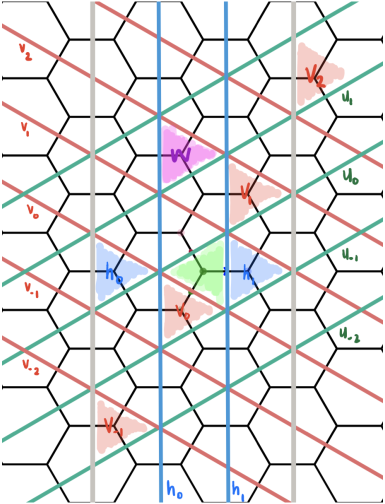

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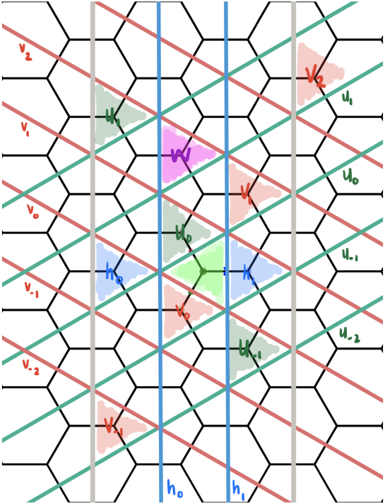

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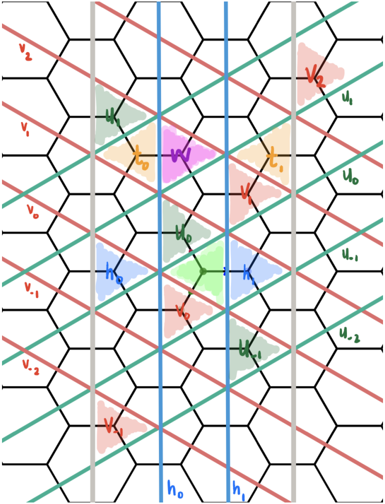

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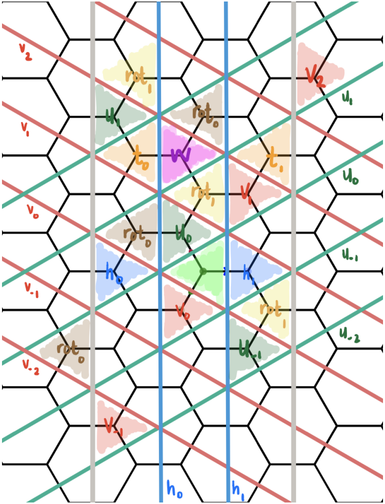

v_2
v_1
v_0
v_{-1}
v_{-2}
rot_1
u_1
rot_0
t_0
w
t_1
rot_1
v_1
rot_0
u_0
h_0
h_1
v_0
rot_1
rot_0
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v_{-1}
v_2
u_1
u_0
u_{-1}
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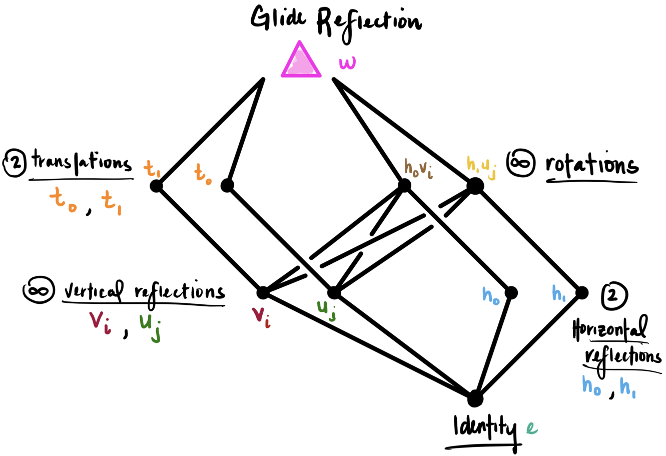

Glide Reflection
w
2 translations
t_0 , t_1
t_1
t_0
h_0v_i
h_1u_j
∞ rotations
∞ vertical reflections
v_i , u_j
v_i
u_j
h_0
h_1
2
Horizontal reflections
h_0 , h_1
Identity e

A New Solution

- This infinite generating set is a poset under left division leading to a normal form that solves the word problem.

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- This infinite generating set is a poset under left division leading to a normal form that solves the word problem.

- GOAL: Write finite state automata that will solve the word problem for $\mathrm{ART}\left(\widetilde{A_2}\right)$ with its classical presentation.

References

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Thank You!

Mahalo!

Small Seifert Fibered Zero Surgery

Peter Johnson

December 2019

University of Virginia

Notation

Let $S^2(\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3})$ be the Seifert fiber space with base orbifold S^2 and 3 critical fibers with corresponding Seifert invariants $\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3}$.

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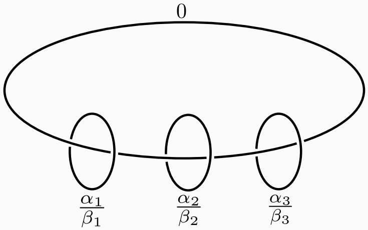
Figure 1: A surgery description of $S^2(\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3})$



Surgery Questions

Question 1
Which $S^2(\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3})$ can be obtained by 0-surgery on a knot in S^3?

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Which $S^2(\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3})$ can be obtained by 0-surgery on a knot in S^3?

Question 2
What obstructions are there to $S^2(\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3})$ being 0-surgery on a knot in S^3?

Examples

Torus knots have Seifert fibered complement. In particular, by work of Moser (1971), 0-surgery on a torus knot is Seifert fibered.

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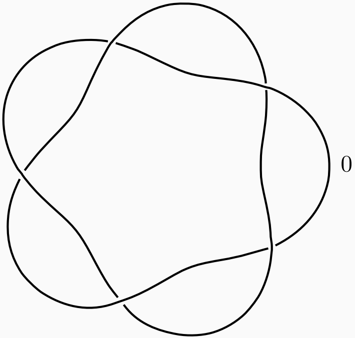
Example (0-surgery on $T_{5,2}$)

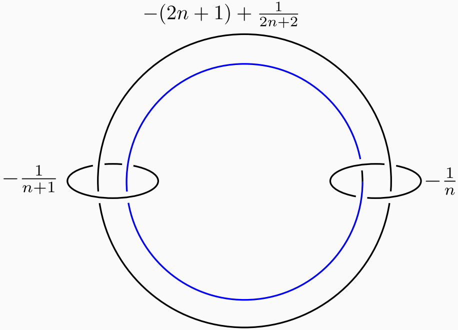
$$S^2\left(\frac{5}{-2}, \frac{2}{1}, \frac{10}{-1}\right) \cong$$



Examples

Theorem (Ichihara - Motegi - Song 2008)
There exists an infinite family of hyperbolic knots K_n with small Seifert fibered 0-surgery, where $n \in \mathbb{Z} \setminus \{0, -1, -2\}$.

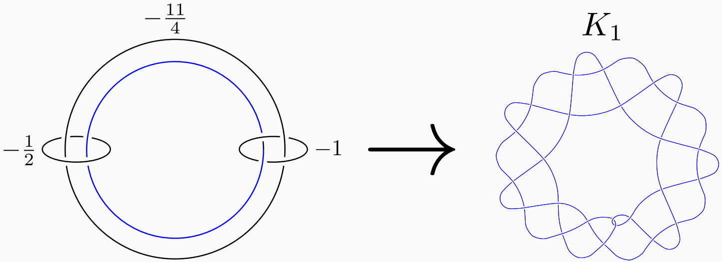
Figure 2: The knot K_n is the image of blue curve after performing the corresponding surgeries on the other 3 link components.



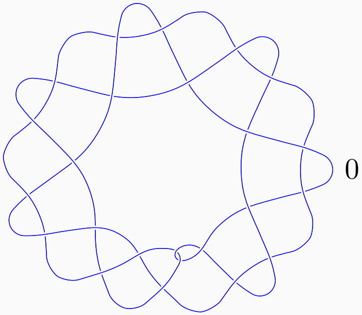
Examples

Example ($n = 1$)

Figure 3: After performing surgery on the link to the left, the image of the blue curve becomes $K_1 \subset S^3$.



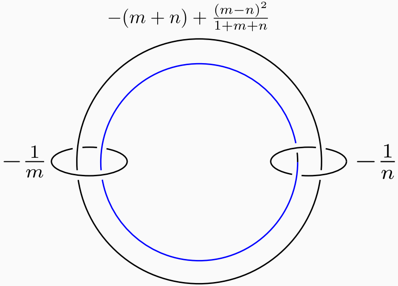
Example ($n = 1$, continued)

$S^2(\frac{3}{2}, -\frac{5}{2}, -\frac{15}{4}) \cong$ 

Examples

Proposition (J. 2019)
There exists an infinite two parameter family of knots $K_{m,n}$ (extending the I-M-S knots) with small Seifert fibered 0-surgery.

Figure 4: The knot $K_{m,n}$ is the image of blue curve after performing the corresponding surgeries on the other 3 link components. Here, $m, n \in \mathbb{Z}$ such that $n \notin \{0, -1\}$, $m \neq 0$, $1 + m + n \neq 0$, and $(m - n)^2$ divides $(1 + m + n)$. Note, $K_{n+1,n} = K_n$.



Basic Algebraic Topological Obstructions
If Y is obtained by 0-surgery on a knot in S^3, then $\pi_1(Y)$ has weight 1, i.e. $\pi_1(Y)$ is normally generated by a single element. Also, $H_1(Y;\mathbb{Z}) \cong \mathbb{Z}$.

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Rohlin Invariant

Theorem (Hedden - Kim - Mark - Park 2018)
If an integral homology $S^1 \times S^2$ has two non-trivial Rohlin invariants, then it is not obtained by surgery on a knot in S^3.

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Theorem (Hedden - Kim - Mark - Park 2018)
For all positive integers k, $S^2(-\frac{2}{1}, \frac{-8k+1}{1}, \frac{-16k+2}{-8k-1})$ is irreducible, has weight 1 fundamental group, and cannot be obtained by 0-surgery on a knot in S^3.

Obstructions

Heegaard Floer Homology

Theorem 1 (Ozsváth - Szabó 2001)
If Y is obtained by 0-surgery on a knot in S^3, then

$$-\frac{1}{2} \leq d_{-1/2}(Y) \qquad \text{and} \qquad d_{1/2}(Y) \leq \frac{1}{2} \tag{1}$$

Obstructions

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Unfortunately, by the following theorem, we cannot use this to obstruct a Seifert fibered homology $S^1 \times S^2$ from being 0-surgery on a knot in S^3.

Theorem 2 (Hedden - Kim - Mark - Park 2018)
Suppose M is homology cobordant to a Seifert fibered homology $S^1 \times S^2$. Then, (1) *also holds for M.*

Obstructions

Work in Progress
A potential strategy to obtain another obstruction:

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Obstructions

Work in Progress
A potential strategy to obtain another obstruction:

- We can prove an analog of the *d*-invariant bounds from Theorem 1 for involutive Heegaard Floer homology.
- However, the analog of Theorem 2 is not clear in the involutive setting. One may hope that, in fact, the analog of Theorem 2 for involutive Heegaard Floer homology does not hold. This would then provide an obstruction to a Seifert fibered homology $S^1 \times S^2$ being 0-surgery on a knot in S^3.

References

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TIGHT CONTACT STRUCTURES ON THE BRIESKORN HOMOLOGY SPEHERES $\Sigma(2,3,6n+1)$

Kürşat Yılmaz

The University of Toledo, Ohio

December 08, 2019

Question

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Can we find the exact number of tight contact structures on a given 3 manifold?

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Can we find the exact number of tight contact structures on a given 3 manifold?

Not always!

Theorem (Mark, Tosun 2018)

The Brieskorn homology spheres $\Sigma(2,3,6n+1)$ *has exactly two tight contact structures for any* $n \geq 1$.

Sketch of Proof:

We start with the basic surgery description of $\Sigma(2,3,6n+1)$. To find the Seifert invariants we begin with solving the equation

$$3(6n+1)b_1 + 2(6n+1)b_2 + 6b_3 = 1$$

for the integers b_1, b_2 and b_3. To make it simple let us take $b_1 = 1, b_2 = -1$ and $b_3 = -n$.

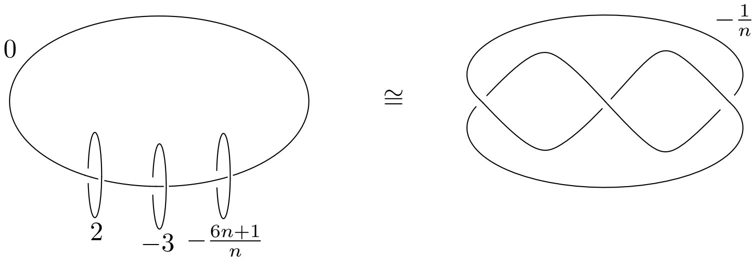

Figure 1: Surgery description of $\Sigma(2,3,6n+1)$

Constructing and Counting the Tight Contact Structures

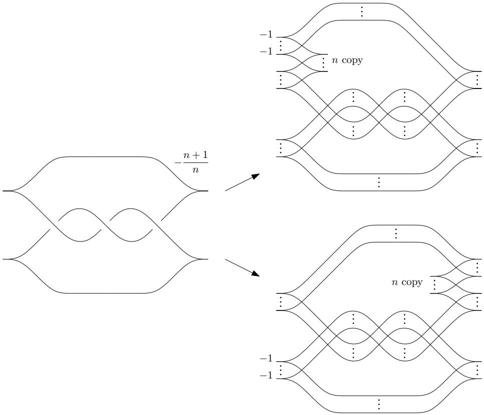

Figure 2: Non-isotopic tight contact structures on $\Sigma(2, 3, 6n + 1)$

Question

How do we find the upper bound?

Question

Question

How do we find the upper bound?

By using Honda's bypass technique!

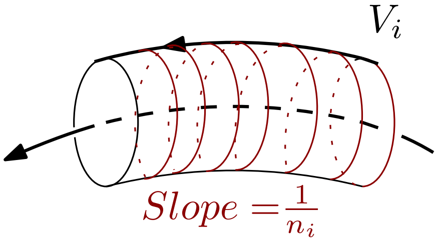

Figure 3: Slope of the dividing curves of abstract solid torus

The attaching maps are can be given as

$$A_1 = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 3 & 1 \\ -1 & 0 \end{pmatrix}, A_3 = \begin{pmatrix} 6n+1 & 6n-5 \\ -n & -n+1 \end{pmatrix}.$$

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Then the corresponding slopes on the boundary of V_i's will be

$$s_1 = \frac{n_1}{2n_1 - 1}, s_2 = -\frac{n_2}{3n_2 + 1}, s_3 = -\frac{nn_3 + n - 1}{(6n+1)n_3 + 6n - 5}.$$

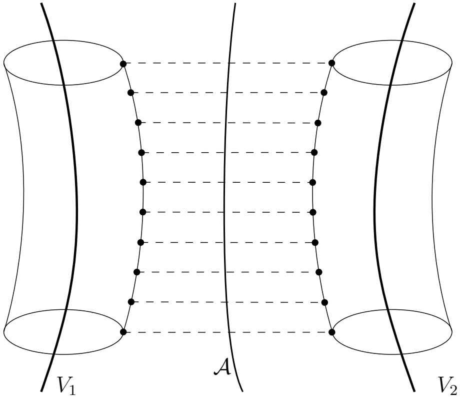

Figure 4: The dividing curve (dashed lines) configuration of the annulus $\mathscr{A}$

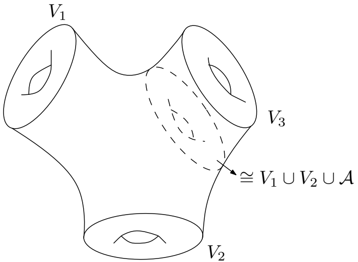

Figure 5: This figure illustrates the isotopy between $\partial(M \setminus (V_1 \cup V_2 \cup \mathscr{A}))$ and $\partial(M \setminus V_3)$.

After configurations we end up with the slopes $s_1 = \frac{2}{5}$ and $s_2 = -\frac{2}{5}$ corresponds to slopes $\frac{1}{n_1} = -\frac{1}{2}$ and $\frac{1}{n_2} = -\frac{1}{2}$ respectively.

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On the other hand, the slope $s_3 = -\frac{1}{5}$ corresponds in coordinates of ∂V_3 to $-\frac{n+1}{n}$ which has continued fraction $[-2, ..., -2]$ (n-times -2) and by the results of Honda we know that the solid torus satisfying this boundary conditions admits exactly two tight contact structures.

References

K. Honda. On the classification of tight contact structures. I. Geom. Topol, 4:309–368, 2000.

P. Ghiggini and S. Schönenberger. On the classification of tight contact structures. In Topology and geometry of manifolds (Athens, GA, 2001), volume 71 of Proc. Sympos. Pure Math., pages 121–151. Amer. Math. Soc., Providence, RI, 2003.

T. E. Mark and B. Tosun. Obstructing pseudoconvex embeddings and contractible Stein fillings for Brieskorn spheres. Advances in Mathematics 335, 878-895, 2018.

Tangle Invariants via Cornered Sutured Floer Homology

Ian Montague

Brandeis University

December 8th, 2019

Theorem

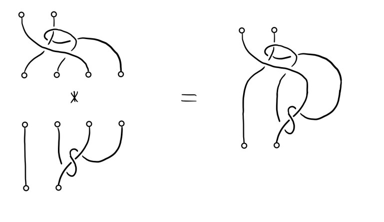
Theorem [M.](paper in progress)

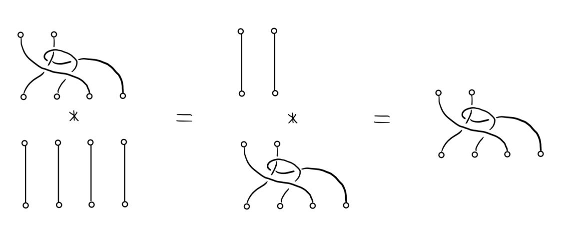
There exists a monoidal functor $CF^- : \mathfrak{Tan} \to 2 - \mathfrak{Mod}$ from the category of tangles to a category of "2-modules", which recovers (a stabilized version of) $gCFL^-(S^3, L)$ for links in S^3.

The Monoidal Category of Tangles

The Category $\mathfrak{Tan}$

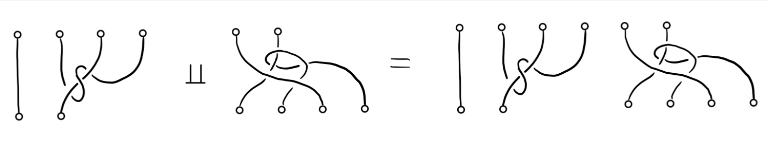
Composition in $\mathfrak{Tan}$ is given by vertical stacking $(*)$:

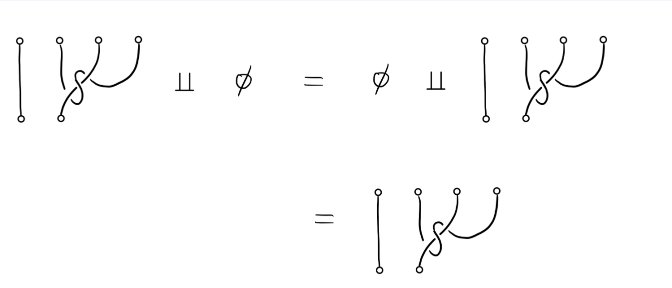




The Category $\mathfrak{Tan}$ (cont.)

$\mathfrak{Tan}$ is also a *monoidal* category under horizontal concatenation $\amalg$:





Tangle Invariants As Functors

Definition

For our purposes, a *link invariant* is map $F : \mathbf{Link} \to \mathbf{R\text{-}Mod}$, (e.g., $R = \mathbb{Z}$, $\mathbb{F}_2$, $\mathbb{F}_2[U]$).

Tangle Invariants As Functors

Definition

For our purposes, a *link invariant* is map F : **Link** $\to$ **R-Mod**, (e.g., $R = \mathbb{Z}$, $\mathbb{F}_2$, $\mathbb{F}_2[U]$).

Categorification

Let $\mathfrak{Bimod}$ be the category where:

- $\mathrm{Ob}(\mathfrak{Bimod})$ = set of dg-algebras $\mathcal{A}$ over R,
- $\mathrm{Mor}(\mathcal{A}, \mathcal{B})$ = set of dg-bimodules over $(\mathcal{A}, \mathcal{B})$.

A *categorification* of F : **Link** $\to$ **R-Mod** is a functor $\mathfrak{F} : \mathfrak{Tan} \to \mathfrak{Bimod}$ such that:

- $\mathfrak{F}(0) = R$
- $H_*(\mathfrak{F}) = F$ when restricted to **Link** $\subset \mathfrak{Tan}$.

What about the Monoidal Structure?

Question

When does a categorified tangle invariant extend to a monoidal functor $\mathfrak{F} : (\mathfrak{Tan}, \amalg) \to (\mathfrak{Bimod}, \otimes)$?

What about the Monoidal Structure?

Question

When does a categorified tangle invariant extend to a monoidal functor $\mathfrak{F} : (\mathfrak{Tan}, \amalg) \to (\mathfrak{Bimod}, \otimes)$?

Answer

It doesn't in general: $\mathfrak{F}(m) \otimes \mathfrak{F}(n) \ncong \mathfrak{F}(m+n)$ for most tangle invariants arising from Floer homology (or Khovanov homology) :'(

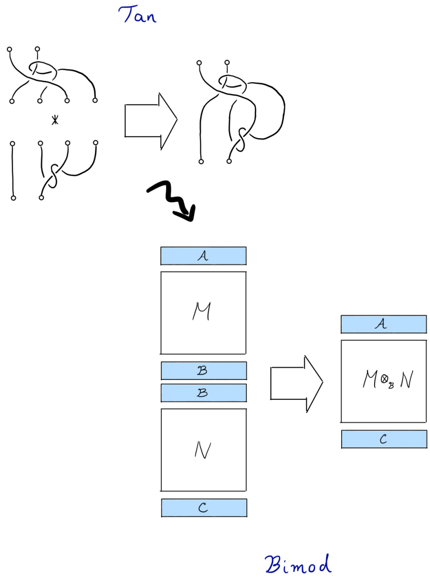
Categorification (cont.)

Idea

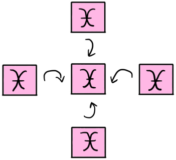
Let's extend our TQFT down one more level:

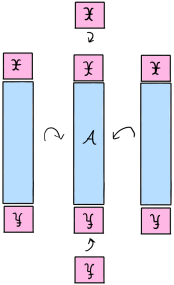
We can replace $\mathfrak{Bimod}$ with a (2-)category $2 - \mathfrak{Mod}$, endowed with a more suitable monoidal structure.

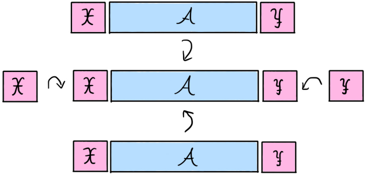
Tan → Bimod



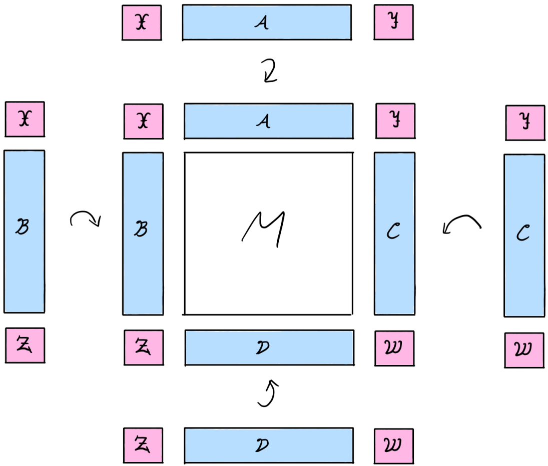
2-Algebras and Algebra-Bimodules



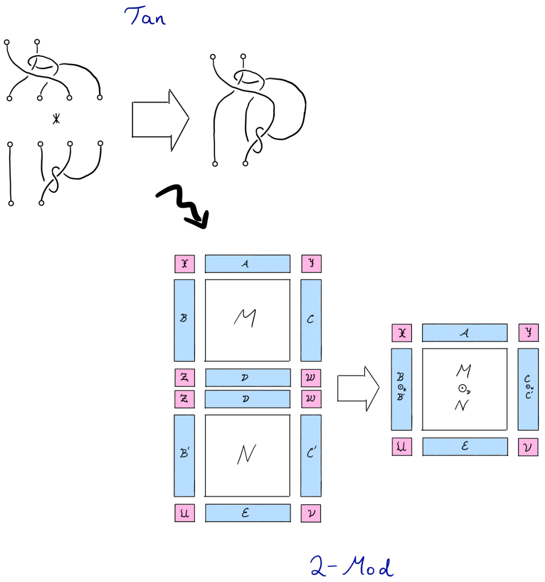




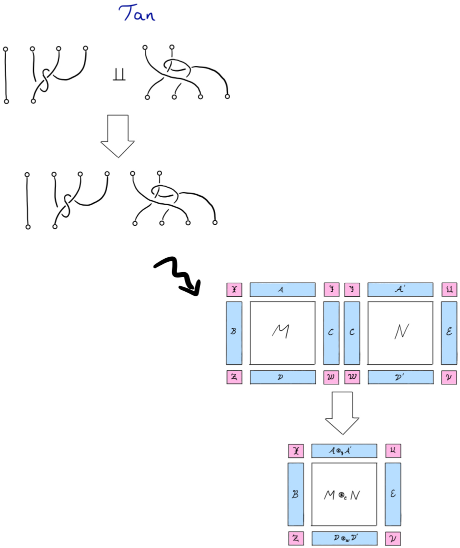
Morphisms in the Category $2 - \mathfrak{Mod}$



Composition in 2 − $\mathfrak{Mod}$



Monoidal Product in $2 - \mathfrak{Mod}$



Knot/Link Floer Homology

Knot/Link Floer Homology

$$HFK^-(S^3, K) = H_*(gCFK^-(S^3, K))$$

is an $\mathbb{F}_2[U]$-module.

$$HFL^-(S^3, L) = H_*(gCFL^-(S^3, L))$$

is an $\mathbb{F}_2[U_1, \dots, U_\ell]$-module.

Tangle Floer Homology

Some Heegaard Floer Tangle Invariants

- Sutured:
  - [**Alishahi-Eftekhary**,′16]
  - [**Zibrowius**,′16]
- Glue Under Vertical Composition:
  - [**Petkova-Vértesi**,′14]
  - [**Ozsváth-Szabó**,′17/′18]

Tangle Floer Homology (cont.)

Idea

- Enhance **Zibrowius'** construction using **Alishahi-Eftekhary's** construction to recover $gCFL^-$ instead of $\widehat{CFL}$.
- Refine this construction so it satisfies the vertical concatenation properties of the **Petkova-Vértesi** and **Ozsváth-Szabó** tangle invariants, i.e., defines a functor $\mathfrak{Tan} \to \mathfrak{Bimod}$.

Tangle Floer Homology (cont.)

Idea

- Enhance **Zibrowius'** construction using **Alishahi-Eftekhary's** construction to recover $gCFL^-$ instead of $\widehat{CFL}$.
- Refine this construction so it satisfies the vertical concatenation properties of the **Petkova-Vértesi** and **Ozsváth-Szabó** tangle invariants, i.e., defines a functor $\mathfrak{Tan} \to \mathfrak{Bimod}$.
- Solving right-hand side of the equation

$$\{\text{bordered sutured Floer homology [Zar 11]}\}$$
$$+\{\text{cornered Heegaard Floer homology [DLM 13]}\}$$
$$= \{\text{cornered sutured Floer homology}\},$$

enhance the above tangle invariant to a monoidal functor $\mathfrak{Tan} \to 2 - \mathfrak{Mod}$.

Tangle Floer Homology (cont.)

Theorem [M.] (paper in progress)

There exists a monoidal functor $CF^- : \mathfrak{Tan} \to 2 - \mathfrak{Mod}$ which recovers (a stabilized version of) $gCFL^-(L, S^3)$ for links in S^3.

Future Research Directions

Other Invariants

Is it possible to construct cornered versions of the Ozsváth-Szabó or Petkova-Vértesi *HF* tangle invariants?

Contact Geometry

Using Honda-Kazez-Matić's *EH* invariant in *SFH* we should be able to define a (relative) LOSS invariant for Legendrian/transverse tangles in $S^2 \times I$.

- How does the LOSS invariant behave under local modifications (e.g., mutation)?
- Does this provide a faster way to compute the LOSS invariant than existing methods (e.g., grid homology)?

Done

Thanks!

On Translation Length of Anosov Maps on Curve Graph of Torus [arxiv:1908.00472]

Sanghoon Kwak (University of Utah)

Joint with

Hyungryul Baik, (KAIST)
Changsub Kim, (KAIST)
Hyunshik Shin, (University of Georgia)

Table of Contents

- **Basic Definitions**
  - Curve Graph - *Stage*
  - (pseudo)-Anosov Mapping Class - *Actor*
- **Main Theorem**
  - Strengthening Masur-Minsky's Result
- **Idea of Proof**
  - Curve Graph of Torus
  - Idea of Proof

Basic Definitions

Curve Graph $C(S)$

- **Surface** $S = S_{g,n}$ of genus $\mathbf{g}$ with $\mathbf{n}$ punctures
- **Curve Graph** $C(S)$ of a Surface $\mathbf{S}$

  Vertices : Isotopy classes of essential simple closed curves

  Edges : Join two vertices if they represent **minimally intersecting** pair of curves.

Curve Graph $\mathcal{C}(S)$

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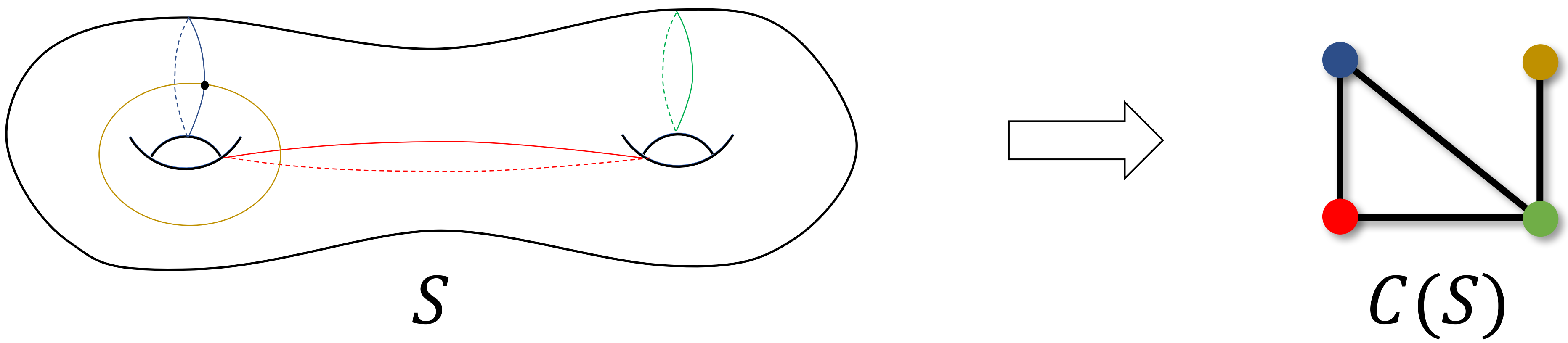
E.g.



Mapping Class Group $Mod(S)$

- **Mapping Class Group** $\boldsymbol{Mod(S)} = \{S \to S: orientation\ preserving\ homeomorphism\}/isotopy$

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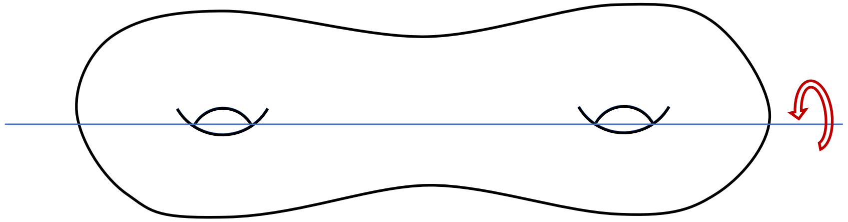
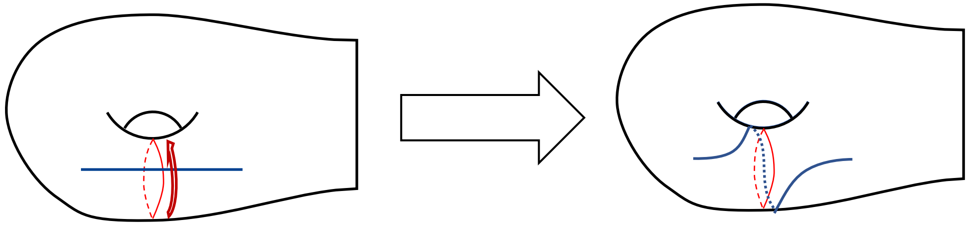
[Nielsen-Thurston Classification, 1988]

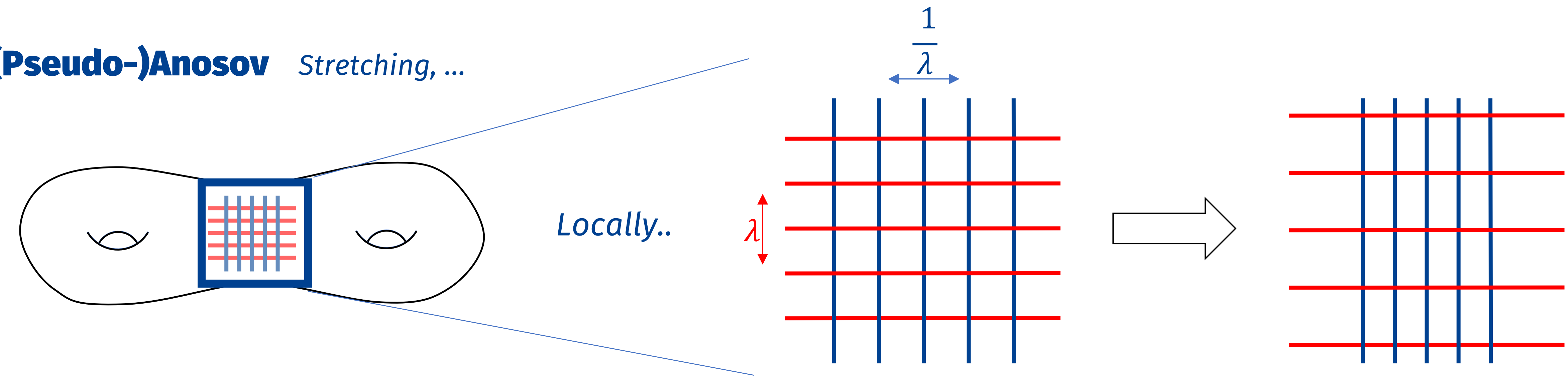
1. Periodic *Rotation, Reflection…*



2. Reducible *Dehn twist, …*



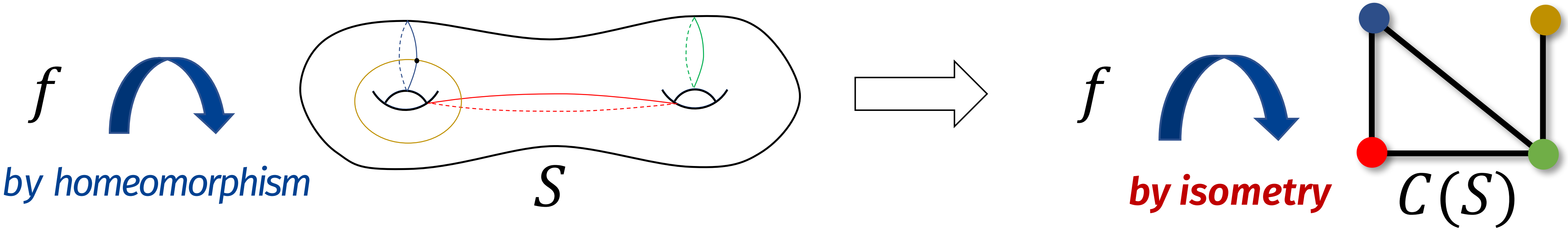
3. (Pseudo-)Anosov *Stretching, …*



Mapping Class Group ↷ Curve Graph

- $Mod(S)$ **acts on** $C(S)$!

  For $f \in Mod(S)$,



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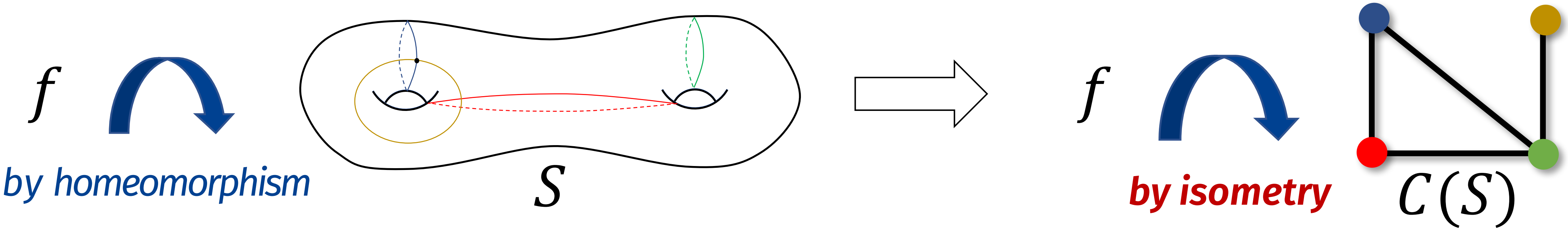

- **Stable Translation Length**

  For $f \in \mathrm{Mod}(S)$, define the **stable translation length** of f as:

$$\mathrm{l}_C(f) = \liminf_{n\to\infty} \frac{d_C(v, f^n(v))}{n},$$

where v is any vertex of $C(S)$. (Note: $l_C(f)$ is independent to choice of v)

Main Theorem

Earlier Works for $l_C(f)$ when S is non-sporadic

Sporadic surface : either [a sphere with $0-3$ punctures] or [a torus with $0-1$ punctures]

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-For non-sporadic surfaces:

Theorem(Masur-Minsky, 1998). Any **pA** map has a **quasi-geodesic axis** in curve graph.
→That is, for any map $f \in Mod(S)$, f acts on a *quasi-geodesic* in $C(S)$, by translation.

Corollary. $l_C(f) > \mathbf{0}$ iff f is **pA**.

Earlier Works for $l_C(f)$ when S is non-sporadic

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-Bowditch further strengthened this result:

Theorem(Bowditch, 2008). There exists a constant $M = M(S)$ only depending on S, such that $l_C(f)$ is **rational with the denominator bounded above $\boldsymbol{M}$.**

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Shackleton(2012), **Webb(2015)**, and **Bell-Webb**(2016; Polynomial-time algorithm)

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But NO literature is found with analogous result for $S = T$(torus).

Main Theorem

Theorem(Baik-Kim-K.-Shin 2019).
Any **Anosov** map has a **geodesic axis** in the curve graph.

→ That is, for any Anosov map $f \in Mod(T)$,
there exists a bi-infinite geodesic in $C(T)$ on which f acts by translation.

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Theorem(Baik-Kim-K.-Shin 2019).
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→ That is, for any Anosov map $f \in Mod(T)$,
there exists a bi-infinite geodesic in $C(T)$ on which f acts by translation.

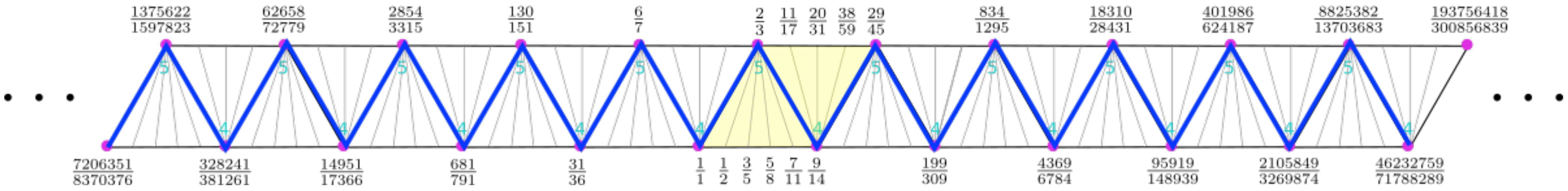
Corollary. $l_C(f) \in \mathbb{Z}^+$ for any Anosov map f.

+Since the proof is constructive,
We devised a polynomial-time algorithm to calculate $l_C(f)$.
Available @ http://samkwak.info/research

Examples(Generated by the Code)

Example 1

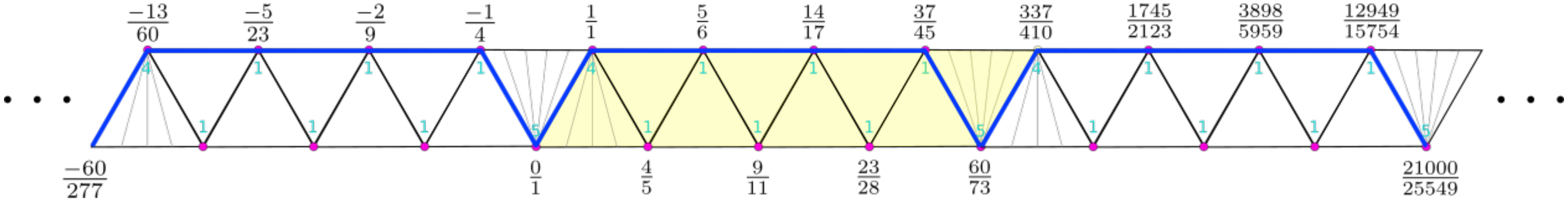
$$z \overset{f}{\longmapsto} \frac{65z-56}{101z-87}$$



∴ Translation Length = 2

Example 2

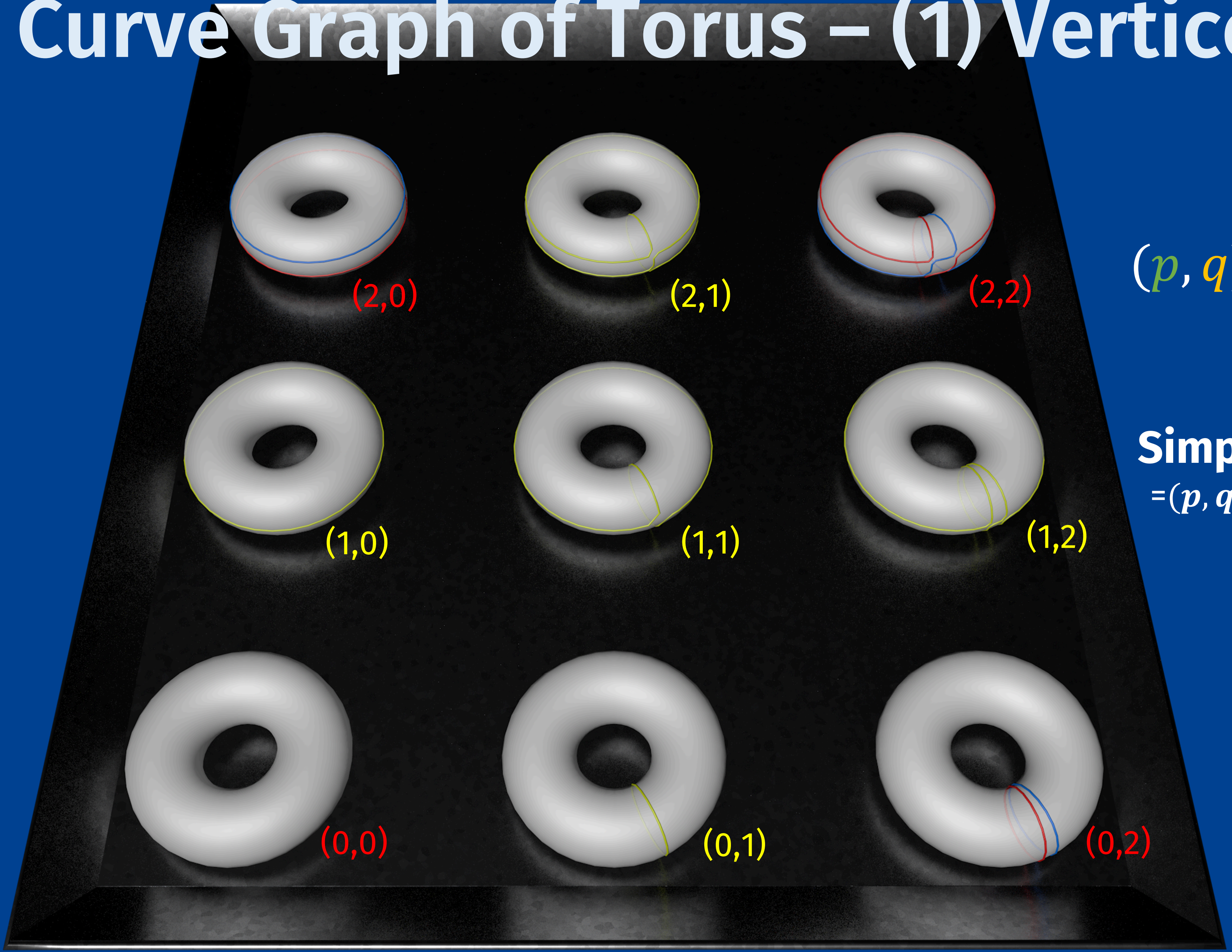
$$z \overset{f}{\longmapsto} \frac{277z+60}{337z+73}$$



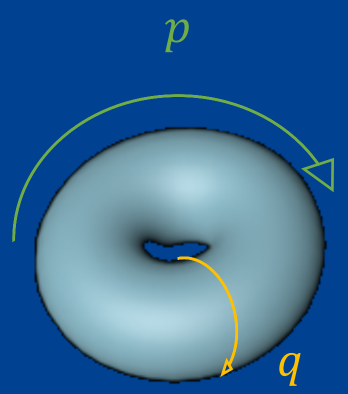
∴ Translation Length = 5

Idea of Proof

Curve Graph of Torus – (1) Vertices

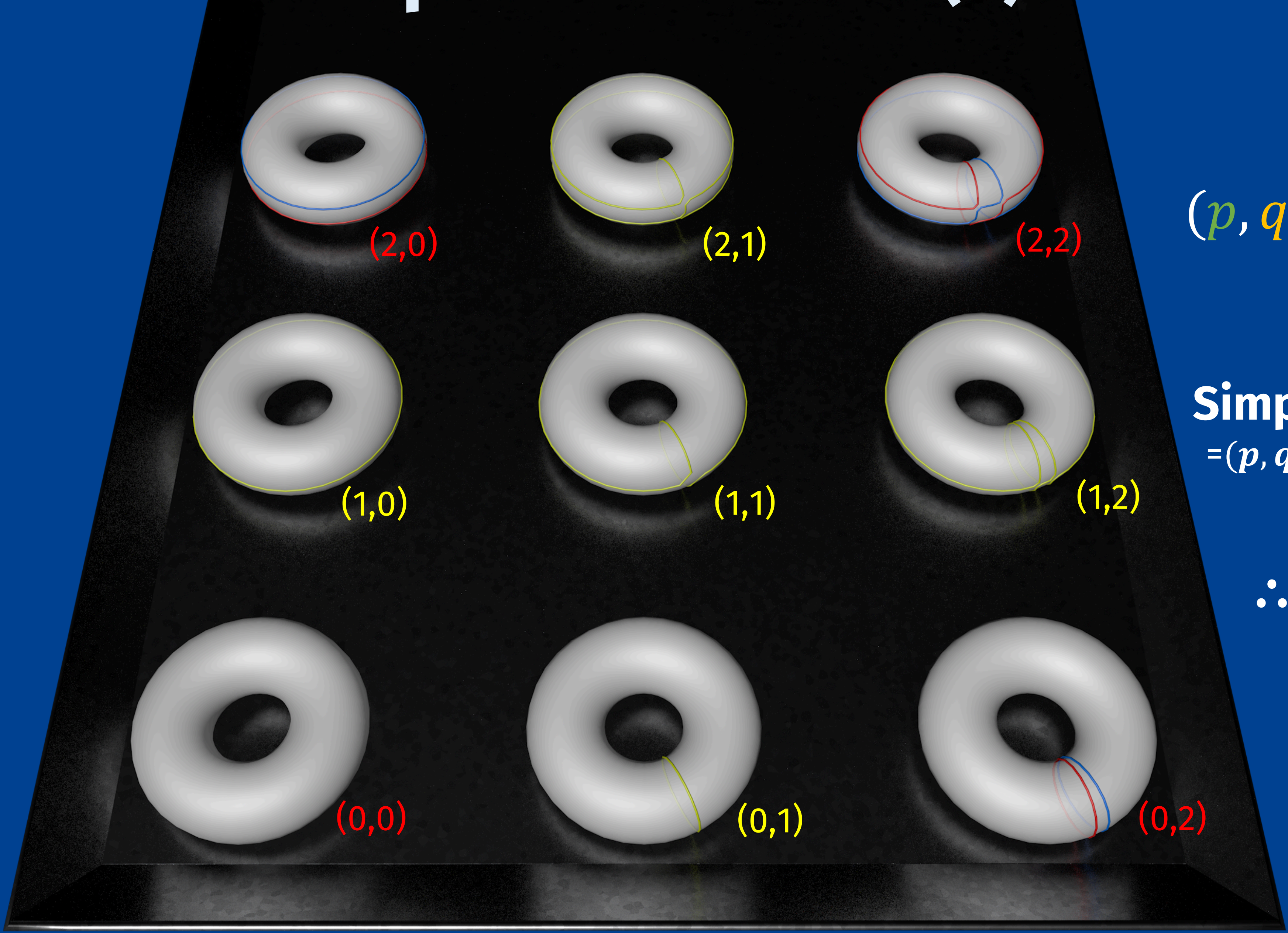


(p, q)-curve :



Simple Closed Curve on Torus
=(p, q)**-curve** with **relatively prime p,q**.

Curve Graph of Torus – (1) Vertices



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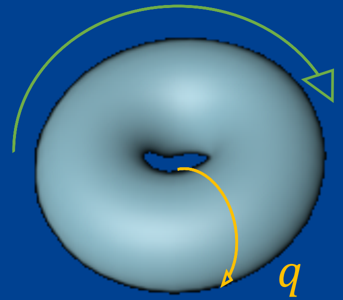
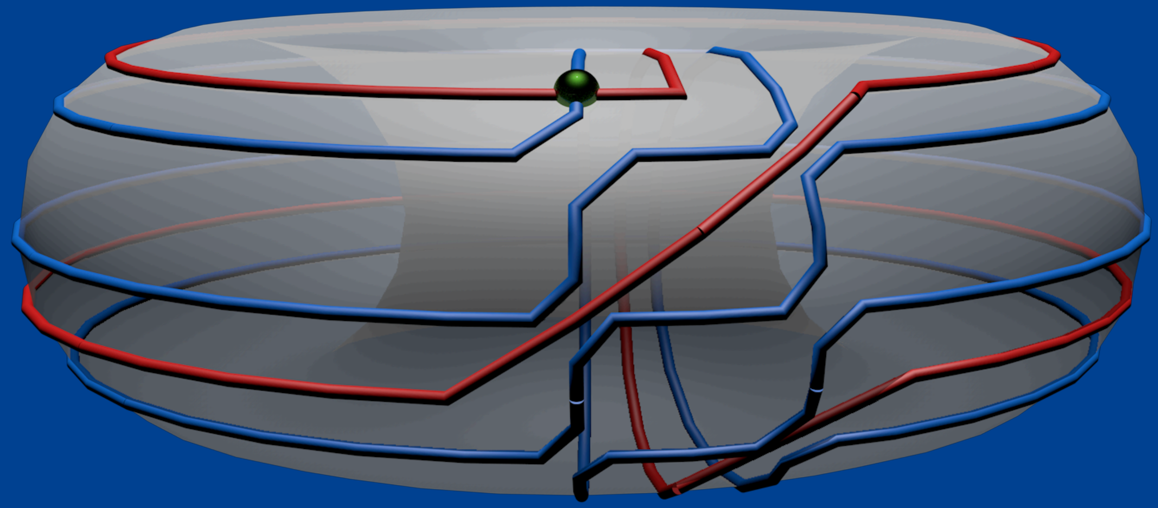


Simple Closed Curve on Torus
=(p, q)**-curve** with **relatively prime p,q**.

$\therefore$ Vertices of $C\,(T)$

$$= \mathbf{Q} \cup \{\frac{\mathbf{1}}{\mathbf{0}}\}$$

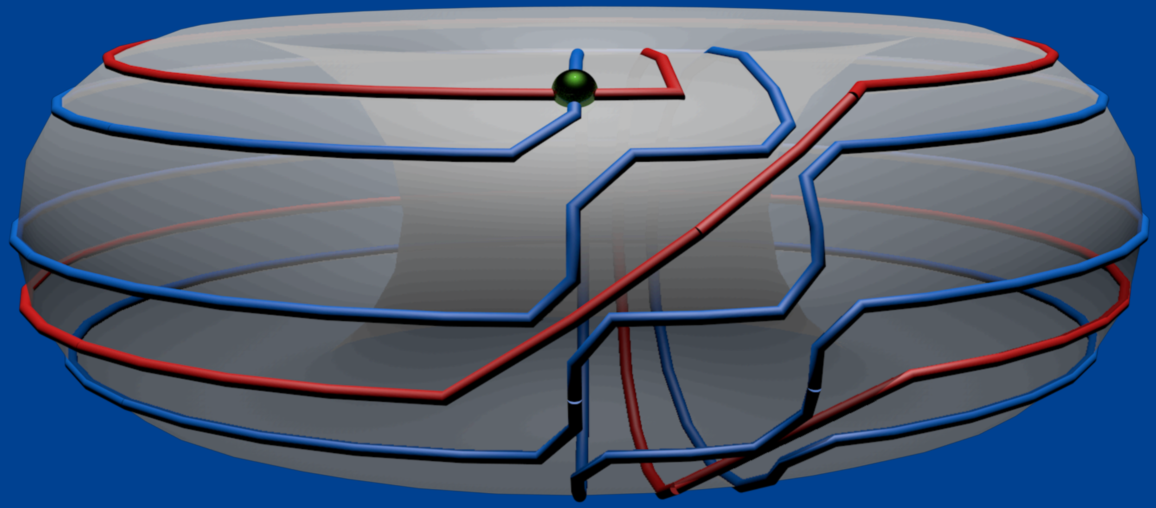
Curve Graph of Torus – (2) Edges



(2,1)-curve & (3,2)-curve intersects at **one** point.

$|(p, q)\text{-curve} \cap (r, s)\text{-curve}| = |ps - qr|$

Curve Graph of Torus – (2) Edges



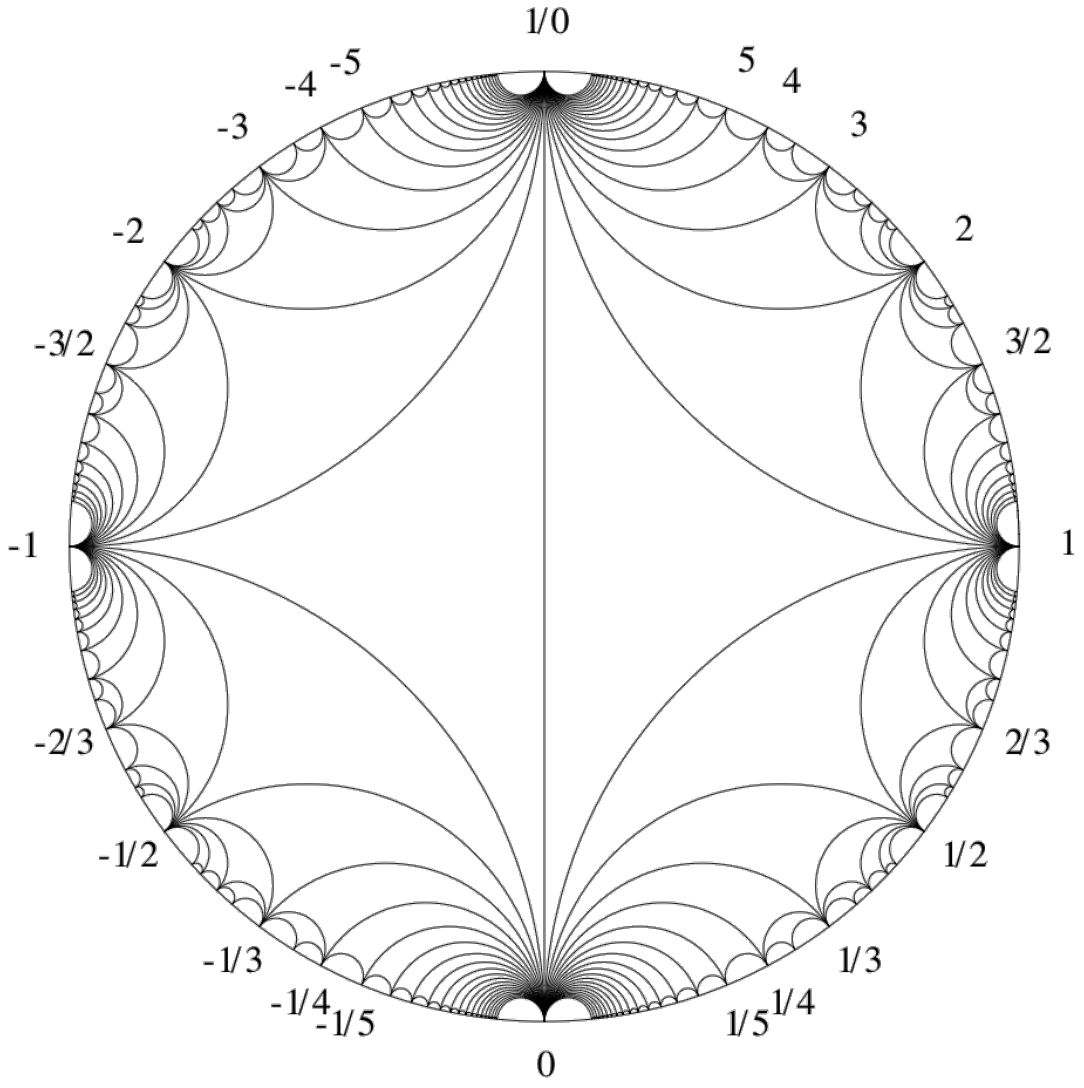
(2,1)-curve & (3,2)-curve intersects at **one** point.

$|(p,q)\text{-curve} \cap (r,s)\text{-curve}| = |ps - qr|$

$\therefore$ We join vertices $\frac{p}{q}$ and $\frac{r}{s}$

if and only if $|\mathrm{ps} - \mathrm{qr}|$**=1**.

∴Curve Graph of Torus = ***Farey Graph!***



Identify $C(T)$ with ***Farey Graph*** F!

Vertices $= \mathbf{Q} \cup \{\frac{\mathbf{1}}{\mathbf{0}}\}$

Edges = Between $\frac{p}{q}, \frac{r}{s}$

with $(|\mathbf{ps} - \mathbf{qr}| = 1)$

Idea of Proof

-Identify Anosov $f \in Mod(T)$ with hyperbolic $f \in PSL_2(\mathbb{Z})$.

-Embed $\mathrm{F} = C(T)$ into Hyperbolic plane $\boldsymbol{H}$.

-$\exists!$ f-Invariant **axis** in $\boldsymbol{H}$.

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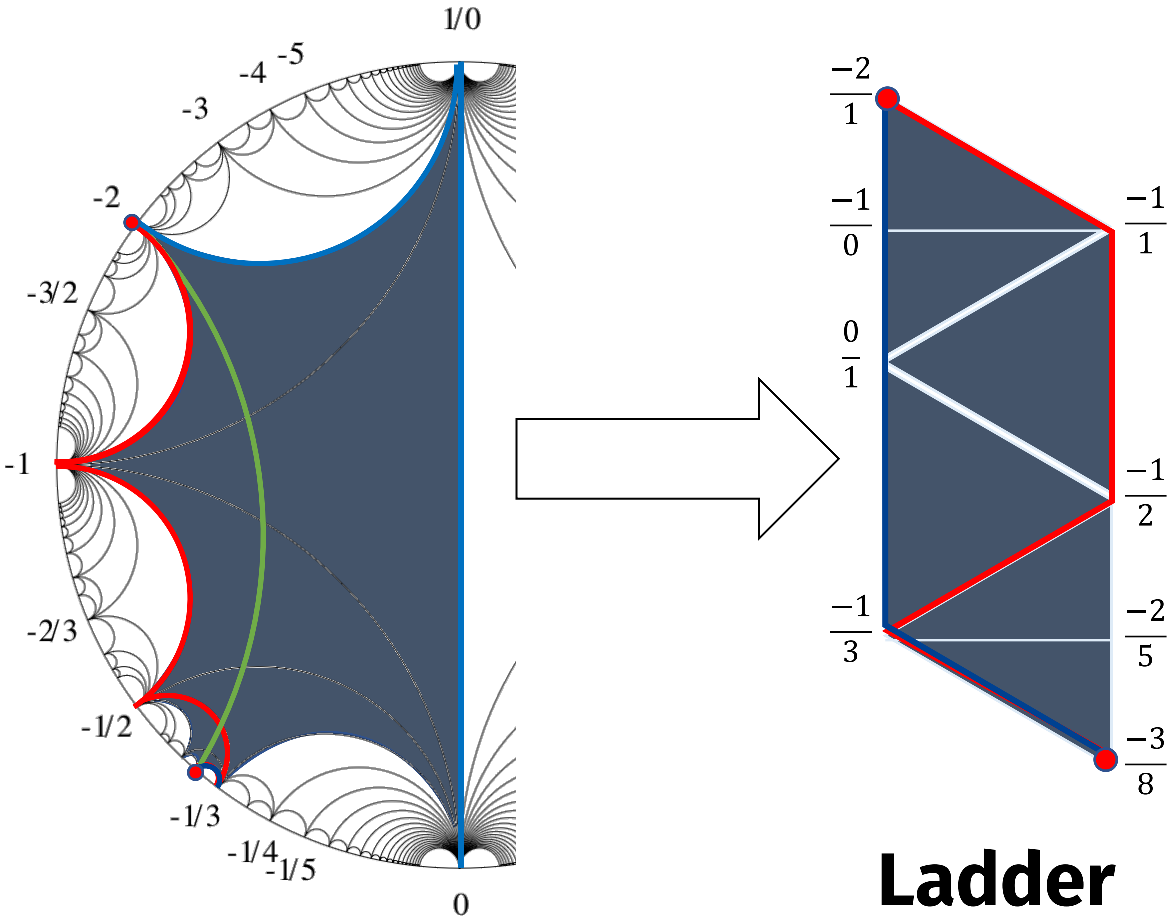
-∃! f-Invariant **axis** in $\boldsymbol{H}$.

-∃! f-Invariant **ladder** $\boldsymbol{L}$ in $\mathbf{F}$.

-∃ f-Invariant **geodesic** $\mathbf{P}$ in $\boldsymbol{L}$.



Ladder

Idea of Proof

-Identify Anosov $f \in Mod(T)$ with hyperbolic $f \in PSL_2(\mathbb{Z})$.

-Embed $\mathrm{F} = C(T)$ into Hyperbolic plane $\boldsymbol{H}$.

-∃! f-Invariant **axis** in $\boldsymbol{H}$.

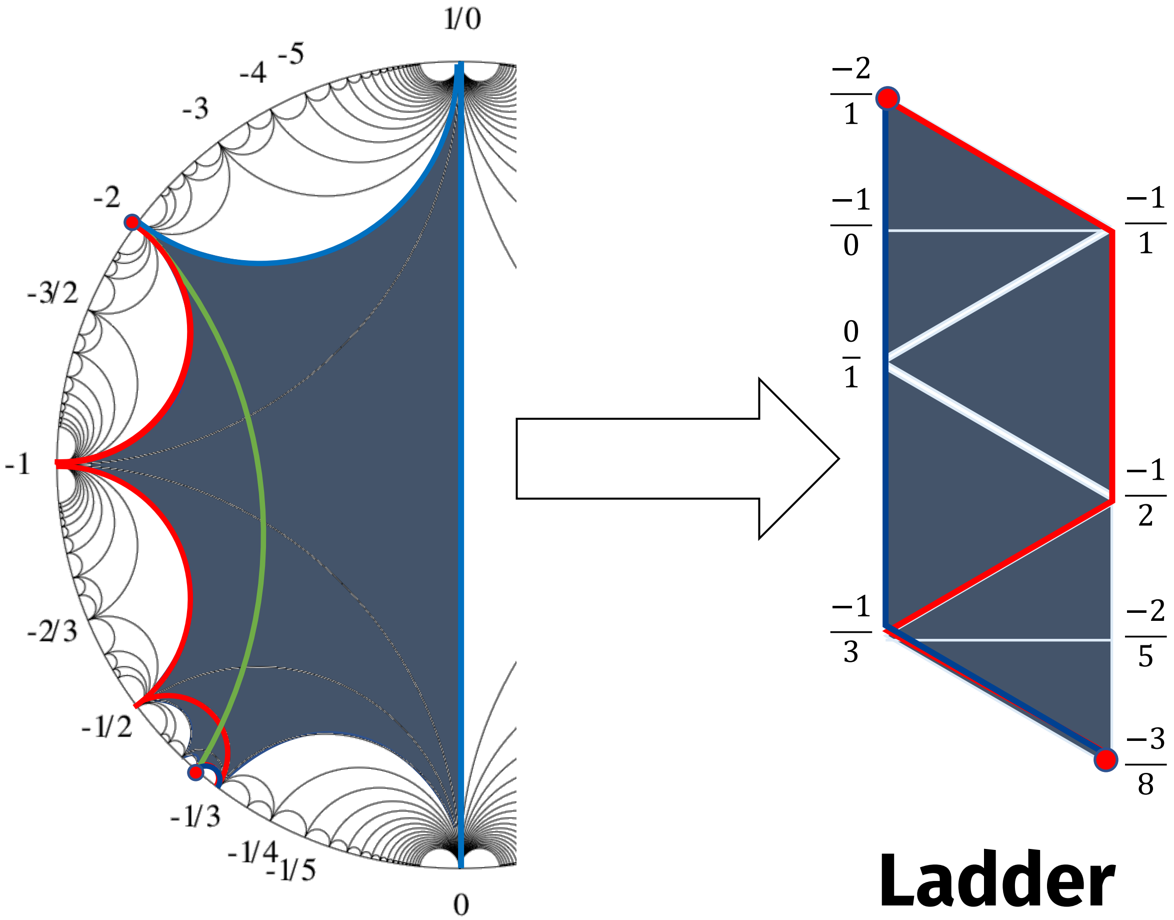
-∃! f-Invariant **ladder** $\boldsymbol{L}$ in $\mathbf{F}$.

-∃ f-Invariant **geodesic** $\mathbf{P}$ in $\boldsymbol{L}$.

-Ladder is geodesically convex.

-$\mathbf{P}$ is f-invariant geodesic in F.

-Q.E.D.



Ladder

Thank you!

Do you have any Questions?

Relative Kirby Diagrams and Casson Tower Factories

Charles Stine (joint with Bob Gompf)

8 December 2019

What are Casson Towers?

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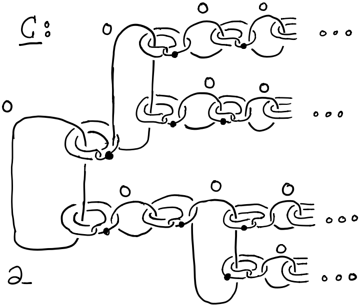

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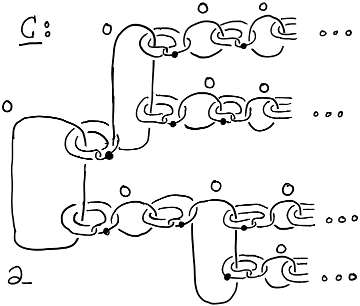

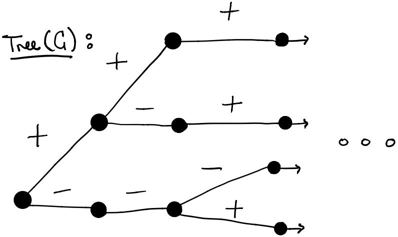

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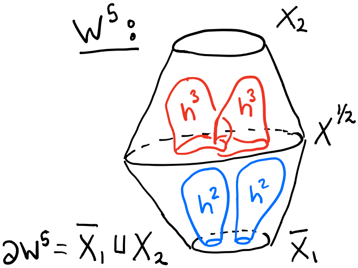

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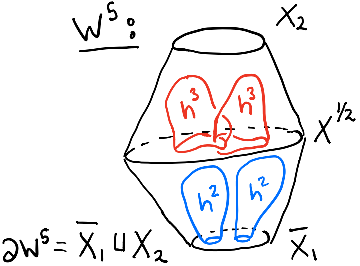
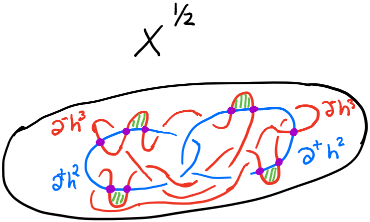
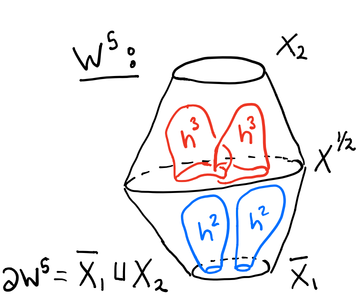
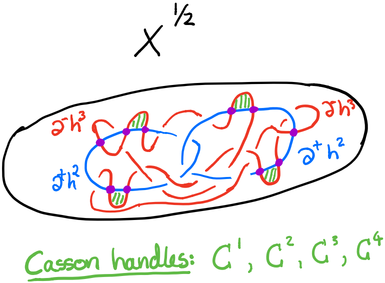




Casson handles: C^1, C^2, C^3, C^4

Where do they appear?





Definition 1

C *is exotic* $\iff \nexists\, (\mathbb{D}^2, \mathbb{S}^1) \overset{C^\infty}{\hookrightarrow} (C, \partial_- C)$

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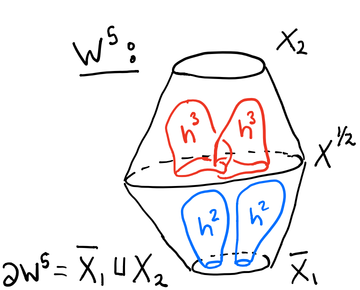

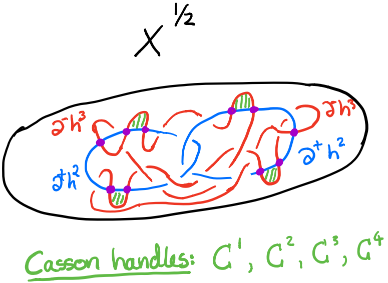

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Question 1

(Open) When is the Casson handle corresponding to a tree exotic?

An Observation:

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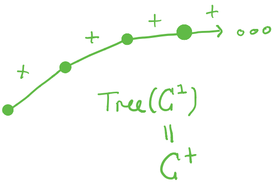

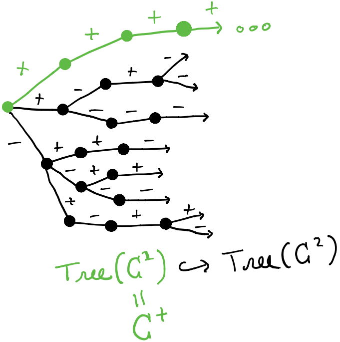

Theorem 1

$$Tree(C^1) \hookrightarrow Tree(C^2) \implies C^2 \hookrightarrow C^1$$

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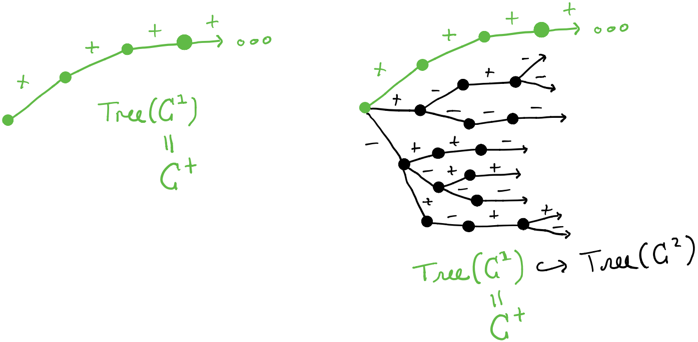

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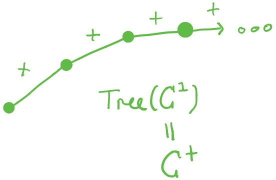
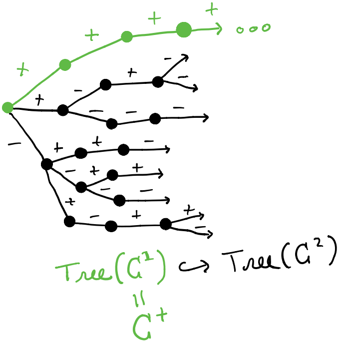




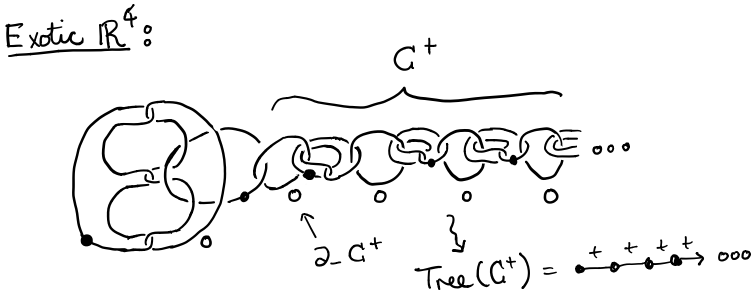
Theorem 1

$Tree(C^1) \hookrightarrow Tree(C^2) \implies C^2 \hookrightarrow C^1$ (Yes, this looks backwards)

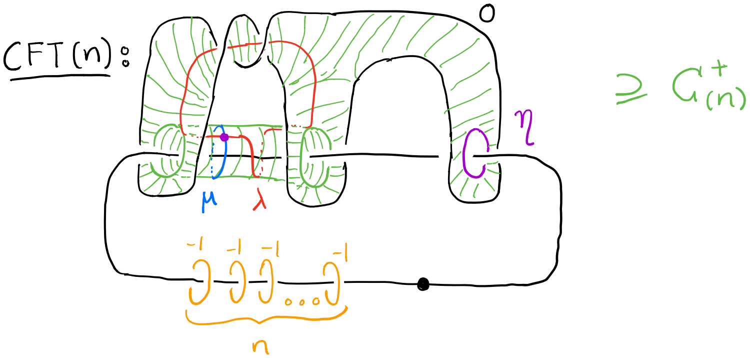
Corollary 1

One branch of C is exotic $\implies$ C is exotic.

Bizaca/Gompf Example



The Casson Tower Factory



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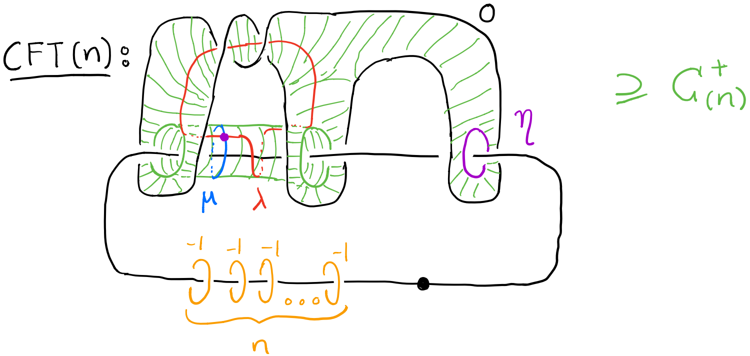

Theorem 2

$CTF(9n\text{-}3) \hookrightarrow E(n)\#\overline{\mathbb{CP}}^2$

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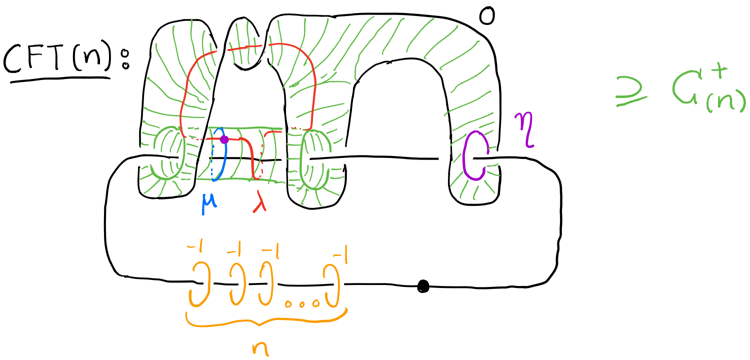


Theorem 2

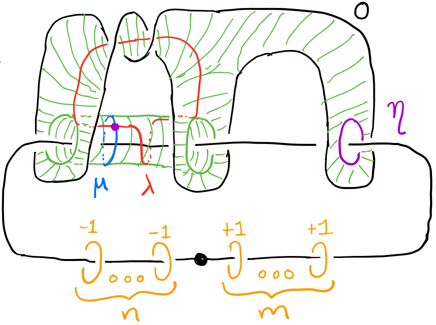
$CTF(9n\text{-}3) \hookrightarrow E(n)\#\overline{\mathbb{CP}}^2$

Corollary 2

C^+ *is exotic. (This takes a little work.)*

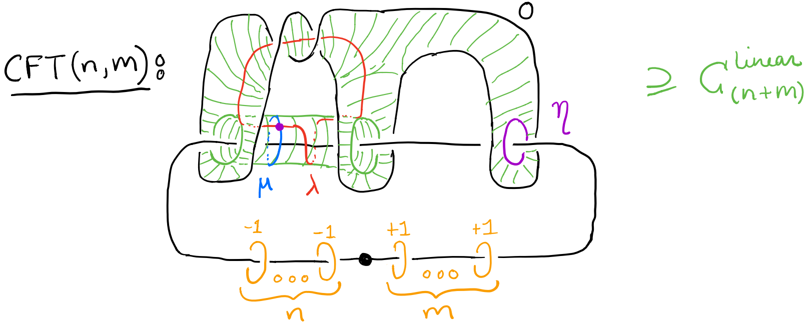
The New Casson Tower Factory

CFT(n,m):



$\supseteq G^{linear}_{(n+m)}$

The New Casson Tower Factory



Proposition 1

$CTF(n, m)$ contains the first $n + m$ stages of every linear Casson handle with n positive and m negative plumbings.

Research Objective

Construct smooth, closed, simply-connected $X(k)$ such that:

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$\Longrightarrow$ All linear Casson handles are exotic.

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Thank you!

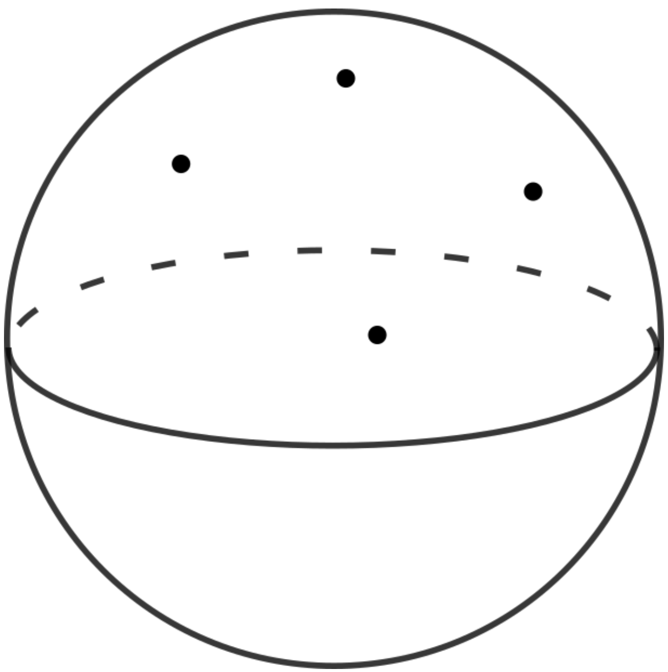
Finite Rigid Sets in the Arc Complex

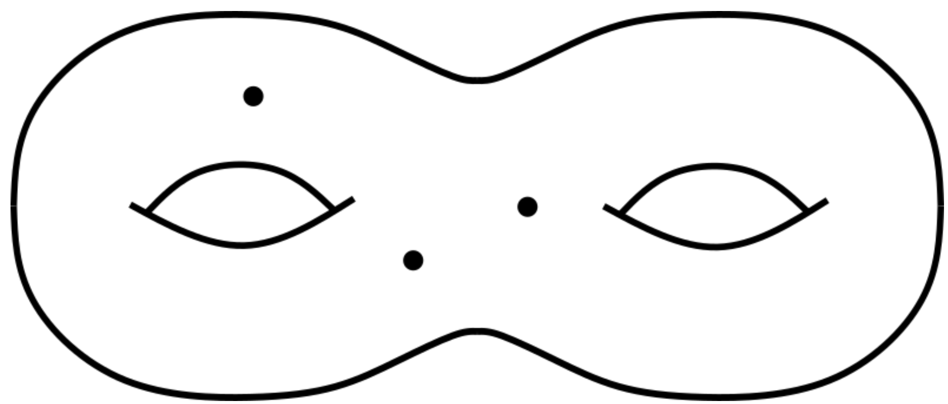
Emily Shinkle

ILLINOIS

Setting

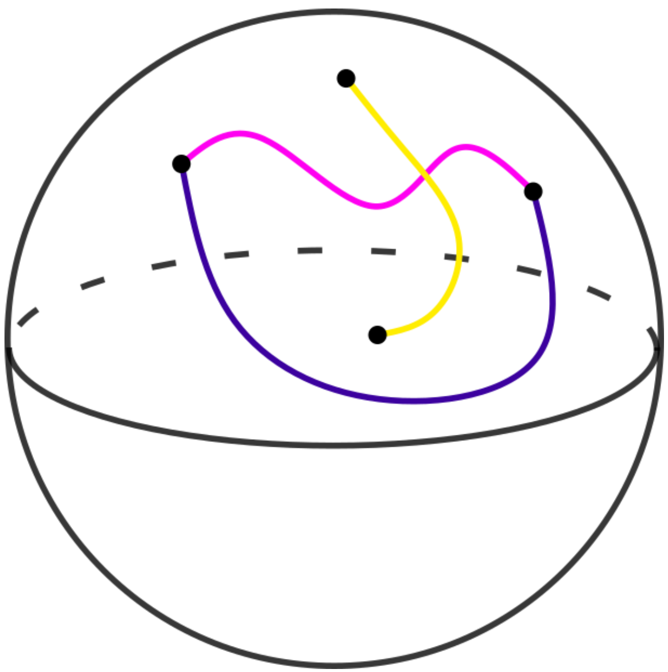
S a closed, connected, orientable, finite-type surface with marked points

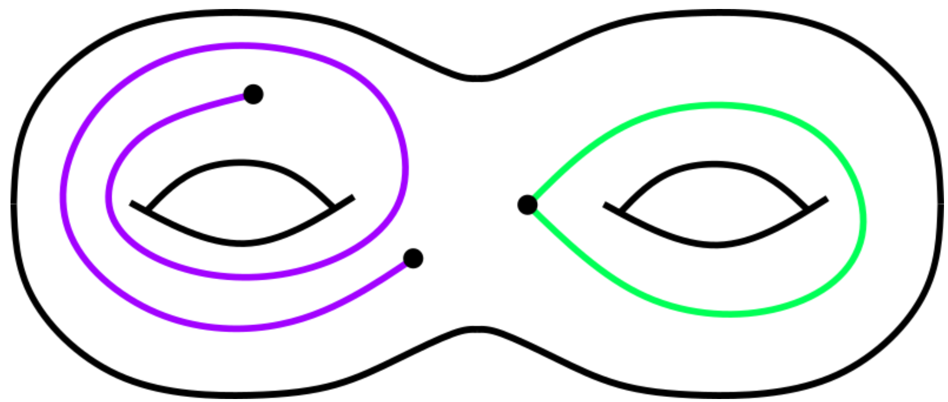




Arcs

Arcs on S are essential paths between marked points with embedded interiors, up to isotopy.





The Arc Complex

The *arc complex* $\mathcal{A}(S)$ of S is a simplicial complex

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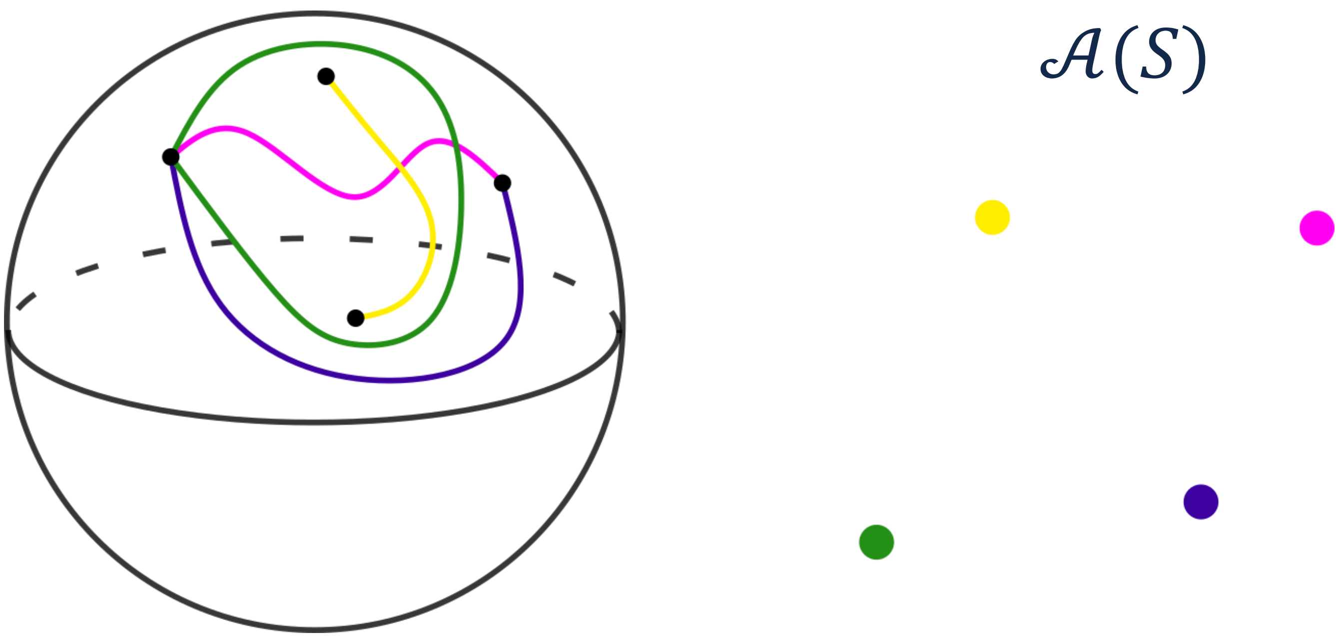
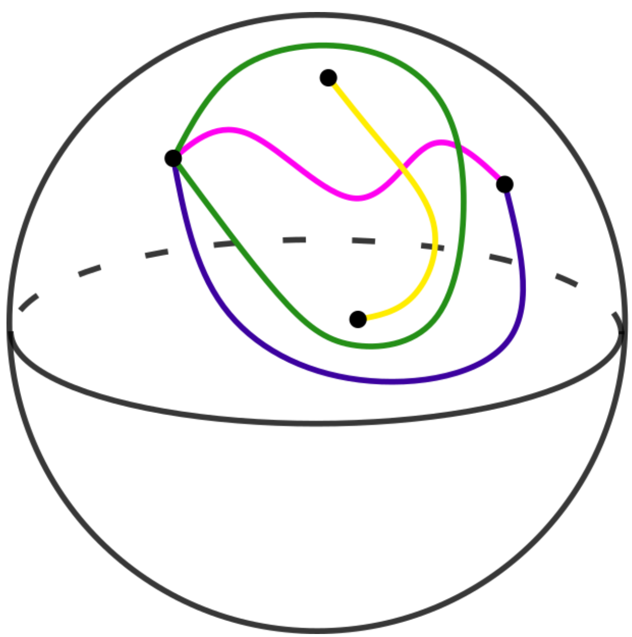
- vertices $\leftrightarrow$ arcs on S



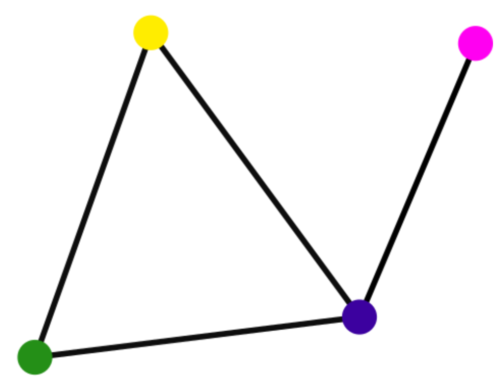
The Arc Complex

The *arc complex* $\mathcal{A}(S)$ of S is a simplicial complex

- vertices $\leftrightarrow$ arcs on S
- k-simplices $\leftrightarrow$ $k + 1$ disjoint arcs



$\mathcal{A}(S)$



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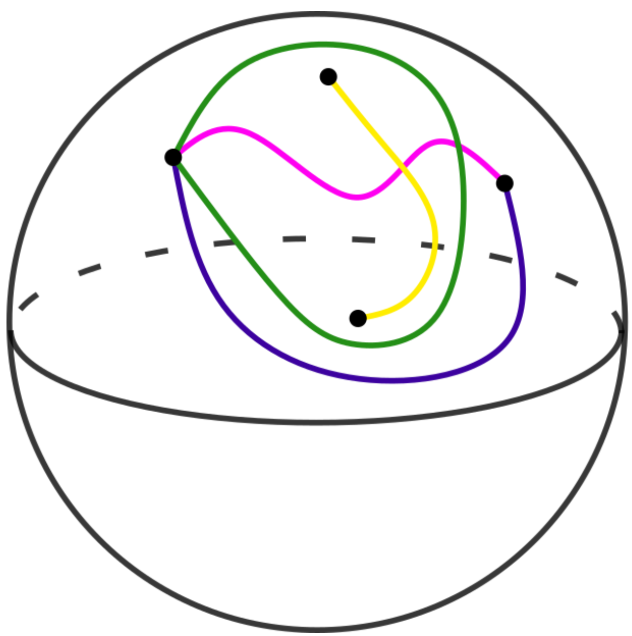

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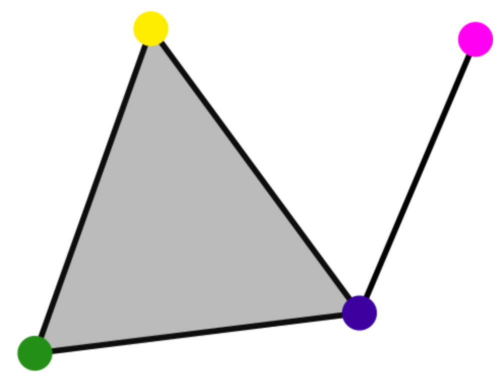

Maps of the Arc Complex

- A homeomorphism $f: S \to S$
  - sends arcs to arcs
  - sends disjoint arcs to disjoint arcs

- Thus, we can define an induced map

$$\tilde{f} \in \mathrm{Aut}(\mathcal{A}(S)).$$

Rigidity of the Arc Complex

Theorem (Irmak-McCarthy, 2010)
Every automorphism

$$\mathcal{A}(S) \to \mathcal{A}(S)$$

is induced by a homeomorphism $S \to S$, unique up to isotopy in most cases.

Rigidity of the Arc Complex

Theorem (Irmak-McCarthy, 2010)
Every automorphism

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is induced by a homeomorphism $S \to S$, unique up to isotopy in most cases.

Corollary: In non-exceptional cases, $\mathrm{Mod}^{\pm}(S) \cong \mathrm{Aut}(\mathcal{A}(S))$.

Strengthening

Theorem (S., 2019)
Every isomorphism

$$\mathcal{A}(S) \to \mathcal{A}(S')$$

is induced by a homeomorphism $S \to S'$,
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Strengthening

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Every isomorphism

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is induced by a homeomorphism $S \to S'$, unique up to isotopy in most cases.

Corollary: $\mathcal{A}(S) \cong \mathcal{A}(S')$ implies $S \cong S'$.

Main Theorem

Theorem (S., 2019)

There is a finite subcomplex $X \subseteq \mathcal{A}(S)$ such that any injection

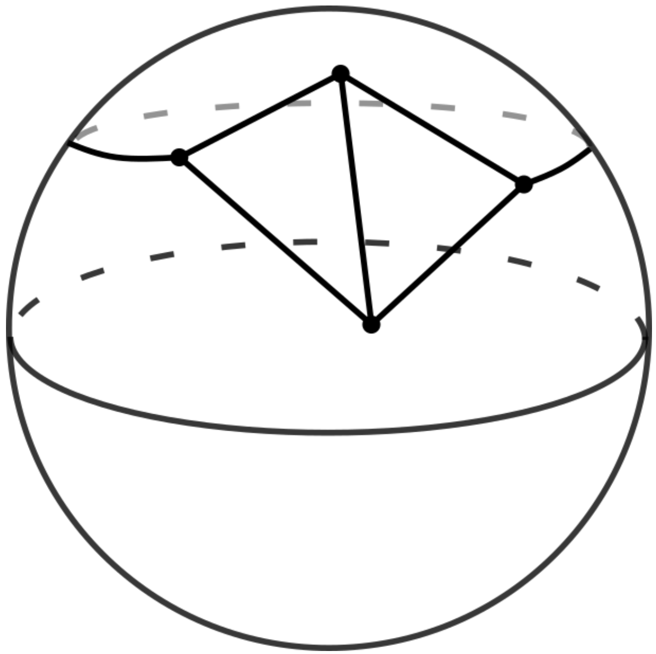
$$X \to \mathcal{A}(S')$$

is induced by a homeomorphism $S \to S'$, unique up to isotopy in most cases, provided $\dim\big(\mathcal{A}(S)\big) = \dim\big(\mathcal{A}(S')\big)$.*

*$S \neq S_{0,3}$

Proof Ideas

- Include a triangulation in X



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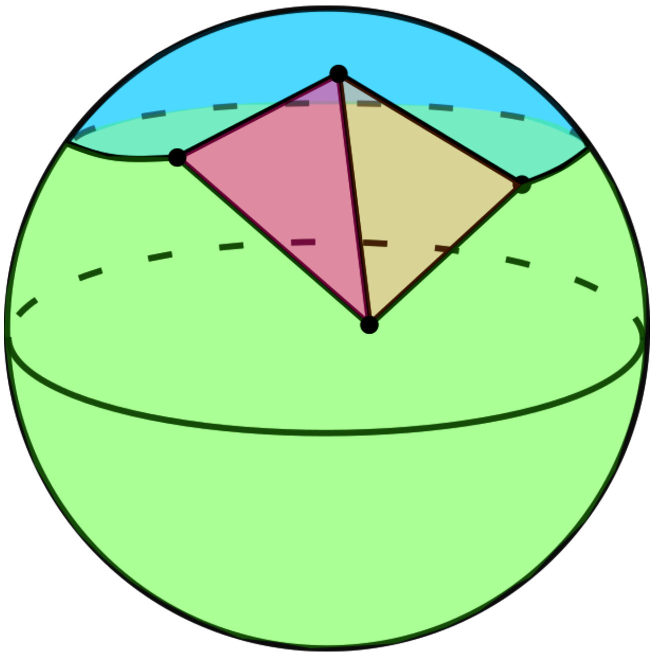

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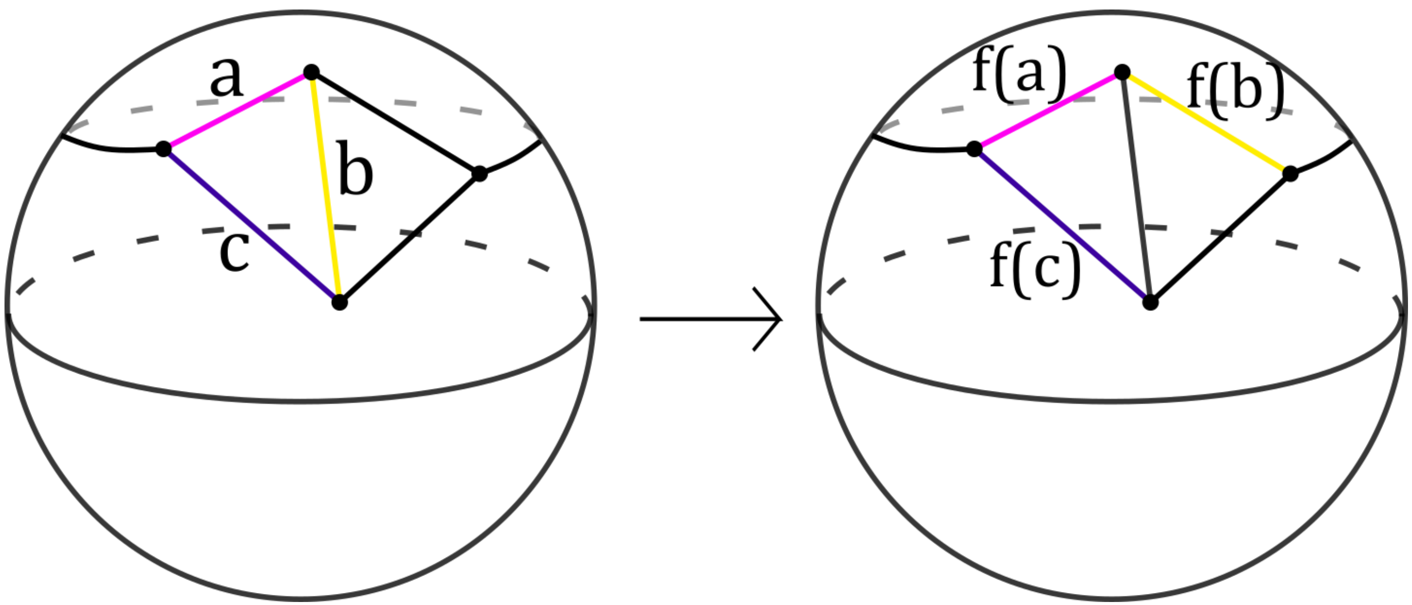
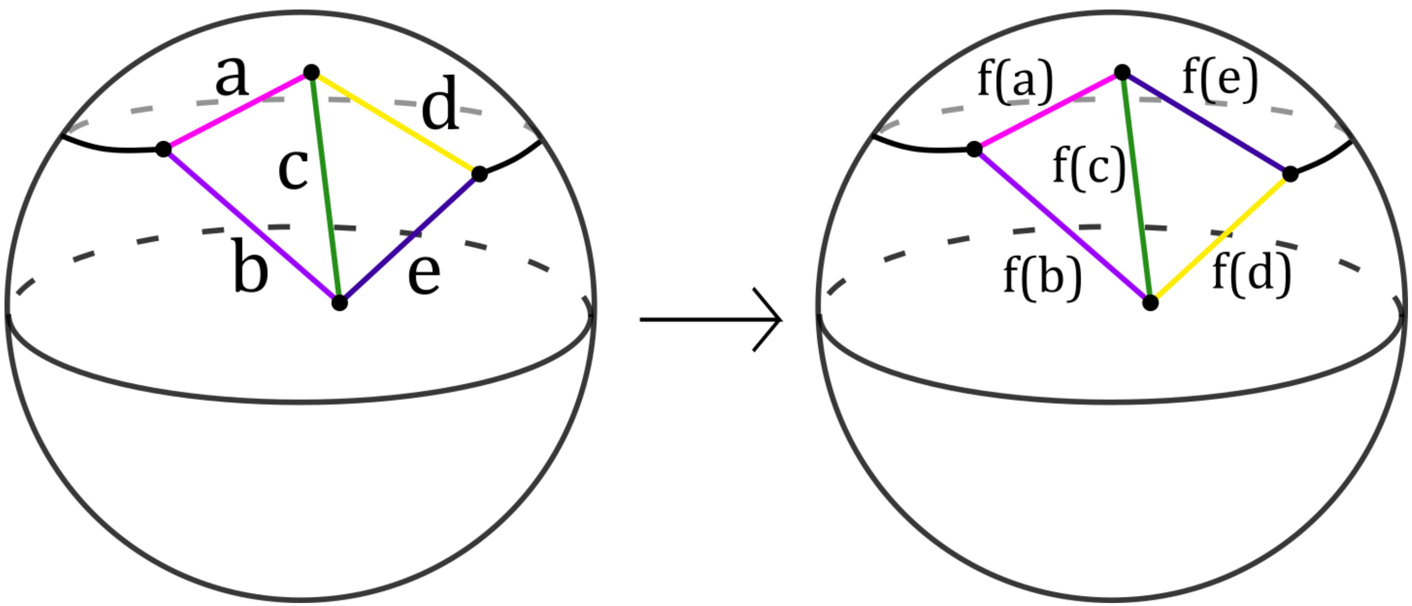
Avoid:



Proof Ideas

- Include a triangulation in X
- Include arcs to guarantee each triangle maps to a triangle
- Include arcs to guarantee orientations are preserved

Avoid:



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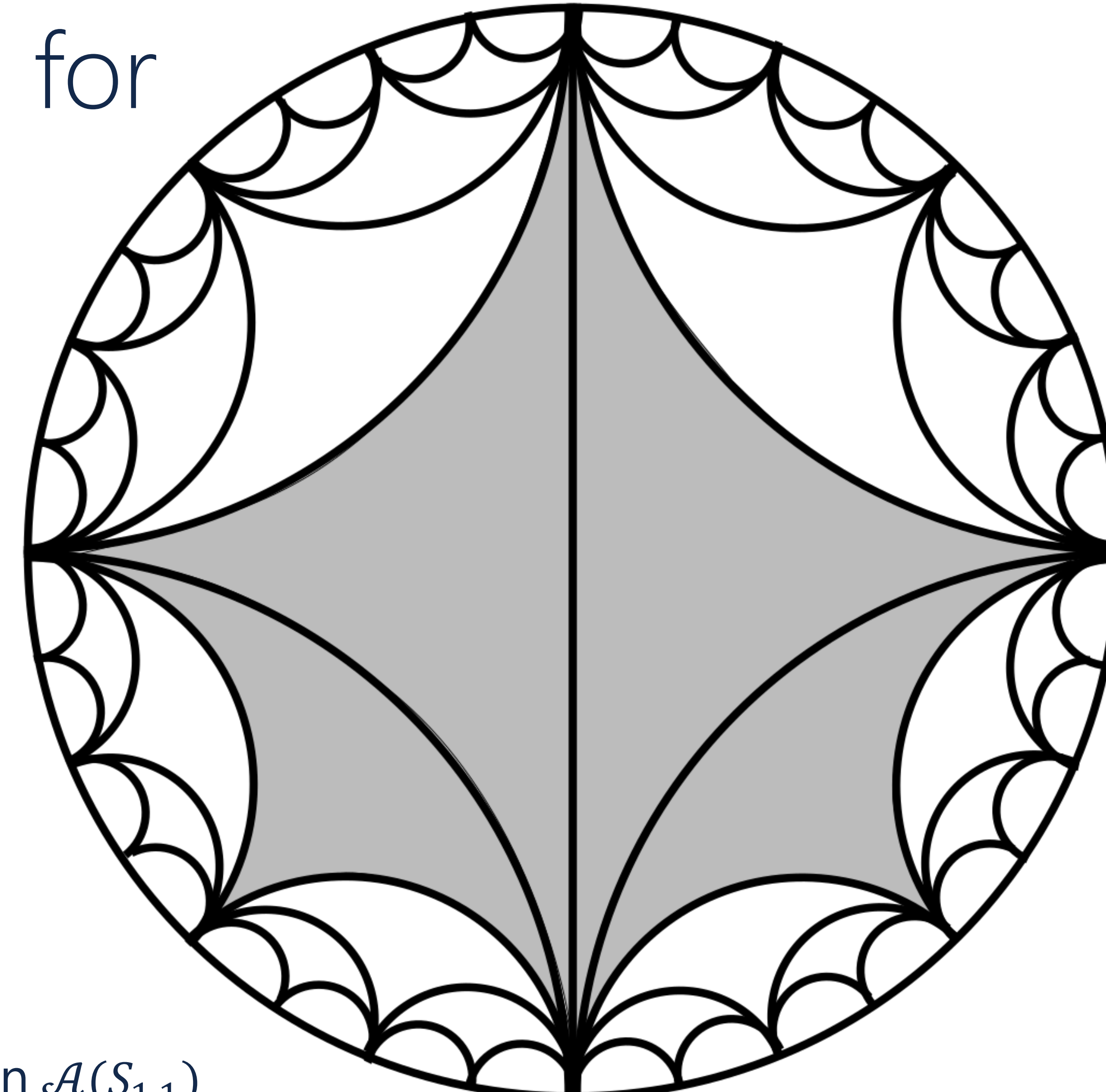

Finite rigid set in $\mathcal{A}(S_{1,1})$