Math 4431 - Fall 2009
Homework 4

Work all these problems and talk to me if you have any questions on them, but carefully write up and turn in **only** *problems 1, 7, 9 and 11.* **Due: November 17**

1) For each i let $X_i = \{0, 1\}$ be the two point set with the discrete topology. Show $\prod_{i=1}^{\infty} X_i$ is the Cantor set.

2) Let C be the standard Cantor set in the unit interval. Let S be a countably infinite subset of $\mathbb{R}$. Given any $\epsilon > 0$ show there is a constant a such that $|a| < \epsilon$ and $h(C) \cap S = \emptyset$ where $h(x) = x + a$ is the translation of $\mathbb{R}$ by a.

3) Let X be a countable, compact, metric space. Then X has an isolated point.

4) Let X be an uncountable, compact, metric space. Show X contains a Cantor set.

5) Let C be a Cantor set in $\mathbb{R}^n$. Show there is a continuous map $f : [0, 1] \to \mathbb{R}^n$ such that $C \subset f([0, 1])$.

6) The product of countably many Cantor sets is homeomorphic to the Cantor set.

7) Let p and q be two points in the Cantor set C. Show there is a homeomorphism $h : C \to C$ such that $h(p) = q$.

8) Prove a connected surface is arc-wise connected. That is, given any two points p and q in a connected surface Σ there is an embedding $f : [0, 1] \to \Sigma$ such that $f(0) = p$ and $f(1) = q$. (An embedding of $[0, 1]$ into a space is called an *arc*.)
Hint: First observe that if x and y can be connected by an arc and y and z can be connected by an arc then so can x and z.

9) Let D be a disk and I be an interval in ∂D. If Σ is a surface and $f : I \to \partial\Sigma$ is an embedding, then show the surface

$$\Sigma \cup_f D$$

is homeomorphic to Σ.
Hint: It might be good to try to show that the space obtained from two disks by gluing them along intervals in their boundary is homeomorphic to a disk. You may assume, as discussed in class, that given any connected component B of $\partial\Sigma$ there is an open set U in Σ that contains B and is homeomorphic to $S^1 \times [0, 1)$.

10) Show that each point in a surface Σ is contained in an open set U that is homeomorphic to $\mathbb{R}^2$.

11) Show that for any connected surface Σ and points p and q in Σ there is a homeomorphism $h : \Sigma \to \Sigma$ such that $h(p) = q$.
Hint: Recall Σ is also arc connected. What if p and q are in an open set homeomorphic to $\mathbb{R}^2$?