Math 500 - Fall 2001
Homework 2

1) Chapter 2 section 17 problem 12.

2) Chapter 4 section 31 problem 5.

3) Let $(X, \mathcal{T})$ be a topological space, U an open set and A an arbitrary set.

1. Show $\overline{A}$ is the intersection of all closed sets containing A.
2. Show that if A is closed then $U \setminus A$ is open.

4) Show $\mathbb{R}^n$ with its standard topology is Hausdorff.

5) Let X be any set with the finite complement topology. When is this space Hausdorff? (Hint: this depends on the cardinality of X.)

6) A set S in a topological space $(X, \mathcal{T})$ is called **dense** if $\overline{S} = X$.

1. Let X be any infinite set with the finite complement topology. Let S be an infinite subset of X. Show that S is dense in X.
2. Let $f, g : X \to Y$ be two continuous functions from X to Y. If Y is a Hausdorff space and $f(x) = g(x)$ on some dense set in X then show that $f = g$ on all of X.

7) Let $Y = \{0, 1\}$ with the discrete topology. Let X be any topological space. Show the following are equivalent:

1. X has the discrete topology,
2. every function $f : X \to Y$ is continuous,
3. every function $f : X \to Z$, where Z is any topological space, is continuous.