

Math 500 - Fall 2001
Homework 5

- 1) Show that if U is an open connected subset of \mathbb{R}^2 , then it is path connected. Hint: Fix an $x_0 \in U$ and show that the set of points in U that can be joined to x_0 by a path is both open and closed in U .
- 2) Show \mathbb{R} and \mathbb{R}^2 are not homeomorphic.
- 3) Show that a closed subspace of a normal space is normal.
- 4) Show that S^1 and $[0, 1]$ are not homeomorphic
- 5) A space X is called *completely normal* if every subspace of X is normal. Prove that X is completely normal if and only if whenever A and B are subsets of X with $A \cap \overline{B} = \overline{A} \cap B = \emptyset$ then there are disjoint open sets U and V such that $A \subset U$ and $B \subset V$.