

Math 500 - Fall 2001 Homework 6

- 1) Show that a connected normal space having more than one point is uncountable. (Hint: use Urysohn's Lemma)
- 2) Show that the any subspace of a separable metric space is separable.
- 3) The Tietze extension theorem is

Theorem 1 *Let X be a normal space and A a closed subspace of X . Then any continuous function $f : A \rightarrow [0, 1]$ can be extended to a continuous function $f : X \rightarrow [0, 1]$.*

Show that Urysohn's Lemma follows from the Tietze extension theorem. (Remark: It is also true but harder that the Tietze extension theorem follows from Urysohn's Lemma. See the book for a proof.)

- 4) Show the continuous image of a separable space is separable.
- 5) Show that a compact Hausdorff space is normal. (You can use the fact we proved in class that a compact Hausdorff space is regular. I stated that such a space was also normal but did not prove it.)