

Math 500 - Fall 2001
Homework 7

1) Is there a continuous surjective map $f : [0, 1] \rightarrow \mathbb{R}^2$? Why or why not.

2) Show there is a continuous surjective map $f : \mathbb{R} \rightarrow \mathbb{R}^2$.

3) If X is a Peano space of more than one point and Y is any Peano space, then show there is a continuous surjective map $f : X \rightarrow Y$.

HINT: Think about problem 1) on Homework # 6.

4) Let (X, d) be a metric space. A function $f : X \rightarrow X$ is *contracting* if there is some constant $\alpha < 1$ such that $d(f(x), f(y)) \leq \alpha d(x, y)$, for all $x, y \in X$. Prove that if X is complete and f is contracting then f is continuous and f has exactly one fixed point. (A fixed point of f is a point $x \in X$ such that $f(x) = x$.)

HINT: Consider the sequence x_n is any point in X and $x_n = f(x_{n-1})$.