Math 618 - Fall 2004
Homework 1

1. Prove the mapping cylinder of $f : X \to Y$ is homotopy equivalent to Y.

2. Show X_1 and X_2 are homotopy equivalent if and only if for every space Y there is a one-to-one correspondence $\psi_Y : [X_1, Y] \to [X_2, Y]$ such that for every continuous map $h : Y \to Y'$ we have
$$h_* \circ \psi_Y = \psi_{Y'} \circ h_*.$$

3. Let X, Y and Z be Hausdorff topological spaces. Suppose that X and Z are locally compact. Then show $C(X \times Z, Y)$ and $C(Z, C(X, Y))$ are homeomorphic spaces. (Recall, $C(X, Y)$ refers to the compact open topology on the space of maps.) HINT: It is probably easiest to use sub-bases for the compact open topology.

4. Show that $\pi_n(X \times Y, (x_0, y_0)) = \pi_n(X, x_0) \oplus \pi_n(Y, y_0)$.

5. Let X be a topological space and A a closed subspace. Show that $\pi_1(A)$ acts on $\pi_n(A), \pi_n(X)$ and $\pi_n(X, A)$ and that the maps in the long exact sequence of homotopy groups for a pair are equivariant with respect to these actions.

6. Let $\pi : \widetilde{X} \to X$ be a covering space. Show $\pi_* \colon \pi_n(\widetilde{X}) \to \pi_n(X)$ is an isomorphism for all $n \geq 2$.

7. Compute $\pi_n(S^1)$ for all n.

8. Compute $\pi_n(\Sigma)$ for all n, where Σ is an oriented surface of genus 2.