Math 618 - Fall 2004
Homework 2

1. Suppose $h : D^{n+1} \to X$ is a continuous map. Let D_1^n be the upper hemisphere of ∂D^{n+1} and D_2^n the lower hemisphere. Let $f_i : D^n \to X$ be the maps defined by restricting h to D_i^n. Show h induces a homotopy of f_1 to f_2 rel boundary.

2. Prove the product of two locally finite CW complexes is again a CW complex.

3. Show that if A is a sub complex of the CW complex X then X/A is a CW complex.

4. If $f : X \to Y$ is a weak-homotopy equivalence then $f_*[A, X] \to [A, Y]$ is a bijection for all CW-complexes A. You can assume X and Y are simply connected and A is a finite CW complex if you like. The statement is true without these extra assumptions, but they make the problem easier and still contain the main ideas.

5. Show if Y is an H-space then $\pi_1(Y)$ acts trivially on $[X, Y]_0$ for any X. (Need to assume (X, x_0) is an NDR-pair.)

6. Suppose (X, A) is a CW-pair and $\pi_n(X, A) = 0$ for all n. Show there is a homotopy $H : X \times [0, 1] \to X$ such that $H(x, 0) = x$, $H(x, 1) \subset A$ and $H(x, t) = x$ for all $x \in A$.

7. Let G be any finitely presented group and H an abelian group on which G acts. Show for any n there is a CW complex X and isomorphisms $\phi_1 : \pi_1(X) \to G$ and $\phi_n : \pi_n(X) \to H$ such that

$$\phi_1(\gamma) \cdot \phi_n(S) = \phi_n(\gamma \cdot S)$$

where $\gamma \in \pi_1(X)$ and $S \in \pi_n(X)$ and $\cdot$ denotes the group action of G on H and of π_1 on pi_n. If you want you can try to show that if H_k is an abelian group for $k = 1, \ldots$ on which G acts you can find a space X such that $\pi_1(X)$ acts on $\pi_n(X)$ as G acts on H_n.

8. Given CW pairs (X, A) and (Y, B) and a cellular map $f : (X, A) \to (Y, B)$ show that

$$f_* \circ h_n = h_n \circ f_*$$

where h_n is the Hurewicz map $h_n : \pi_n(X, A) \to H_n(X, A)$ or $h_n : \pi_n(Y, B) \to H_n(Y, B)$.

9. With notation as in the previous problem show

$$\partial \circ h_n = h_{n-1} \circ \partial$$

where $\partial : \pi_n(X, A) = \pi_{n-1}(A)$ on the left hand side of the equation and on the right $\partial : H_n(X, A) \to H_{n-1}(A)$.