Math 6441 - Spring 2021
Homework 6

Read Chapter 3.1 and 3.2 in Hatcher's book.
Work all these problems and talk to me if you have any questions on them. Write up and turn in all the problems. **Due: April 19.**

1. A cochain $\phi \in C^1(X; G)$ can be thought of as a function from paths in X to G. Show that if $\delta\phi = 0$ then
   (a) $\phi(\gamma * \eta) = \phi(\gamma) + \phi(\eta)$,
   (b) if γ is homotopic to η rel endpoints, then $\phi(\gamma) = \phi(\eta)$,
   (c) ϕ is a coboundary if and only if $\phi(f)$ only depends on the endpoints of f, for all f,
   (d) Show that the discussion above gives a homomorphism $H^1(X; G) \to Hom(\pi_1(X); G)$ Note: the universal coefficients theorem says that this map is an isomorphism if X is path connected.
2. Show that a map $f : S^n \to S^n$ has degree n if and only if $f^* : H^n(S^n; \mathbb{Z}) \to H^n(S^n; \mathbb{Z})$ is multiplication by n.
3. In Hatcher's book, the cohomology of $\mathbb{R}P^n$ is computed. Use this computation (you do not have to reproduce it) to show that there is no map $\mathbb{R}P^n \to \mathbb{R}P^m$ inducing a nontrivial map $H^1(\mathbb{R}P^m; \mathbb{Z}/2) \to H^1(\mathbb{R}P^n; \mathbb{Z}/2)$ if $n > m$.
4. Let M_g be a surface of genus g. Let X be the wedge product of g copies of the torus T^2. Problem 1 in Section 3.2 of Hatcher's book gives a quotient map $M_g \to X$. Compute the map induced on cohomology and determine the cup product structure on M_g in terms of the cup produce structure of T^2 (which we worked out in class).
5. Show that any map $S^4 \to S^2 \times S^2$ must induce the zero map on H^4. Show that this is not necessarily true for maps $S^2 \times S^2 \to S^4$.
6. For $S^n \times S^m$ compute all cap products (don't forget the case when $n = m$).