**Math 6441** **Midterm Exam** **Spring 2020**

Name: ______________________________

Signature: ____________________________

**Instructions:** Print your name and sign your signature to indicate that you accept the honor code.

*Instructions:* Complete three of the four problems below, and **circle** the numbers of the three problems you want graded in the box below – not circled problems will **not** be graded and if four numbers are circled, then only the first three will be graded. (Each problem is out of 10 points.)

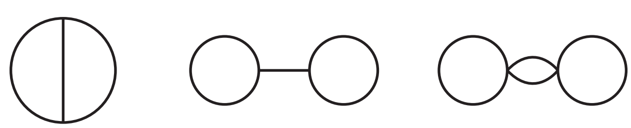
| problem | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| score | | | | |

Please note that a complete solution of a problem is preferable to partial progress on several problems.

**Note:** You can use result and computations from class, but you must make it clear what results you are using in your arguments.

*Good Luck*

1. Show the first two figures below (left to right) are homotopy equivalent, but they are not homotopy equivalent to the third.



2. If $X$ is a space that has $\mathbb{R}^k$ as a covering space, then show that any map $S^n \to X$, $n \geq 2$ is homotopic to a constant map.

3. Let $Y = D^2$, $Z = S^1 \times S^1$, and $Z$ the result of gluing $\partial Y = S^1$ to $\{pt\} \times S^1$ in $Z$. Use Van Kampen's theorem to compute the fundamental group of $X$.

4. Use covering spaces to show that the free group of rank 2 has a normal subgroup of index 3 and a non-normal subgroup of index 3. If the free group is generated by elements a and b then give explicit generators for the subgroups.

Problem number ____

Problem number ____

Problem number ____

Problem number ____