SMOOTH STRUCTURES ON COLLARABLE ENDS OF 4-MANIFOLDS

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Abstract. We use Furuta's result, usually referred to as "10/8-conjecture", to show that for any compact 3-manifold M the open manifold $M \times \mathbb{R}$ has infinitely many different smooth structures. Another consequence of Furuta's result is existence of infinitely many smooth structures on open topological 4-manifolds with a topologically collarable end, provided there are only finitely many ends homeomorphic to it. We also show that for each closed spin 4-manifold there are exotic $\mathbb{R}^4$'s that can not be smoothly embedded into it.

There is a well known (and still unsolved at the time of writing) conjecture in four dimensional topology known as the *11/8 conjecture*. Namely, if X is a closed spin topological four-manifold, then for an appropriately chosen basis for its second homology the intersection matrix has the form $(\oplus_k E_8) \oplus (\oplus_l H)$, where E_8 is the negative definite 8 by 8 intersection matrix of the "E_8 plumbing", and H is a 2 by 2 hyperbolic matrix,

$$H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

We consider the case when X is a *smooth* manifold. By Rochlin's theorem the number of copies of E_8, k, is even. Further, the 11/8 conjecture asserts that the number of hyperbolics, l, in the intersection form of X, satisfies the relation

$$l \geq \frac{3}{2}k.$$

The numbers in the "11/8" refer to the quotient of the Euler characteristic and the signature of the manifold in the in the case of the equality $l = \frac{3}{2}k$. The conjecture is known to be true in the case $k = 2$, see Theorem 2 of [D], where the minimal possible l is realized by the K3 surface. Recently Furuta has proven the "10/8" inequality, that is, for a given number of E_8's, k, the number of hyperbolics, l, has to satisfy the inequality $l > k + 1$, see [S].

In this work we use Furuta's result, or more precisely a weaker inequality $l \geq k$, to prove the existence of infinitely many different smooth structures on $M \times \mathbb{R}$, where M is an arbitrary compact three-manifold. Furthermore, we show that 4-manifolds with a topologically collarable end that satisfy the conditions of Theorem 5 have infinitely many smooth structures. Our construction of these smooth structures is based on Gompf's [G1] end-sums with exotic $\mathbb{R}^4$'s. The main building block in our constructions is a well known exotic $\mathbb{R}^4$, which we will denote by $\mathcal{R}_1$, and can be obtained from the K3 surface. Our smooth structures are obtained as end-sums of a given open manifold with different number of copies of $\mathcal{R}_1$ and, consequently, we are producing only countable families of smooth structures. The infinite end-sum of copies of $\mathcal{R}_1$ and their mirror images is reminiscent to the universal exotic $\mathbb{R}^4$ constructed by Freedman and Taylor [FT], although our construct is probably not "universal" itself, it has the same known non embedding properties of the universal $\mathbb{R}^4$.

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Construction and properties of $\mathcal{R}_n$. Let Y denote the standard K3 surface. Its intersection form introduces a decomposition of the second homology, $H_2(Y) \cong (\oplus_2 E_8) \oplus (\oplus_3 H)$. According to Casson [C] the six elements of $H_2(Y)$ that span the $\oplus_3 H$ summand can be represented by three wedges of two immersed 2-spheres and the extra intersections can be grouped into pairs consisting of one with a positive and one with a negative sign. For each of these $\pm$-pairs of intersections there is a loop with a framed neighborhood such that if a 2-handle is attached to it with the prescribed framing then there is an ambient isotopy inside the union of a neighborhood of the 2-spheres and the 2-handle that eliminates the given pair of intersections. Although, in general, such a 2-handle can not be embedded ambiently, Casson has shown that we can ambiently cap the loop by a Casson handle. We fix an open regular neighborhood of the immersed wedges of 2-spheres and denote its union with the capping Casson handles by N. Since any Casson handle can be embedded into the standard 2-handle, N smoothly embeds into $\sharp_3(S^2 \times S^2)$. From Freedman's celebrated result [F] it follows that each Casson handle has a core that is locally flat topologically embedded 2-disc. The complement of the wedges of 2-spheres and the (topological) cores of the Casson handles in $\sharp_3(S^2 \times S^2)$ is an exotic $\mathbb{R}^4$, that is, a manifold homeomorphic but not diffeomorphic to the standard $\mathbb{R}^4$. We denote this manifold by $\mathcal{R}_1$. We select an arbitrary 3-manifold inside N that separates the end of N from the wedges of spheres and topological core discs. It also separates $\sharp_3(S^2 \times S^2)$ and the closure of the component contained in $\mathcal{R}_1$ we denote by K_1. So, K_1 is a compact manifold with boundary contained in $\mathcal{R}_1$. Note that there are neighborhoods of ends of $\mathcal{R}_1$ and the interior of K_1 that are smoothly embedded into Y and that each has a separating topologically embedded locally flat 3-sphere.

For an integer $n > 1$ we define $\mathcal{R}_n$ to be end-sum of n copies of $\mathcal{R}_1$, $\mathcal{R}_n = \natural_n \mathcal{R}_1$. Each copy of $\mathcal{R}_1$ inside $\mathcal{R}_n$ contains a copy of K_1. We use the connecting neighborhoods of arcs from the end-sum to form a boundary connected sum of the copies of K_1, $K_n = \natural_n K_1$, so K_n is a compact manifold with boundary embedded into $\mathcal{R}_n$. Equivalently, we can construct $\mathcal{R}_n$ by working with a connected sum of n copies of the K3 surface: let Y_n denote connected sum of n copies of the K3 surface, $H_2(Y_n) \cong (\oplus_{2n} E_8) \oplus (\oplus_{3n} H)$. Each copy of the K3 surface in Y_n contains a copy of N. We assume that the 4-balls affected by the construction of the connected sums were disjoint from the copies of N, so we connect together the copies of N by open regular neighborhood of arcs, each passing through one of the $S^3 \times I$ tubes that connect the K3 surfaces in Y_n. We call the resulting open connected manifold N_n. As in the case of single K3 surface, we can smoothly transplant N_n into $\sharp_{3n}(S^2 \times S^2)$. It is easy to see that the complement of the wedges of 2-spheres and topological cores in $\sharp_{3n}(S^2 \times S^2)$ is diffeomorphic to $\mathcal{R}_n$. Again, as in the case of single K3 surface, there is a neighborhood A_n of the end of the interior of K_n that is contained in N_n and, therefore, in Y_n. We note that there is a topological locally flat embedding of the 3-sphere that separates A_n.

We also define $\mathcal{R}_\infty = \natural_\infty \mathcal{R}_1$ and $\mathcal{R}_* = \mathcal{R}_\infty \natural -(\mathcal{R}_\infty)$, where $-(\mathcal{R}_\infty)$ denotes the "mirror image" of $\mathcal{R}_\infty$, the same underlying manifold, but with opposite orientation.

Theorem 1. *If X is a closed smooth spin 4-manifold, then there is an integer $m > 0$ such that for any integer $n \geq m$ $\mathcal{R}_n$ and K_n can not be smoothly embedded into X or into any closed smooth spin 4-manifold with the same intersection form as X.*

Proof. As we have noted above, $H_2(X) \cong (\oplus_{2k} E_8) \oplus (\oplus_l H)$. Let m be any integer with the property $2m > l - 2k$. We claim that when $n \geq m$ $\mathcal{R}_n$ cannot be smoothly embedded into X. Let us assume that there is such an embedding. From the image of this embedding we remove the complement of A_n in K_n and denote the resulting open manifold by X_0. The end of X_0 has a neighborhood diffeomorphic to A_n. We have constructed above an embedding of A_n via N_n into Y_n, the connected sum of n copies of the K3 surface. We cut out from Y_n the component of $N_n - A_n$ that contains the wedges of immersed 2-spheres and denote the resulting manifold $Y_{n,0}$. We form a closed smooth manifold Z by identifying the ends of X_0 and $Y_{n,0}$. Since the left out piece contained the generators of the second homology summand, $H_2(Y_{n,0}) \cong \oplus_{2k} E_8$. Note that using the topological locally flat 3-sphere separating A_n it is easy to see that $H_2(Z) \cong H_2(X) \oplus H_2(Y_{n,0})$, so

$$H_2(Z) \cong (\oplus_{2k+2n} E_8) \oplus (\oplus_l H)$$

with $2k + 2n > l$ which is impossible by Furuta's result. $\square$

An obvious consequence of this theorem is

Corollary 2. *For $1 \leq m, n \leq \infty$ and $n \neq m$, $\mathcal{R}_n$ and $\mathcal{R}_m$ are not diffeomorphic. $\mathcal{R}_\infty$ and $\mathcal{R}_*$ can not be smoothly embedded into any closed smooth spin 4-manifold. $\mathcal{R}_n$, K_n, where $n > 0$, $\mathcal{R}_\infty$ and $\mathcal{R}_*$ can not be embedded into any negative definite smooth 4-manifold. Furthermore, $\mathcal{R}_n \natural -(\mathcal{R}_n)$ and $\mathcal{R}_*$ do not embed into any definite closed smooth 4-manifold.*

Note that since the universal $\mathbb{R}^4$ from [FT] contains any exotic $\mathbb{R}^4$, it itself can not be embedded into a closed smooth spin 4-manifold.

Proof. From the construction it follows that $\mathcal{R}_n$ can be embedded into $\sharp_{3n}(S^2 \times S^2)$, but not into $\sharp_{2n}(S^2 \times S^2)$; otherwise, as in the proof of Theorem 1 we could construct a closed spin 4-manifold that would contradict Furuta's result. We take $\mathcal{R}_\infty$ to be

$\mathcal{R}_1$. Let us assume that we have chosen nondiffeomorphic $\mathcal{R}_{n_i}$, for $i \leq k$. We choose an integer $n_{k+1} > \frac{3}{2}n_k$. Then $\mathcal{R}_{n_{k+1}}$ does not smoothly embed into $\sharp_{3n_k}(S^2 \times S^2)$ and so it is not diffeomorphic to any $\mathcal{R}_{n_i}$, for $i \leq k$. Therefore, there are infinitely many smoothly distinct $\mathcal{R}_n$'s. Moreover, no two of $\mathcal{R}_n$'s are diffeomorphic, since each $\mathcal{R}_n$ is end sum of n copies of $\mathcal{R}_1$, the equation $\mathcal{R}_n = \mathcal{R}_m,\ n < m$, would imply that for any $k > m$, $\mathcal{R}_k = \mathcal{R}_l$ for some $n \leq l \leq m$. So it would follow that there are only finitely many distinct $\mathcal{R}_n$'s.

Since $\mathcal{R}_\infty$ and $\mathcal{R}_*$ contain every $\mathcal{R}_n$ it follows from Theorem 1 that $\mathcal{R}_\infty$ does not embed into any spin closed smooth 4-manifold. Thus it follows that $\mathcal{R}_\infty$ can not be diffeomorphic to any $\mathcal{R}_n$, for $1 \leq n < \infty$.

If $\mathcal{R}_n$ or K_n could be embedded into a negative definite smooth 4-manifold, we could excise all hyperbolics from the connected sum of n K3 surfaces and reglue the complement of the embedded $\mathcal{R}_n$ inside a negative definite manifold. The resulting closed 4-manifold would be a smooth negative definite 4-manifold whose intersection form would contain E_8 summands. It is well known algebraic result that such forms can not be diagonalized over the ring of integers so the existence of such a manifold is impossible by Donaldson's theorem (see Theorem 1 in [D], to remove the restriction on the fundamental group one needs to use Seiberg-Witten theory). Therefore, $\mathcal{R}_n$, for $n > 0$, and so $\mathcal{R}_\infty$ and $\mathcal{R}_*$ can not be embedded into any negative definite simply-connected smooth 4-manifold.

Finally, since $-\mathcal{R}_n$ does not embed into a positive definite smooth 4-manifold, $\mathcal{R}_n \natural - (\mathcal{R}_n)$ and $\mathcal{R}_*$ do not embed into any definite closed smooth 4-manifold. $\square$

Remark. Following an argument of Freedman, see [G1], we may find in each $\mathcal{R}_n$ uncountable many nondiffeomorphic exotic $\mathbb{R}^4$'s sharing the same nonembedding property as $\mathcal{R}_n$. Namely, in $\mathcal{R}_n$ there is a neighborhood of the end that is homeomorphic to $S^3 \times (0, \infty)$ and disjoint from K_n. For each $0 < t < \infty$ let $\mathcal{R}_{n,t}$ denotes the complement of $S^3 \times [t, \infty)$. It follows from the work of Taubes [T] on periodic ends that no two of $\mathcal{R}_{n,t}$, $0 < t < \infty$, are diffeomorphic.

$\mathcal{R}_*$ shares another non embedding property with the universal $\mathbb{R}^4$ from [FT]: it can not be embedded into the interior of a smooth 4-cell.

Proposition 3. *If $e : B^4 \hookrightarrow X$ is a locally flat topological embedding of the 4-ball into a smooth 4-manifold X, then $\mathcal{R}_*$ can not be smoothly embedded into the interior of $e(B^4)$.*

Proof. We follow Freedman and Taylor's proof of the same property for the universal exotic $\mathbb{R}^4$. The first step is to embed X into $\mathbb{C}P^2 \sharp_n (S^2 \times S^2)$, for some n large enough. So if X had a boundary we double it, then surger homotopicaly non trivial loops away from $e(B^4)$ and form the connected sum with sufficient numbers of copies of $\mathbb{C}P^2$ and $\overline{\mathbb{C}P}^2$ to obtain a closed smooth simply connected manifold with the required intersection form. By adding copies of $S^2 \times S^2$ the manifold becomes diffeomorphic to $\mathbb{C}P^2 \sharp_n (S^2 \times S^2)$. The second homology class of a complex projective line inside $\mathbb{C}P^2 \subset \mathbb{C}P^2 \sharp_n (S^2 \times S^2)$ can be represented by an embedded surface in the complement of $e(B^4)$. By adding extra copies of $S^2 \times S^2$ if necessary, we can find in the complement of $e(B^4)$ a smooth 2-sphere representing the same homology class. Blowing down this sphere produces an embedding of $e(B^4)$ into

$\sharp_n(S^2 \times S^2)$. It follows from Corollary 2 that there can not be a smooth embedding of $\mathcal{R}_*$ into $e(B^4)$. $\square$

Smooth structures on $M^3 \times \mathbb{R}$. Gompf has used his end-sum construction to produce uncountably many smooth structures on $S^3 \times \mathbb{R}$. Recently F. Ding [Di] has extended this result to produce an uncountable family of smooth structures $M^3 \times \mathbb{R}$, when M^3 is a closed connected orientable 3-manifold that admits a smooth embedding into $\sharp_n \overline{\mathbb{C}P}^2$, the connected sum of complex projective planes with negative orientation. Ding's argument can be used to extend his result to the cases where $\sharp_n \overline{\mathbb{C}P}^2$ is replaced by any definite closed smooth 4-manifold and also to include the 3-manifolds with boundary whose doubles have the required embedding property. Although the authors do not know of any classification of 3-manifolds with respect to existence of the embeddings into definite 4-manifolds, there are some well known examples of 3-manifolds, most notably the Poincare sphere, that do not smoothly embed into a definite closed 4-manifold.

Theorem 4. *Let M^3 be a compact 3-manifold. Then $M \times \mathbb{R}$ has infinitely many different smooth structures.*

Proof. We prove the theorem first for the case when M is closed and orientable. It is well known that any orientable closed compact 3-manifold embeds into $\sharp_k(S^2 \times S^2)$, for a k large enough, see [K]. Briefly, such a manifold bounds a 4-dimensional handlebody with a 0- and 2-handles and such that all attaching framings are even. It is an easy exercise in Kirby's link calculus to show that the double of this handlebody is diffeomorphic to $\sharp_k(S^2 \times S^2)$ (where k is the number of the 2-handles in the handlebody). So, we fix such k for the given 3-manifold M. Obviously, a bicollared neighborhood of M gives an embedding of $M \times \mathbb{R}$ into $\sharp_k(S^2 \times S^2)$. From Theorem 1 it follows that there is an integer n_1, such that $\mathcal{R}_{n_1}$ can not be smoothly embedded into $\sharp_k(S^2 \times S^2)$. We form an end-sum of $M \times \mathbb{R}$ with $\mathcal{R}_{n_1}$. Clearly $(M \times \mathbb{R}) \natural \mathcal{R}_{n_1}$ can not be diffeomorphic with $M \times \mathbb{R}$ since the former can not be smoothly embedded into $\sharp_k(S^2 \times S^2)$. However, by its construction, $\mathcal{R}_{n_1}$ smoothly embeds into $\sharp_{n_1}(S^2 \times S^2)$. Let $k_2 = k + n_1$. We fix an embedding of $M \times \mathbb{R}$ into the first k summands of $\sharp_{k_2}(S^2 \times S^2)$ and an embedding of $\mathcal{R}_{n_1}$ into the remaining n_1 copies. We may assume that the both embedding miss the connecting tube between the first k and the last n_1 copies of $S^2 \times S^2$. Now, we can perform the end-sum of $M \times \mathbb{R}$ and $\mathcal{R}_{n_1}$ ambiently inside $\sharp_{k_2}(S^2 \times S^2)$. The next step is to select an integer n_2 large enough so that $\mathcal{R}_{n_2}$ does not embed into $\sharp_{k_2}(S^2 \times S^2)$. The new smooth structure, $(M \times \mathbb{R}) \natural \mathcal{R}_{n_1} \natural \mathcal{R}_{n_2} = (M \times \mathbb{R}) \natural \mathcal{R}_{(n_1+n_2)}$ is different from the previous two and it embeds into $\sharp_{k_3}(S^2 \times S^2)$, where $k_3 = k_2 + n_2$. We can obviously iterate the construction to obtain a sequence of smooth structures, $(M \times \mathbb{R}) \natural \mathcal{R}_{n_i}$.

If M is an orientable compact 3-manifold with the boundary we work with its double, DM. The double of M, and so M itself, can be embedded into $\sharp_k(S^2 \times S^2)$ and the same construction produces a sequence of smooth structures on the double. We perform each end-sum on the same copy of $M \times \mathbb{R}$ in the $DM \times \mathbb{R}$ and so we obtain a sequence of smooth structures $(M \times \mathbb{R}) \natural \mathcal{R}_{n_i}$ that are all standard near $\partial M \times \mathbb{R}$. Since each of these smooth manifolds requires a different minimal number "n" to embed into $\sharp_n(S^2 \times S^2)$, they are all different.

Finally, if M is a non orientable compact 3-manifold it has a two-fold cover N that is an orientable 3-manifold. N smoothly embeds into $\sharp_k(S^2 \times S^2)$ for some k. We choose n_1 as before, so that $\mathcal{R}_{n_1}$ does not embed into $\sharp_k(S^2 \times S^2)$. We form the end-sum $(M \times \mathbb{R})\natural\mathcal{R}_{n_1}$ and cover it by $(N \times \mathbb{R})\natural_2\mathcal{R}_{n_1}$. By the construction $(N \times \mathbb{R})\natural_2\mathcal{R}_{n_1}$ does not embed into $\sharp_k(S^2 \times S^2)$ so it is not diffeomorphic to $N \times \mathbb{R}$. Since a diffeomorphism between $(M \times \mathbb{R})\natural\mathcal{R}_{n_1}$ and $M \times \mathbb{R}$ would lift to their homotopically equivalent 2-fold covers it can not exist. As in the previous cases we obtain a sequence of different smooth structures on $M \times \mathbb{R}$. $\square$

Remark. All the smooth structures on $M \times \mathbb{R}$ constructed above have one of the two ends standard. Obviously, by end-summing $M \times \mathbb{R}$ with $\mathcal{R}_n$'s on the both sides one can obtain doubly indexed countable family of smooth structures.

Smooth structures on manifolds with collarable ends. If an 4-manifold has countably many ends, each having a neighborhood of the infinity that is *topologically* collarable as $S^3 \times \mathbb{R}$, then Gompf has shown that the manifold possesses uncountably many different smooth structures [G2]. Ding [Di] has extended this results to include manifolds whose ends are topologically collared by $M^3 \times \mathbb{R}$ where M^3 embeds into $\sharp_n \overline{\mathbb{C}P}^2$.

Theorem 5. *Let X be an open topological four manifold with at least one topologically collarable end, i.e. an end homeomorphic to $M \times \mathbb{R}$, for some closed 3-manifold M. If X has more than one end we assume that for one of the topologically collarable ends there are only finitely many other ends that have this topological type. Then, X has infinitely many different smooth structures.*

Simple examples of manifolds to which the theorem applies are the interiors of compact 4-manifolds with boundaries.

Proof. Let X be a 4-manifold with a single end. Since any open 4-manifold is smoothable we assume that X has a smooth structure. Next we form $X\natural\mathcal{R}_\infty$. We fix a homeomorphism h between $M \times \mathbb{R}$ and a neighborhood of the end of $X\natural\mathcal{R}_\infty$ such that the nth copy of K_1 is embedded into $M \times (n, n+1)$. For any $t \in \mathbb{R},\ t \geq 1$, we define X_t as $(X\natural\mathcal{R}_\infty) - h(M \times [t, +\infty))$.

We claim that if $t < s$ and $M \times (t, s)$ contains a copy of K_1 then X_t and X_s cannot be diffeomorphic. From this it follows that X has infinitely many smooth structures. To prove this claim we assume that there are two positive real numbers, $t,\ s$, satisfying the properties above, but such that X_t is diffeomorphic to X_s. Then there is a neighborhood of the end of X_s, denoted by V that does not intersect the copy of K_1 in $M \times (t, s)$. The diffeomorphism between X_s and X_t identifies another copy of V as a neighborhood of the end of X_t. We glue to the end of X_s infinitely many copies of the region between the two copies of V in X_s. Using this smooth structure we find separating smoothly embeddings of 3-manifolds: $M_1 \hookrightarrow M \times (-1, 0)$ and $M_2 \hookrightarrow V \subset X_t$. For $i = 1, 2$, M_i bounds a smooth simply connected spin 4-manifold Z_i. We glue together Z_1, Z_2 and the piece of $M \times (-1, t)$ bounded by $M_1 \coprod M_2$. The resulting closed smooth manifold Z is spin (but not necessarily simply connected) and by adding copies of the K3 surface with appropriate orientation we may assume that Z has signature zero. Therefore

Z has the same intersection form as a connected sum of hyperbolics. Next we find n large enough so that K_n does not smoothly embed into a closed 4-manifold with the same intersection form as Z. Note that we can embed K_n in our newly constructed periodic end by ambiently connecting together n copies of K_1. Also, by the construction there is a copy V passed the embedded K_n. Now we can use M_2 in this copy of V to construct a smooth manifold homotopically equivalent to Z and containing K_n, which by our choice of n is impossible.

In the case that X has more than one end we have assumed that at least one of them is homeomorphic to $M \times \mathbb{R}$, for some 3-manifold M and furthermore we have assumed that there are finitely many ends of X homeomorphic to $M \times \mathbb{R}$. We repeat the construction from above on this end and obtain a sequence of smooth structures X_n. Since there can be only finitely many diffeomorphisms that permute the ends homeomorphic to $M \times \mathbb{R}$ the sequence X_n contains infinitely many different smooth structures on X. $\square$

Final comments and questions. The method used in the proof of Theorem 5 could also be used to prove Theorem 4 and part of Corollary 2. However, the given proofs are more natural and instructive. Moreover, the proof of Theorem 4 can be used to show that $M \times \mathbb{R}$ has infinitely many smooth structures if M is an open 3-manifold with all ends collarable. Namely, one can trim all the ends to obtain a compact 3-manifold with boundary. Obviously Theorem 4 is true if M is a 3-manifold such that $M \times \mathbb{R}$ is homeomorphic to $N \times \mathbb{R}$, where N is a 3-manifold with collarable ends. An example of such an M is the Whitehead manifold whose end is not collarable, but whose product with the real line is $\mathbb{R}^3 \times \mathbb{R}$. This leads to the question whether for any 3-manifold M with finitely generated homology there is a 3-manifold N as above.

All our results produce only countable families of smooth structures. However, by shaving the end of any member of these families we can produce an uncountable family of smooth structures. Either there are uncountably many different smooth types between them or the end of the original smooth structure is periodic. Although our methods fail to distinguish between them we conjecture that the former is the case.

References

[C] A. Casson, *Three lectures on new infinite construction in 4-dimensional manifolds (notes prepared by L. Guillou)*, A la Recherche de la Topologie Perdue (L. Guillou and A. Marin, eds.), Progress in Mathematics, vol. 62, Birkhäuser, 1986, pp. 201–244.

[D] S. Donaldson, *The orientation of Yang-Mills moduli spaces and 4-manifolds topology*, J. Diff. Geom. **26** (1987), 397–428.

[Di] F. Ding, *Smooth structures on some open 4-manifolds*, Topology **36** (1997), 203–207.

[F] M. Freedman, *The topology of 4-dimensional manifolds*, J. Diff. Geom. **17** (1982), 357–453.

[FT] M. Freedman and L. Taylor, *A universal smoothing of four-space*, J. Diff. Geom. **24** (1986), 69–78.

[G1] R. Gompf, *An infinite set of exotic R^4's*, J. Diff. Geom. **21** (1985), 283–300.

[G2] R. Gompf, *An exotic menagerie*, J. Diff. Geom. **37** (1993), 199–223.

[K] R. Kirby, *The topology of 4-manifolds*, Lecture Notes in Math. 1374, Springer-Verlag, 1989.

[S] D. Salamon, *Spin geometry and Seiberg-Witten invariants*, preprint, 1996.

[T] C. Taubes, *Gauge theory on asymptotically periodic 4-manifolds*, J. Diff. Geom. **25** (1987), 363–430.

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